

Robust Predictive Control Based on LMI Optimization for Active Suspension System

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Abstract. The vehicle will inevitably be vibrated in the process of running. The suspension is an important device to ensure the comfort and safety of the vehicle. In order to study the control method of the suspension, we simulate a quarter of the active suspension device which uses longitudinal acceleration as a measurement index. we compare the control effect of traditional PID control with robust predictive control. Through simulation, we find that the traditional PID control can realize the stability control of the active suspension without constraints. However, on the one hand, it brings serious jitter, on the other hand, it is not suitable for the actual constrained situation. On this basis, we consider the robust predictive control with constraints. By means of simulation, we obtain that the robust predictive control can produce less jitter under the premise of achieving the stable control of the constrained system, which is more suitable for practical engineering requirements.

1. Introduction

Vehicles will cause vibrations when it is running, which is not conducive to safety and comfort. In order to improve the stability and comfort of the vehicle, vehicles usually use a suspension system to suppress the vibration. The traditional vehicle suspension system consists of springs and a damping system. It does not require external control, but only dissipates or temporarily stores energy. It is also called a passive suspension system [1]. This system has poor adaptability, and it limits the improvement of vehicle vibration damping performance. So, it is a deadlock for traditional passive suspension system because of its fixed characteristics of springs and dampers. Good tune and design of a passive suspension can, to some extends, optimize and trade off riding comfort quality and stability, but cannot eliminate these conflicts completely [2]. The proposal of the active suspension provides a new method for vehicle vibration damping. In the 20th century, the idea of active suspension was put forward. Later, it was widely used in trains and automobile vehicles [3-7].

In recent years, controllable suspension systems have attracted considerable attentions by researchers and automobile industries, because it can overcome some limitations of traditional passive suspension systems. The controllable suspension system is always divided into active and semi-active system. Active suspension systems have greater advantages than passive suspension systems. Mainly reflected in that the vehicle system can be adjusted with changes in speed, road surface by self-regulation. The active suspension system is more suitable to use the control algorithm to achieve stability and simulation verification than the semi-active suspension system. Therefore, we mainly discuss the control design of the active suspension system here.

PID control methods are widely used in industry [8]. As the earliest practical algorithm, it has been



widely applied. The modernization control designs PID as a module, this module not only can greatly save the time, but also can facilitate the adjustment. The PID controller consists of a proportional unit (P), an integral unit (I), and a derivative unit (D), which can meet the control requirements of most linear systems, but PID is not suitable for some lag systems and high-order systems. Here we use PID control algorithm to design active suspension control, aiming to test the active suspension performance under the control of PID algorithm.

Linear H_∞ control theory is a very important method to solve the robust control of linear systems. When the traditional constraint H_∞ control method is used to attain controllers that satisfy both time-domain hard constraints and robust performance [9], the upper bound of the external disturbance energy must be assumed a priori, so it is not possible to coordinate constraints and satisfy system performance. Considering this question, we use LMI-based state feedback control strategy to ensure the efficiency of the algorithm. By ensuring the system's H_∞ performance under the conditions of system dissipation, the rolling time-domain closed-loop system can coordinate control performance and system constraints real-time.

2. Active suspension system modelling

This design considers a two-degree-of-freedom 1/4 car model. First, based on the assumption that the mass distribution coefficient of the car body design tends to be between 0.8 and 1.2, it can be considered that the body vibrations are independent from each other before and after the vibration, and the complete decoupling of the front and rear wheels can be achieved. With a symmetric excitation input, we can build a two-degree-of-freedom 1/4 suspension model. The physical model shown in Figure 1 was constructed from the load changes of the active suspension and the physical structural features of the suspension system. The model has the following features:

- Under the premise of correctness, the description parameters of the system are simplified.
- It is convenient for us to model.
- The model can basically represent the physical structure of the active suspension.

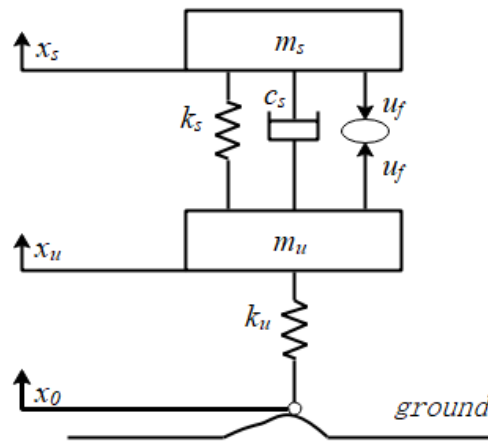


Figure 1. Two-degree-of-freedom 1/4 car model

Where m_s and m_u represent the sprung and unsprung mass. $x_s - x_u$ is the suspension travel stroke, $x_u - x_0$ is the tire movement displacement, x_0 is the road surface longitudinal displacement caused by the road surface roughness; u_f is the main power, provided by the vehicle's internal hydraulic transmission device, and can be regarded as the control input.

According to Newton's second law, we can establish the dynamic equation:

$$m_s \ddot{x}_s + c_s (\dot{x}_s - \dot{x}_u) + k_s (x_s - x_u) - u_f = 0 \quad (1)$$

$$m_u \ddot{x}_u - c_s (\dot{x}_s - \dot{x}_u) - k_s (x_s - x_u) + k_u (x_u - x_0) + u_f = 0 \quad (2)$$

Simplify the formulae of (1) and (2) to obtain the following formula:

$$\ddot{x}_s = \frac{1}{m_s} [-c_s (\dot{x}_s - \dot{x}_u) - k_s (x_s - x_u) + u_f] \quad (3)$$

$$\ddot{x}_u = \frac{1}{m_u} [c_s (\dot{x}_s - \dot{x}_u) + k_s (x_s - x_u) - k_u (x_u - x_0) - u_f] \quad (4)$$

In the above equation, when $u_f = 0$, the active suspension model is actually called the passive suspension model [10]. At the same time, the control input is also limited:

$$|u_f(t)| \leq u_{\max}, \quad \forall t \geq 0. \quad (5)$$

Combining (3) and (4) with the constraint problem, the state space model of the active suspension can be constructed. In addition, we define the state variables: $x = (x_s - x_u, \dot{x}_s, x_u - x_0, \dot{x}_u)$. If the ground unevenness causes the vertical velocity to be regarded as interference, the state space model can be described as:

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 & 0 & -1 \\ -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & 0 & \frac{c_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & \frac{c_s}{m_u} & -\frac{k_u}{m_u} & -\frac{c_s}{m_u} \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \end{pmatrix} \omega(t) + \begin{pmatrix} 0 \\ \frac{u_{\max}}{m_s} \\ 0 \\ -\frac{u_{\max}}{m_u} \end{pmatrix} u(t) \quad (6)$$

According to the vehicle model, the nominal values of the physical parameters of equation (6) are as follows:

$$m_s = 320\text{kg}, \quad m_u = 40\text{kg}, \quad k_s = 18\text{kN/m}, \quad c_s = 1\text{kN} \cdot \text{s/m}, \\ k_u = 200\text{kN/m}, \quad x_{\max} = 0.08\text{m}, \quad u_{\max} = 1.5\text{kN}.$$

In order to simulate vehicles passing through pits or bags, we assume that the system's disturbance input is:

$$\omega(k) = \begin{cases} \frac{\pi VA}{L} \sin \frac{2\pi V}{L} kT_s, & 0 \leq k \leq \frac{L}{VT_s}, \\ 0, & k > \frac{L}{VT_s}, \end{cases} \quad (7)$$

The corresponding parameter attributes in the formula are shown in Table 1:

Table 1. Parameter attributes

parameter	unit	value
V	km/h	60
A	m	0.1
L	m	5
T_s	s	0.02

Combining people's feelings of comfort, we chose longitudinal acceleration of the body to describe the ride comfort [11]:

$$y(t) = \begin{pmatrix} -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & 0 & \frac{c_s}{m_s} \end{pmatrix} x(t) + \frac{u_{\max}}{m_s} u(t) \quad (8)$$

3. H_∞ robust predictive control

This chapter first introduces the unconstrained H_∞ control, then introduces the time-domain constrained H_∞ control, and finally derives the rolling time domain constrained H_∞ control. Robust predictive control after linear matrix inequality optimization can coordinate constraints and system performance [12]. The LMI optimizes the state feedback control strategy, and then refreshes the system state at the sampling time, so that the system reaches the rolling optimization control.

3.1. Unconstrained H_∞ control

First, regardless of the time domain constraints, the state feedback is $u = Kx$. The closed-loop system that has been discretized is described as:

$$\begin{aligned} x(k+1) &= A_{cl}x(k) + B_1\omega(k) \\ z_1(k) &= C_{1,cl}x(k) + D_1\omega(k) \end{aligned} \quad (9)$$

The matrix $P(Q = P^{-1})$ is introduced according to the Schur complement formula. Combined with the system dissipation conditions, we transform the matrix. Finally, we make it satisfy the following inequality:

$$\begin{pmatrix} Q & O & * & * \\ 0 & \gamma I & * & * \\ AQ + B_u Y & B_1 & Q & 0 \\ C_1 Q + D_{1u} Y & D_1 & 0 & \gamma I \end{pmatrix} > 0 \quad (10)$$

Where the discrete closed-loop H_∞ norm from ω to z_1 is smaller than γ , $Y = KQ$

By solving the above matrix inequality, the system can realize unconstrained H_∞ control.

3.2. Time domain constraint conversion and constrained H_∞ control

The parameter e_ν is the standard base vector of $\mathbb{R}^{p_2}$. Considering the time-domain constraints, the system satisfies the terminal domain of the standard MPC stability, so Q and Y satisfy (11) while also satisfying inequality:

$$\begin{pmatrix} z_{2\nu, \max}^2 & e_\nu^T (C_2 Q + D_{2u} Y) \\ * & Q \end{pmatrix} > 0, \quad \nu = 1, 2, \dots, p_2 \quad (11)$$

3.3. Rolling time domain H_∞ control algorithm

Applying the rolling optimization principle, we will solve the LMI optimization problem updated by the latest measurement state $x(k)$ at each sampling time k , and add a constraint to re-dissipate the closed-loop system. Namely:

$$\begin{pmatrix} p_0 - p_{k-1} + x(k)^T P_{k-1} x(k) & x(k)^T \\ x(k) & Q \end{pmatrix} \geq 0 \quad (12)$$

Combining the unconstrained H_∞ control algorithm with time domain constraints, and we add the rolling optimization, robust predictive control can be achieved out and the system can have self-optimizing performance.

4. Experimental evaluation

We hope that the active suspension system can achieve the function of tracking disturbances stably and being able to predict road surface disturbances at time to achieve a function of noise reduction. Therefore, we start from the unconstrained control algorithm and gradually consider adding time domain constraints and rolling time domain optimization, eventually, the system can achieve a certain degree of robustness.

By solving the linear matrix inequality of the above optimization problem, an optimal algorithm for robust predictive control can be realized. The simulation data uses the data given in the previous

chapter. The disturbance input is added at 20s~35s. Figure 2 shows the input as external disturbance, the simulation results are in Figure 3. In order to reflect the superiority of robust predictive control in active suspension systems, we added PID control for comparison, the parameters are set through multiple tests to obtain the optimal tuning result: $K_p = 3000, K_i = 1200, K_d = 3$. (Both simulations are in the same disturbance input in Figure 2):

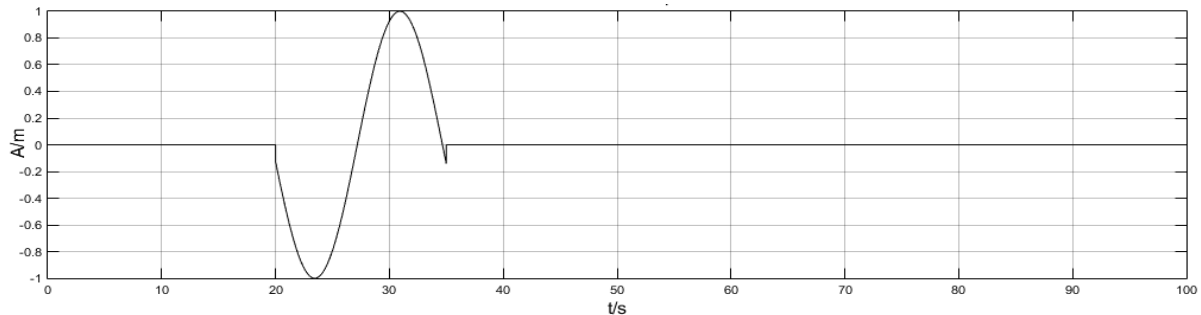


Figure 2. Disturbance input

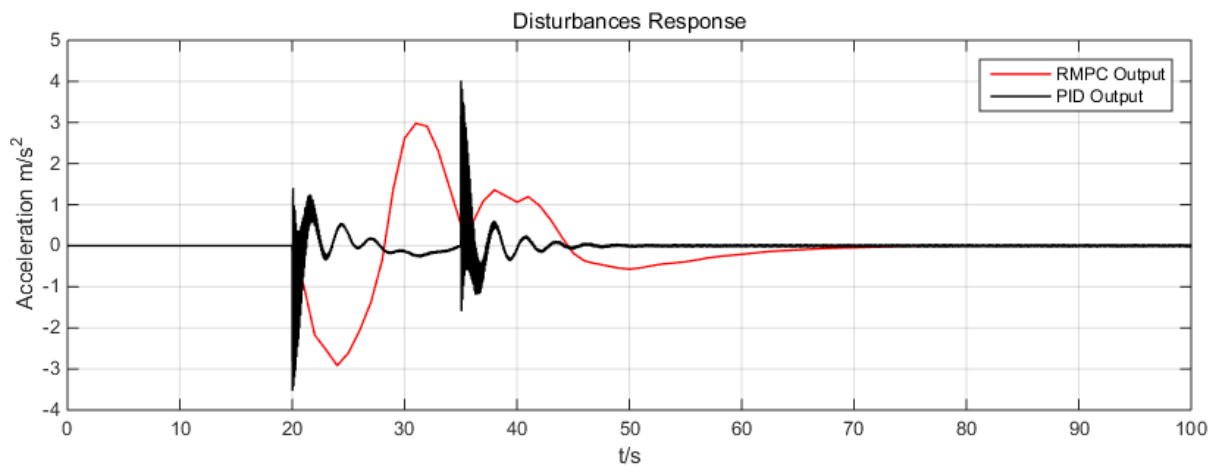


Figure 3. Robust predictive active suspension control and PID output after optimization

Analysis of the results, using PID regulation can adjust the longitudinal acceleration and be stability finally. However, during the adjustment process, the PID adjustment will cause the waveform to vibrate and the acceleration to change drastically. During normal PID adjustment, this process can be ignored. However, in the active suspension system, passengers will feel extremely uncomfortable if this happens. As a result, PID regulation cannot solve active suspension control well.

It can be seen from the simulation figure that the optimized rolling time domain H_∞ control method can effectively solve the control problem of the active suspension system. There is no sudden change in acceleration and system stability is better than PID regulation. Since the LMI optimization is performed online, the closed-loop system can coordinate the contradictory relationship between the control performance and the system constraints in real time and obtain the best possible control quality under the premise of ensuring the control constraints.

5. Conclusion

First, we construct a state space model for active suspension. Then we used PID control and H_∞ robust predictive control to adjust. Among them, the traditional PID adjustment results show that the system can achieve stability, and the adjustment time is within the acceptable range. However, in the disturbance rejection process, the system output is vibrated violently, which is unacceptable for a suspension system. Through the robust predictive control after optimization, we find that the system stability is outstanding, and it can realize online optimization to feedback the system state in real time.

More importantly, the accuracy of robust predictive control is also better than PID control. From the simulation image, it can solve the active suspension problem very well.

Similarly, in the high-precision control cases such as active suspension, we cannot rely on simple control methods such as PID, so we need introduce many advanced and stable control strategies to solve such problems.

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