

Design Method of Three-Rotation and Translation Spatial Full Compliant Mechanism

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Abstract. According to the kinematics analysis of 4-RRCR spatial parallel mechanism with three rotation and translation(3R1T), establish differential Jacobian matrix relationship with input and output of four degree of freedom(DOF) spatial parallel mechanism, and used to design space of 3R1T full compliance mechanism. In the design, the method of solving the model of by method of Isotropic Material with Penalization(SIMP) is optimality criteria (OC) method. Through static simulation analysis, the compliance mechanism of obtained by flexible hinge replacement method and by the topology optimization method are respectively in HyperWorks®. The simulation analysis results show us the 3R1T full compliance mechanism design by topology optimization method has same motion characteristics as the traditional parallel mechanism, and the compliance mechanism design by topology optimization method global stiffness and material utilization are better than the compliance design by flexible hinge replacement method.

1. Introduction

Parallel mechanism was first put forward by Hunt[1] in 1978, parallel mechanism has received extensive attention and development. In the research of parallel mechanism process, it is found that the 6 DOF parallel mechanism is not competent in some situation, and the use of 6 DOF parallel mechanism the control system is not only very complex, but also too many assembly manufacturing errors seriously affecting the kinematic accuracy. Use the less DOF parallel mechanism can not only reduce the error which the impact of manufacturing assembly, but also simplify the control system, so it can have a greater utilization. Therefore, seek less DOF parallel mechanism has become the focus of many scholars other than 6 DOF parallel mechanism, and obtained a lots of results. In a long period, about 4 DOF and 5 DOF parallel mechanism research was almost blank until ZLATANOV[2] and Fang[3] put forward the type synthesis of it. On this basis, scholars put forward a comprehensive method of 4 DOF symmetric parallel mechanisms and analysed [4-12]. Subsequently, the 4 DOF compliant mechanism developed rapidly.

The compliant mechanism is enriched on the basis of parallel mechanism. There are usually two methods to design compliant mechanism: pseudo-rigid body and topology optimization. The pseudo-rigid body method uses flexure hinge to replace the kinematic pair in the parallel mechanisms. This process is characterized that assembly free, lubrication free and high accuracy by direct matching of kinematic pairs on the parallel mechanisms. However, using flexure hinge replace kinematic pair has a



prominent problem with lead to serious stress concentration, which makes it difficult to meet the requirement of vibration simulation and fatigue life of a parallel mechanism. The method of topology optimization is used topology optimization algorithm to realize compliant mechanism. According to the process of motion operating conditions of mechanism, obtained the hidden hinge compliant parallel mechanism, because there are no obvious flexure hinge, the material distribution of compliance mechanism is uniform, and the stress concentration phenomenon is solved and improved compared with that obtained by pseudo-rigid body method.

Based on the research results of ZLATANOV[2] and Li[11] et al established the Jacobian matrix of spatial 3R1T parallel mechanism by screw theory. Based on the research, the topological optimization of spatial 3R1T parallel mechanism is completed by using the Jacobian matrix, and obtain the spatial 3R1T fully compliant parallel mechanism.

2. Parameter Extraction of Spatial Parallel Mechanism .

The parallel mechanism of 4-RRCR mainly consist of three parts: the moving platform, the fixed platform, and chain of connection moving/fixed platform, each chain contains three rotation pair and a cylindrical pair, these motion pair in space position arrangement as shown in Figure , for a chain, the two axis of rotation pair near fixed platform was parallel each other, the axis of cylindrical and axis of rotation pair which connect moving platform extension line was handed over to a point which locate line of center moving platform and center of fixed platform.

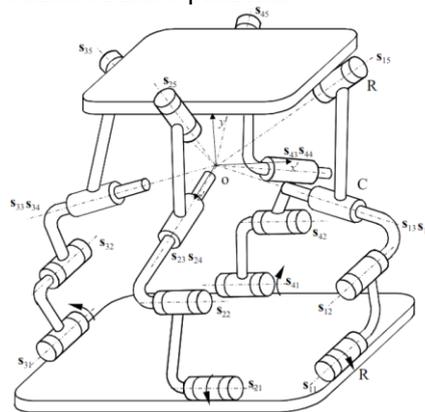


Figure 1 4-RRCR parallel prototype mechanism

For Figure 1, used screw theory to analysis chain of parallel mechanism and obtain the motion spiral system, take point of axis of cylindrical pair and axis of rotation pair extension line wan handed as origin of coordinates, set up coordinate system \$o'-x'y'z'\$,because of the moving platform is only in the space around the X, Y, Z axis rotation and along Y axis translation, the moving platform center along the X, Z axis movement displacement and speed value equal to zero. By screw theory, the speed screw of moving platform in the coordinate \$o'-x'y'z'\$ is:

$$v = (\omega_x, \omega_y, \omega_z; 0, v_y, 0)^T$$

Note the jacobian matrix of the \$i\$ th is \$J_i\$, and then \$v = J_i \dot{\theta}_i\$, where \$\dot{\theta}_i = (\dot{\theta}_{i1}, \dot{\theta}_{i2}, \dot{\theta}_{i3}, \dot{\theta}_{i4}, \dot{\theta}_{i5})\$, which is the velocity of the \$i\$ th branch joint. By Li[11] research results, the expression of the screw \$S_{ij}\$ of each chain joint was obtained, and constructs each branched chain kinematic spiral \$J_i\$, and the screw obtained was as follows :

$$\begin{aligned}
 J_1 &= (\$_{11} \ \$_{12} \ \$_{13} \ \$_{14} \ \$_{15}) \\
 &= \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{11} & m_{12} \\ 0 & 0 & 0 & n_{11} & n_{12} \\ 0 & 0 & 0 & 0 & 0 \\ b_{11} & b_{12} & b_{13} & 0 & 0 \\ c_{11} & c_{12} & c_{13} & 0 & 0 \end{pmatrix} \quad (1)
 \end{aligned}$$

$$\begin{aligned}
 J_2 &= (\$_{21} \ \$_{22} \ \$_{23} \ \$_{24} \ \$_{25}) \\
 &= \begin{pmatrix} 0 & 0 & 0 & l_{21} & l_{22} \\ 0 & 0 & 0 & m_{21} & m_{22} \\ 1 & 1 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 \\ b_{21} & b_{22} & b_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (2)
 \end{aligned}$$

$$\begin{aligned}
 J_3 &= (\$_{31} \ \$_{32} \ \$_{33} \ \$_{34} \ \$_{35}) \\
 &= \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{31} & m_{32} \\ 0 & 0 & 0 & n_{31} & n_{32} \\ 0 & 0 & 0 & 0 & 0 \\ b_{31} & b_{32} & b_{33} & 0 & 0 \\ c_{31} & c_{32} & c_{33} & 0 & 0 \end{pmatrix} \quad (3)
 \end{aligned}$$

$$\begin{aligned}
 J_4 &= (\$_{41} \ \$_{42} \ \$_{43} \ \$_{44} \ \$_{45}) \\
 &= \begin{pmatrix} 0 & 0 & 0 & l_{41} & l_{42} \\ 0 & 0 & 0 & m_{41} & m_{42} \\ 1 & 1 & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & 0 & 0 \\ b_{41} & b_{42} & b_{43} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (4)
 \end{aligned}$$

This paper, the driving mechanism of the parallel mechanism is designed to be the mobile pair of the branched chains, remove J_1 and J_3 in the fourth row, and remove J_2 and J_4 in the sixth row and inverse respectively, take out the line of driver, finally, obtain the inverse matrix of jacobian matrix of the whole mechanism, as follows:

$$G = \begin{pmatrix} \bar{G}_{11} & \bar{G}_{12} & \bar{G}_{13} & \bar{G}_{15} \\ \bar{G}_{21} & \bar{G}_{22} & \bar{G}_{23} & \bar{G}_{25} \\ \bar{G}_{31} & \bar{G}_{32} & \bar{G}_{33} & \bar{G}_{35} \\ \bar{G}_{41} & \bar{G}_{42} & \bar{G}_{43} & \bar{G}_{45} \end{pmatrix}$$

where:

$$\begin{aligned}
 \bar{G}_{11} &= -\frac{b_{11} \times c_{12} - b_{12} \times c_{11}}{b_{11} \times c_{13} - b_{13} \times c_{11} - b_{12} \times c_{13} + b_{13} \times c_{12}} \\
 \bar{G}_{12} &= 0
 \end{aligned}$$

$$\begin{aligned}\bar{G}_{13} &= -\frac{c_{11} - c_{12}}{b_{11} \times c_{13} - b_{13} \times c_{11} - b_{12} \times c_{13} + b_{13} \times c_{12}} \\ \bar{G}_{15} &= \frac{b_{11} - b_{12}}{b_{11} \times c_{13} - b_{13} \times c_{11} - b_{12} \times c_{13} + b_{13} \times c_{12}} \\ \bar{G}_{21} &= 0 \\ \bar{G}_{22} &= -\frac{a_{21} \times b_{22} - a_{22} \times b_{21}}{a_{21} \times b_{23} - a_{23} \times b_{21} - a_{22} \times b_{23} + a_{23} \times b_{22}} \\ \bar{G}_{23} &= -\frac{b_{21} - b_{22}}{a_{21} \times b_{23} - a_{23} \times b_{21} - a_{22} \times b_{23} + a_{23} \times b_{22}} \\ \bar{G}_{25} &= \frac{a_{21} - a_{22}}{a_{21} \times b_{23} - a_{23} \times b_{21} - a_{22} \times b_{23} + a_{23} \times b_{22}} \\ \bar{G}_{31} &= -\frac{b_{31} \times c_{32} - b_{32} \times c_{31}}{b_{31} \times c_{33} - b_{33} \times c_{31} - b_{32} \times c_{33} + b_{33} \times c_{32}} \\ \bar{G}_{32} &= 0 \\ \bar{G}_{33} &= -\frac{c_{31} - c_{32}}{b_{31} \times c_{33} - b_{33} \times c_{31} - b_{32} \times c_{33} + b_{33} \times c_{32}} \\ \bar{G}_{35} &= \frac{b_{31} - b_{32}}{b_{31} \times c_{33} - b_{33} \times c_{31} - b_{32} \times c_{33} + b_{33} \times c_{32}} \\ \bar{G}_{41} &= 0 \\ \bar{G}_{42} &= -\frac{a_{41} \times b_{42} - a_{42} \times b_{41}}{a_{41} \times b_{43} - a_{43} \times b_{41} - a_{42} \times b_{43} + a_{43} \times b_{42}} \\ \bar{G}_{43} &= -\frac{b_{41} - b_{42}}{a_{41} \times b_{43} - a_{43} \times b_{41} - a_{42} \times b_{43} + a_{43} \times b_{42}} \\ \bar{G}_{45} &= \frac{a_{41} - a_{42}}{a_{41} \times b_{43} - a_{43} \times b_{41} - a_{42} \times b_{43} + a_{43} \times b_{42}}\end{aligned}$$

So, the jacobian of the organization is. $J = \bar{G}^{-1}$.

3. Model Design of 3R1T Spatial Compliance Mechanism

3.1 Domain Area and Parameter for Topology Optimization

The full compliance mechanism modeling by SIMP method for topology optimization and iterate solved method utility OC algorithm, select element density of model as design variable to establish optimal design function. Finally, the element density of material tends to 0 or 1 by introducing a penalty factor, thus, can achieve the goal of removing material.

According to branch chain and mobile/fixed platform position relationship in space of 4-RRCR spatial mechanism, got design domain shown in Figure 2 of topology optimization. The basis of fixed platform set a no-design area with size is $180 \times 180 \times 5 \text{mm}^3$, the fixed platform is rectangular with size is $130 \times 130 \text{mm}^2$, on the upside of fixed platform has a rectangular as moving platform with size is $60 \times 60 \text{mm}^2$. In order to prevent singularity location in drive process, the moved platform has counterclockwise rotation of 2° relative to fixed platform, domain area between moved platform and fixed platform set as design area (shown blue area in Figure 3), and, according to spatial parallel mechanism drive direction and position set aside space to install piezoelectric ceramic, in piezoelectric ceramic install space has a range around area as no-design (shown red area in Figure 3). The load and constraint shown arrow with yellow color and green area respectively in Figure 3. The material choose stainless steel 304 as design material of 3R1T full compliance mechanism, it Young's modulus is

$E=2.05 \times 10^5 F/mm^3$, Poisson ratio is $\mu=0.3$, material density is $\rho = 7.85 \times 10^{-9} t/mm^3$, given driving force $F_1 = F_2 = F_3 = F_4 = 5000N$ with direction is piezoelectric axis direction. The volume fraction of topology optimization constraint was equal 0.2.

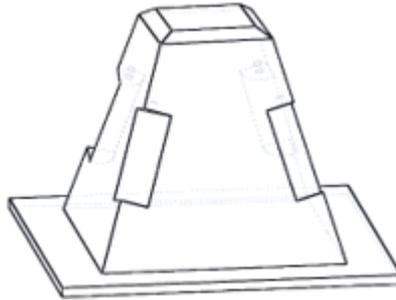


Figure 2 compliance mechanism design model

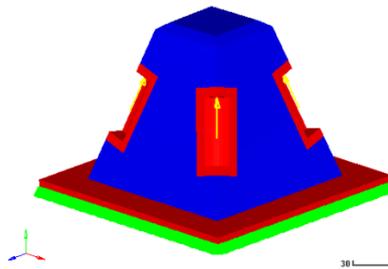


Figure 3 Spatial 3R1T topology design area and boundary condition

Through 16-step iteration computer of the spatial 3R1T compliance parallel mechanism, the final topology optimization is shown in Figure 4.

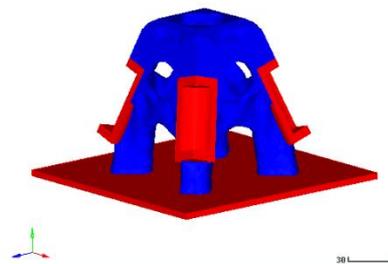


Figure 4 Spatial 3R1T full compliance parallel topology optimization results

In the process of iteration, the curve of relation between the objective function and iteration step shown in Figure 5, in Figure 5 showed that the flexibility of objective function is fastest decline in 0-6 step, and converging to the 7 step.

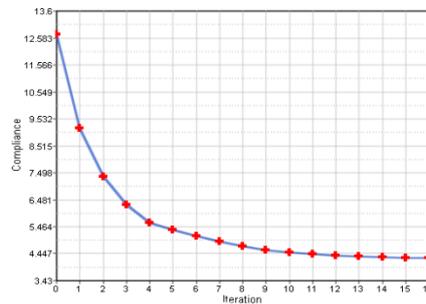


Figure 5 The objective function varies with the number of iterations during iteration

3.2 Analysis result

Meshing smoothing for spatial 3R1T compliance mechanism by design of topology optimization, export the model to CAD software erect modeling again, later let model static simulation analysis again in HyperWorks®, get the stress nephogram of before optimization and after optimization respectively showed in Figure 6.

Analysis stress of 3R1T full compliance mechanism of design by topology optimization method and design by flexible hinge replacement method respectively, the analysis result shown that: for spatial compliance mechanism design by topology optimization method, the model maximum stress reduced 10.78% compare to optimal before, and the model minimum stress decreased by more than 400% compare to optimal before. Compare with compliance mechanism design by flexible hinge replacement method,

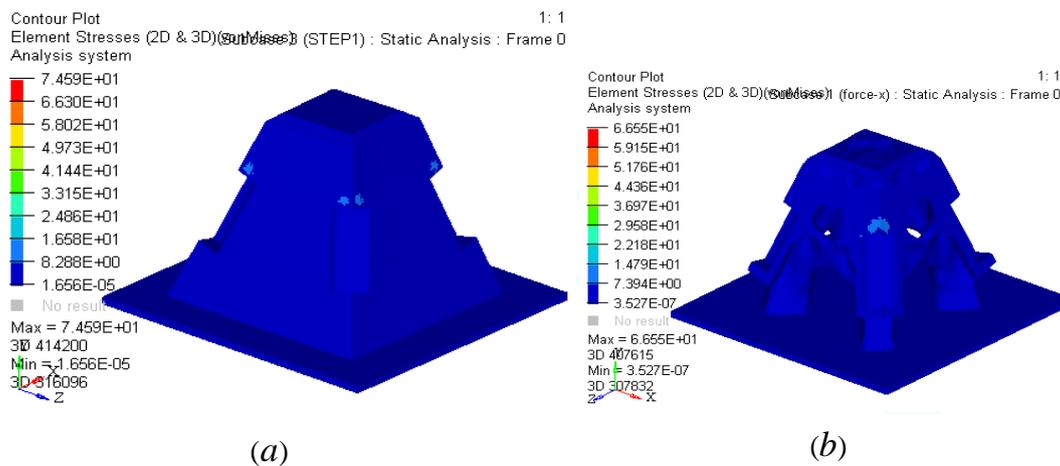


Figure 6 Model analysis of Stress nephogram. (a) Optimized before stress distribution cloud. (b) Optimized after stress distribution cloud

the model maximum stress value decreases by 75%. So, utility topology optimization method to design compliance mechanism can improve material utilization rate, compare with compliance mechanism design by flexible hinge replacement method, the global structure material distribution more reasonable.

Based on the dynamic analysis of compliance mechanism design by topology optimization method, select the center of moving platform translation and rotation displacement in the motion as moving platform translation and rotation displacement. The rotation displacement of X, Y, Z with axis and translation displacement with Y axis shown in Figure 7 respectively.

From the Figure 7 shown that, the central of 3R1T compliance mechanism in space displacement respectively is $R_x = -5.151 \times 10^{-6} \text{ rad}$, $R_y = 2.404 \times 10^{-7} \text{ rad}$, $R_z = -5.631 \times 10^{-6} \text{ rad}$, and $D_y = 2.685 \times 10^{-4} \text{ mm}$, the result data shown that spatial 3R1T compliance mechanism design by topology optimization method can achieve displacement with characteristics of small scale, realize the requirement of

micro/nano displacement characteristics.

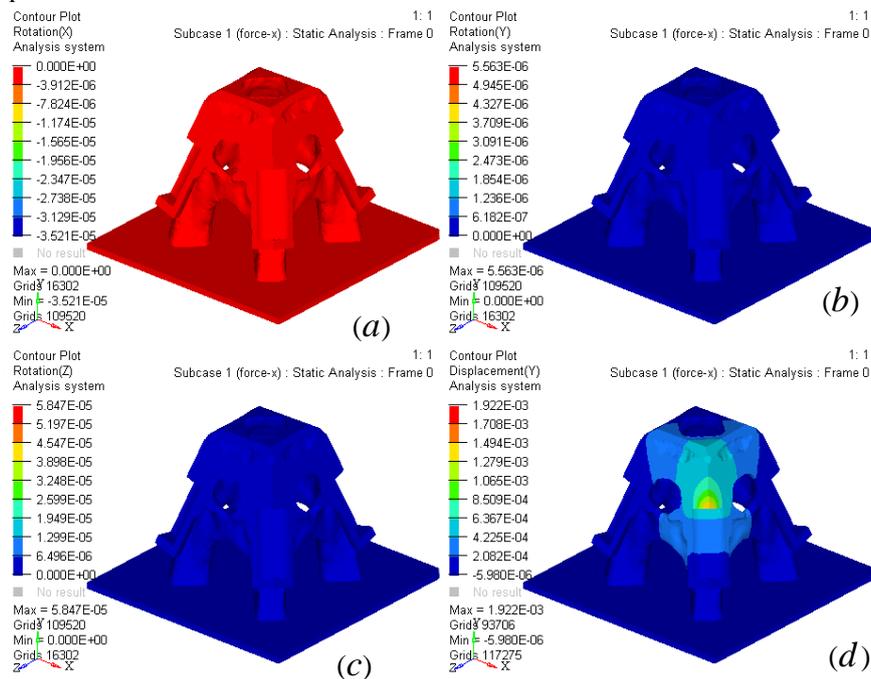


Figure 7 The four nano-scale displacements of the spatially compliant mechanism. (a) Rotational displacement about X axis. (b) Rotational displacement about Y axis. (c) Rotational displacement about Z axis. (d) Move the displacement along the Y axis.

4. Conclusion

In this paper, the jacobian matrix relation is added to topology optimization method. Through the study of the differential kinematics analysis of parallel mechanism for input and output jacobian matrix, combine the modeling method of SIMP and solving method of OC algorithm, finally, obtain a spatial compliance mechanism with same differential movement characteristics of the prototype of parallel mechanism, and achieve micro/nano meter displacement. This method can not only realize the differential motion characteristics of the compliance mechanism, but also can realize the global stiffness distribution of the compliance mechanism and improve the utilization efficiency of the material. The method can also be utilized in the process of other spatial compliance mechanism design.

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