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Vibration Tracking Control of Piezoelectric Bimorph Bender in Novel Reciprocal State Space Form

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Abstract. In this paper, the deflection of a piezoelectric bimorph bender is controlled to track given time varying reference command solely utilizing direct acceleration measurement without integration. Novel estimator that directly applies acceleration measurement and controller that applies estimated state derivative signals have been developed in reciprocal state space (RSS) form. Simulations for the augmented system of both closed loop system and estimator have been carried out to successfully verify the proposed methods. The design approach in this paper is applicable to smart structures with accelerometers as sensors.

1. Introduction

Piezoelectric bimorph bender is a smart structure and can serve as both sensor and actuator in many electromechanical applications such as acoustic sensor, force sensor, vibration sensor [1], energy harvesting [2-3], minute robotic [4], and actuators [5]. Typically, the structure of a piezoelectric bimorph bender [6-7] is an elastic plate sandwiched between two piezoelectric plates and is mounted in a cantilever arrangement. In this paper, an accelerometer is installed on a piezoelectric bimorph bender as sensor. A novel estimator is designed based on the direct acceleration measurement. In addition, a novel tracking controller based on estimated state derivative is designed so that the deflection of the bimorph piezoelectric plate can track the time varying reference command. However, the proposed design objectives are not achievable applying traditional state feedback algorithms in the state space form as follows.

$$\dot{x} = \overline{A}x + \overline{B}u \tag{1a}$$

$$u = -kx \tag{1b}$$

$$y = Cx \tag{1c}$$

, where x, u and y are state, control and measurement, respectively. The reason is that acceleration measurement only can be modelled as state derivative. Therefore, people cannot apply available state related measurement feedback algorithms without using numerical integrations which will increase complexity and implement cost of controller.

To directly utilize state derivative measurement in controller and estimator designs, the first author proposed the following reciprocal state space (RSS) form [8].

$$x = A\dot{x} + Bu \tag{2a}$$

$$u = -k\dot{x} \tag{2c}$$

$$y = C\dot{x} \tag{2c}$$

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The name of RSS form was given because the open loop poles of system (2a) are the reciprocals of the eigenvalues of matrix A. In this paper, the vibrational tracking control of a piezoelectric bimorph plate is carried out in RSS form applying state derivative measurement in design.

2. Modelling of a smart piezoelectric bimorph bender

The smart piezoelectric bimorph bender in this research consists of a composite plate with piezoelectric actuation and a micro-machined accelerometer as sensor. We developed discrete model of the smart plate for both dynamic analysis and control designs of transverse vibration. The candidate composite plate in [9] has laminated design angles of [-55.66/43.62/4.18] and has six graphite/epoxy piles with length, width and thickness of 0.305, 0.076 and 0.134×10^{-2} m, respectively. We model the smart plate in such a way that, 0.4 mm piezoelectric PVDF (polyvinylidene fluoride) material is distributed on both upper and lower surfaces of composite plate and one electrode serving as actuator is installed on upper surface while one micro-accelerometer serving as sensor is attached on the lower surface. Applying Hamilton's law and using the first four shape functions in [10] and the physical properties of PVDF in [11], three sub-models are first built for the structural, electrical and coupling subsystems and then assembled to get the resultant discrete model as follows.

$$\begin{bmatrix} M_s+M_{pz} & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \ddot{u} \\ \ddot{u}_e \end{bmatrix}+\begin{bmatrix} K_s+K_{pz} & -K_{u\phi} \\ K_{\phi u} & K_{\phi\phi} \end{bmatrix}\begin{bmatrix} u \\ u_e \end{bmatrix}=\begin{bmatrix} 0 \\ \Gamma q \end{bmatrix} \tag{3a}$$

$$y=C_a\ddot{u} \tag{3b}$$

, where M_s and K_s are mass and stiffness matrices of the composite plate while M_{pz}, K_{pz}, $K_{u\phi}$ and $K_{\phi\phi}$ are PVDF mass and PVDF stiffness, coupling and capacitance matrices, respectively.

In addition, Γ is related to the location of electrode and electric shape function which consistent with prescribed voltage/charge boundary conditions. Furthermore, u, u_e and Γq are the generalized displacement vector, voltage across the PVDF and surface charge, respectively. Based on the location of micro-accelerometer and the first four mechanical shape functions in [10], one can get the sensor matrix C_a and acceleration measurement y. Details of modelling methods and techniques for smart plate are available in [12]. From the second equation in (3a), one can have

$$u_e=K_{\phi\phi}{}^{-1}\Gamma q-K_{\phi\phi}{}^{-1}K_{\phi u}u \tag{4}$$

Substituting (4) into first equation in (3a), we have the following system in second order form.

$$(M_s+M_{pz})\ddot{u}+(K_s+K_{pz}+K_{u\phi}K_{\phi\phi}{}^{-1}K_{\phi u})u-K_{u\phi}K_{\phi\phi}{}^{-1}\Gamma q=M\ddot{u}+Ku+Df=0 \tag{5}$$

, where $D=-K_{u\phi}K_{\phi\phi}{}^{-1}\Gamma$ and $f=q$.

3. Estimator design in RSS form

Defining $x^T=[u \;\; \dot{u}]^T$, system equation (5) is expressed in reciprocal state space form as follows.

$$x=\begin{bmatrix} 0 & -K^{-1}M \\ I & 0 \end{bmatrix}\dot{x}+\begin{bmatrix} -K^{-1}D \\ 0 \end{bmatrix}f=A\dot{x}+Bf \tag{6a}$$

$$y=[0 \;\; C_a]\dot{x}=C\dot{x} \tag{6b}$$

To directly utilize the sensed acceleration signal without numerical integration, the following estimator in RSS form is proposed.

$$\hat{x}=A\dot{\hat{x}}+Bf+L(y-\hat{y}) \tag{7a}$$

$$\hat{y}=C\dot{\hat{x}} \tag{7b}$$

, where $\hat{x}$ and $\dot{\hat{x}}$ are the estimated state and estimated state derivative, respectively.

The estimated state error is then defined as:

$$e=\hat{x}-x \tag{8}$$

Subtracting (6a) from (7a), we get the following error dynamics of estimator in RSS form.

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$$e = (A - LC)\dot{e} \tag{9}$$

Therefore, once the estimator gain L in (9) is properly designed, the estimation error will converge to zero. Consequently, full estimated state derivative feedback gain *k* in (10) is designed so that the closed loop system of smart piezoelectric bimorph bender is stable, namely, all system eigenvalues have negative real parts.

$$f = -k\dot{\hat{x}} \tag{10}$$

Combining (6)-(10), an augmented system equation of state and estimator error is obtained as follows.

$$x_t = \begin{bmatrix} x \\ e \end{bmatrix} = \begin{bmatrix} A - Bk & -Bk \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = A_t \dot{x}_t \tag{11}$$

Since A_t in (26) is an upper triangle matrix, full estimated state derivative feedback gain *k* and estimator gain L can be designed separately. The separation principle also holds for systems in reciprocal state space form. For the reciprocal state space system in (11), the eigenvalues of $A - Bk$ should be designed as the reciprocals of desired closed loop eigenvalues while the eigenvalues of $A - LC$ should be designed as the reciprocals of desired estimator eigenvalues.

4. Tracking control design in reciprocal state space form

The following tracking controller in (14) can work for the RSS system in (12-13) in following discussion.

Plant: $x = A\dot{x} + Bf$ (12)

Performance output: $z = Hx$ (13)

Tracking controller: $f = -k\dot{\hat{x}} + Nr$ (14)

, where *x*, $\hat{x}$, *y*, *u* and *r* are state vector, estimated state vector, sensor measurement vector, control vector and reference command vector, respectively while *z* is the system performance output of interest, which must track the given reference command *r*. In (14), feedback gain *k* is first design to build a stable closed loop system, then feedforward gain *N* is designed to track the reference command. Note that if feedback gain *k* is not properly desgn and the closed loop system is not stable, no matter what feedforward gain *N* is designed, the performance output *z* can never well track the reference command *r*.

In this paper, pole placement method in RSS form [13] is applied to design feedback gain *k* to ensure that the closed loop system is stable. When we applies pole placement design for RSS system in (12), the gain *k* must be designed such that the eigenvalues of $(A - Bk)$ are reciprocals of the desired closed loop eigenvalues. In addition, LQR [13] and sliding mode control [14] in RSS form developed by the first author can also be used to build a stable closed loop system.

When the closed loop system is stable $(\dot{\bar{x}}(\infty) = 0$ and $\dot{\bar{e}}(\infty) = 0)$ we have steady state as

$$\bar{x} = (A - Bk)\dot{\bar{x}} + BK\dot{\bar{e}} + BNr_0 = BNr_0 \tag{15}$$

When the steady state of tracking error is zero, we have

$$\bar{\varepsilon}(\infty) = r_0 - H\bar{x} = r_0 - HBNr_0 = (I - HBN)r_0 = 0 \tag{16}$$

Consequently, we have feedforward gain *N* as

$$N = (HB)_{right}^{-1} \tag{17}$$

, where $(HB)_{right}^{-1}$ is the right inverse of matrix *HB*.

Suppose that *HB* is a $m \times n$ full rank matrix and $m \le n$. We have

$$(HB)_{right}^{-1} = (HB)^T (HB(HB)^T)^{-1} \tag{18}$$

Since feedforward gain *N* in (17) is independent of feedback gain *k*, the feedback gain *k* and feedforward gain *N* can be designed separately.

5. Numerical Example

The matrices of the model of piezoelectric bimorph bender in (5) are modelled as follows.

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$$M = 10^{-3} \times \begin{bmatrix} 287 & 4 & 0 & 0 \\ 4 & 367 & 0 & 0 \\ 0 & 0 & 24.72 & 0.189 \\ 0 & 0 & 0.189 & 23.267 \end{bmatrix}, K = \begin{bmatrix} 38.1 & -0.2 & -13.4 & -16.4 \\ -0.2 & 1510.3 & 96.9 & -167.1 \\ -13.4 & 96.9 & 659.0 & -849.6 \\ -16.4 & -167.1 & -849.6 & 5250.9 \end{bmatrix},$$

$$D^T = [348.1431 \quad 424.7985 \quad 0 \quad 0],\ C_a = [1.315 \quad 0.27 \quad 0.631 \quad 0.081]$$

Suppose that the first generalized displacement is of interest, the performance output matrix H is then selected as follows.

$$H = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

Substituting the above matrices to the reciprocal state space system in (6a). One can verify that the open loop system has zero damping and cannot track step command and time varying command $2\sin 5t$ by applying feedforward gain N alone as shown in figure 1 and 2.

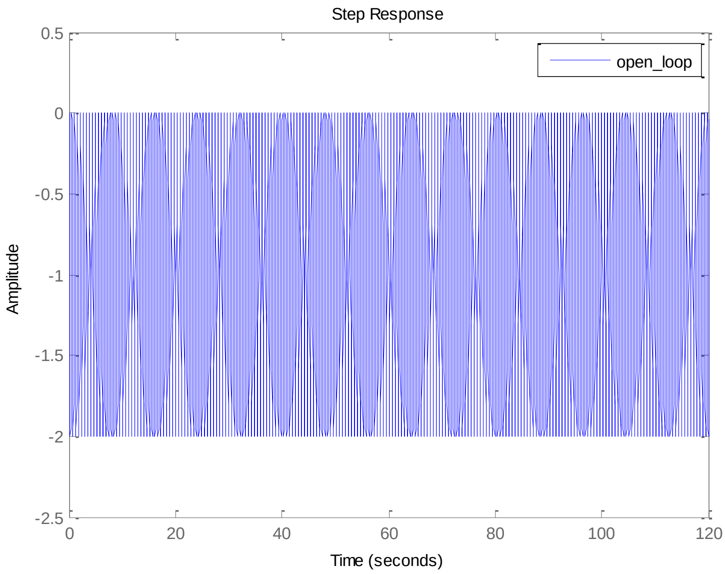

Figure 1. Open loop step tracking

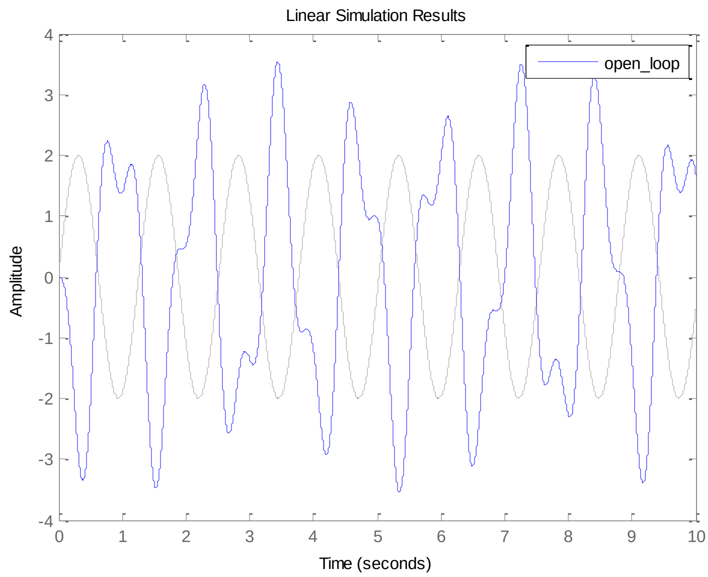

Figure 2. Open loop $2\sin 5t$ tracking

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The desired poles of closed loop system are selected as $-1\pm 2i$, $-2\pm 2i$, $-3\pm 4i$, $-4\pm 6i$ for both estimator and controller. Therefore, feedback gain k in (14) and estimator gain L in (7a) are designed through pole placement method in such a way that the eigenvalues of $(A-Bk)$ and $(A-LC)$ are the reciprocals of those desired poles. Consequently, the augmented system in (11) is built for simulation.

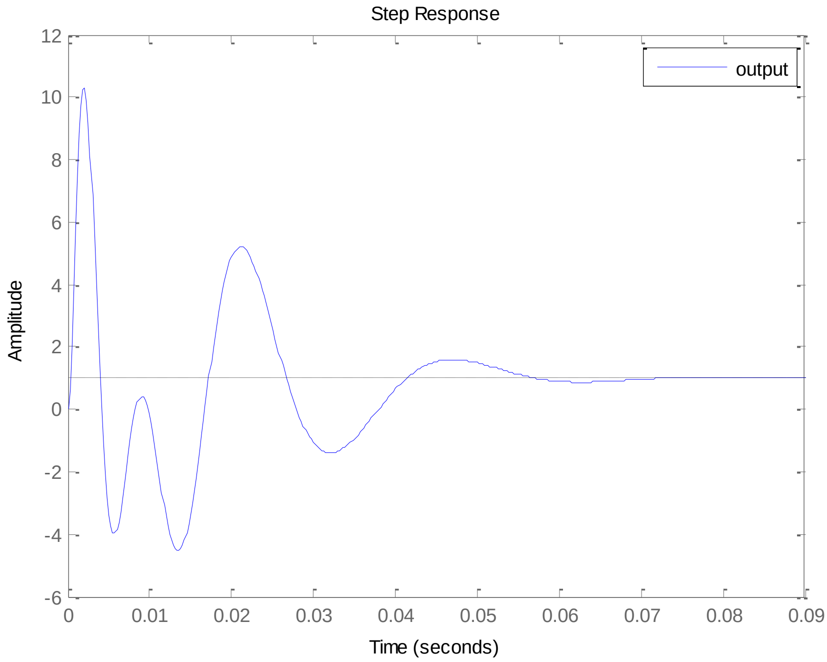

Figure 3. Closed loop step tracking

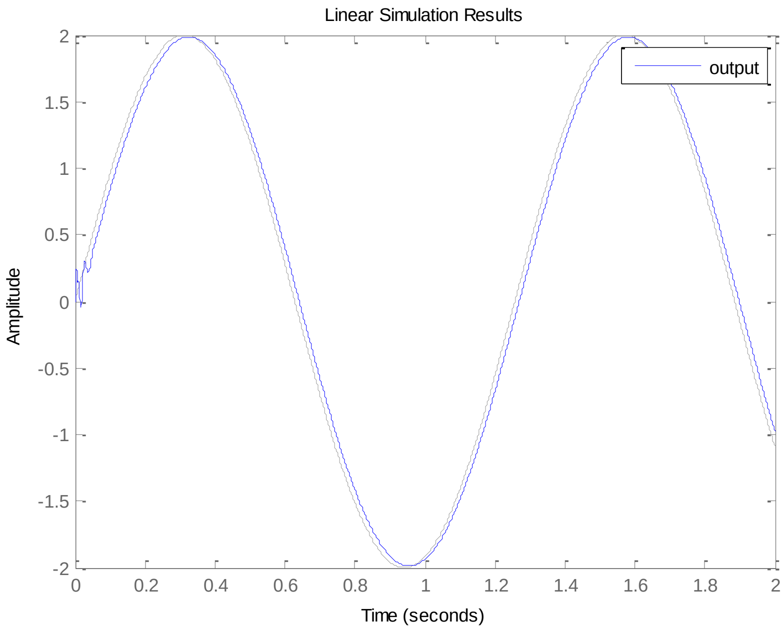

Figure 4. Closed loop $2\sin 5t$ tracking

As anticipated, the proposed design can well track step command and time varying command $2\sin 5t$ as shown in figure 3 and 4. Therefore, the proposed design approach is successfully verified.

6. Conclusion

In this paper, the designs of novel estimator that directly employs acceleration measurement and tracking controller that applying estimated state derivative feedback in RSS form have been introduced. Simulations based on a model of piezoelectric bimorph bender have been performed to verify the proposed design approach. The piezoelectric bimorph bender that can deflect itself by following time varying reference command can be used as a lifting surface and communication antenna of aerospace vehicles and in many other applications. The proposed design approach is also suitable for applications of other smart structure with accelerometers as sensors.

7. Acknowledgments

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