Empirical models in the investigations of internal combustion engines - limitations and examples of application

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Abstract. The empirical model of engine is determined by set of functions that constitute dependences approximating the measurement results of engine parameters corresponding to the adopted control value system. The use of appropriate approximation method, enabling the description of parameters using polynomial functions with several variables, ensures a large convergence of the calculated results with the current running engine operation parameters. However, the adequacy of such a model is limited to range of parameter variability and engine regulation taken into account during the identification experiment. With the right range selection of factors variability, which have controlling effect on engine operation, those approximation functions can be used to calculate the average of the parameters in the whole field of engine operation. However, the number of factors affecting the engine's properties is very high. The impact of some may be very significant, others less than a measurement uncertainties or production and regulatory inaccuracies. The paper presents an example of empirical model that was used to optimize the parameters of selection for turbochargers to engine, obtaining a high convergence of calculations with the results of the experiment, but not exceeding 5%. By determining the set of inputs for baseline model, the current knowledge of the phenomena occurring in the cylinder of the piston engine and the results of the preliminary comparative tests were based on. The principle of "redundancy" was followed, striving to adopt as many factors as possible, whose effects on engine performance indicators were considered to be dominant. In order to determine the final form of the approximation function, verification of the correlation between pre-accepted variables was carried out using the sensitivity analysis. The influence of single factors and their interactions on output value in the tested range of factor variability was evaluated using sensitivity indicators and to determine them, a statistical method of regression analysis was used.

1. Introduction

Nowadays mathematical modelling is a basic research tool used to analyse the operating processes of an internal combustion engine and associated with it systems. It is the result of the increasing knowledge of thermodynamic and gasodynamic processes taking place in the particular phases of the engine's working cycle and the development of numerical methods for solving sets of equations of the adopted mathematical description. However, if there is an insufficient knowledge of cause and effect relationships between factors (input values) and engine performance parameters (output values), it is necessary to conduct the experimental tests. The insertion of the data, experienced from the modelled engine, allow to obtain the required correlation of mathematical description and reality. The experimental data can also be used to determine the experimental engine model, showing the relationship between parameters and factors without a formal analysis of cause and effect relationships. Such models are created through approximation of parameter values' sets obtained during an experimental investigations, using multidimensional regression methods [**Błąd! Nie można odnaleźć źródła odwołania.**, **Błąd! Nie można odnaleźć źródła odwołania.**, **Błąd! Nie można odnaleźć źródła odwołania.**, **Błąd! Nie można odnaleźć źródła odwołania.**]. If we assume that the processes occurring in the engine and in the systems cooperating with it are dependent on a certain set of independent factors X $\{x_1, x_2, \ldots, x_i\}$, then based on the results of measurements of parameters Z $\{z_1, z_2, \ldots, z_j\}$ obtained for the appropriately selected values of X, the experimental model of the engine is a function of factors X and coefficients B $\{\beta_1, \beta_2, \ldots, \beta_k\}$:

$$\hat{z} = f(x_1, x_2, \ldots x_i, \beta_1, \beta_2, \ldots \beta_k) \tag{1}$$

The form of the regression function (1) can be arbitrarily chosen. The results of the research presented in the available literature (among others [1, 7, 8]) indicate the possibility of the correct approximation of the same measurement results with the different functions. However, by selecting the form of the function, the possibility of determining unknown coefficients, the adequacy of the obtained model and the practicality of its usage should be taken into account. An important problem is the selection of an appropriate system of the X factors. For practical reasons, it is necessary to limit the number of the measurements. It is intentional to use the systemic test methods that take into account the simultaneous influence of all factors on the engine. This means that you need to take the measurements in accordance with the methods of the experiment planning. In this case, system value of the X factors results from the usage of the experiment plan. It is necessary to take into account the technical limitations related to the possibility of engine regulating and its proper functioning at all values of X factors resulting from the adopted plan of the experiment. The range of variability of X factors should be selected in such way that it is possible to determine the parameter values in the area of engine operation covered by the experimental research. The later use of the model for further research is limited to this area.

In spite of these limitations, these types of models can be used for a comparative assessment of the parameters with different variants of engine design and associated systems, especially in the case of modernization of the already existing engines with well-recognized characteristics. This paper presents an example of using an experimental model to assess the rightness of the selection of the turbochargers for the SW 680 engine with a forced induction system.

2. Planning of the experiment during the selection of turbochargers for the engine

2.1. Selection of descriptive variables of the model

The purpose of model building was to use it for the research involving the shaping of the engine torque characteristics when the design parameters of turbochargers change. Analysing the relations between the engine operation indicators, it can be concluded that to determine the torque of a motor with a particular structure, it is sufficient to know the engine thermal efficiency, volumetric efficiency and mechanical losses of the engine. In the turbocharged engine, it is also necessary to know the exhaust gas temperature, which allows to calculate the operation parameters of the combustion turbine, and then – from the power balance with the compressor – the supercharging parameters. The following set of Z $\{z_1, z_2, z_3, z_4\}$ parameters (output values) for the experimental model of the turbocharged engine was adopted:

- z_1: engine thermal efficiency (η_c),
- z_2: volumetric efficiency (η_v),
- z_3: exhaust gas temperature (T_t),
- z_4: average friction pressure (p_m).

The dependences known from the literature were used to describe the average friction pressure (p_m). In order to determine the parameters z_1, z_2, z_3, the tests were carried out in accordance with the principles of the experiment planning. The number of the factors affecting the engine's operating parameters is very high. The impact of some parameters may be very significant and of others parameters less than measurement uncertainties or production and regulatory inaccuracies. By the determining of the set of the input values, it was based on the current knowledge about the processes occurring in the cylinder of the piston engine and the results of the preliminary comparative tests [**Błąd! Nie można odnaleźć źródła odwołania.**]. The principle of "redundancy" was followed, striving to adopt as many factors as possible, whose effects on the engine performance indicators were considered to be dominant. The final verification of the significance of the factors was carried out during subsequent elimination tests. The following set X $\{x_1, x_2, x_3, x_4, x_5\}$ of the factors (input values) was selected:

- x_1: engine speed (n),
- x_2: supercharging pressure (p_{ba}),
- x_3: supercharging temperature (T_{ba}),
- x_4: excess air ratio (λ),
- x_5: average exhaust gases pressure (p_{g1}).

The separation of the factors x_2, x_3, x_5 gave the opportunity to assess the impact of the turbocharger design on the charging conditions on which the engine's operating parameters depend.

The experimental researches were carried out according to the multi section quasi-uniform plan, whose usefulness in the testing of the internal combustion engines was demonstrated, inter alia, in the [**Błąd! Nie można odnaleźć źródła odwołania.**, **Błąd! Nie można odnaleźć źródła odwołania.**]. The layout of this plan assumes five levels of the input values determined for normed values in such a way that the lower and upper real values have been transformed to the values from the range [-2,378, +2,378]. It enables the approximation of the motor parameters through the second degree polynomials and ensures the stability of the estimation of the regression function inaccuracy. At the input value i = 5, this plan is characterized by the following number of system input value: $n_k = 32$ in the core plan, $n_0 = 10$ in the central point of the plan, $n_\alpha = 10$ in the star points of the plan. The total number of the layouts is n = 52 [**Błąd! Nie można odnaleźć źródła odwołania.**]. For each plan layout, the measurements were carried out with the number of the measurement repetition r = 3. The ranges of the factor variability and the approach used for their normalization are presented in Table 1.

Table 1. The ranges of the variability for the input variables and assigned to them levels of values resulting from the experiment plan.

Factor's name	Designation	Range of variation	Levels of the factor values for the normed values $\hat{x}$ ($\alpha_{rot} = 2,378$)				
		$x_{min} - x_{max}$	$-\alpha$	-1	0	+1	$+\alpha$
Engine speed n [rpm]	x_1	1000–2200	1000	1350	1600	1850	2200
Excess air ratio λ	x_2	1,3–2,7	1,3	1,71	2	2,29	2,7
Supercharging pressure p_{ba} [MPa]	x_3	0,1–0,18	0,1	0,123	0,14	0,157	0,18
Supercharging temperature T_{ba} [K]	x_4	320–380	320	337	350	363	380
Average exhaust gases pressure p_{g1} [MPa]	x_5	0,11–0,19	0,11	0,133	0,15	0,167	0,19

Due to particular difficulties with obtaining planned conditions of the engine operation during the experimental tests, the measurements were conducted for the values of factors differing from the levels determined in the experimental design/ plan. These differences were then taken into account by rescaling of the values for the standardized inputs (corresponding to the obtained actual lower x_{min} and upper x_{max}

values of the factors), for which the parameters of the approximation polynomials were calculated. The normalization of the x_i actual input values was carried out in such a way that the lower and upper values were transformed to normalized values $\hat{x}_i$ from the range [-1, +1]. The actual values were transformed according to the formula:

$$\hat{x} = \frac{2(x - x_{śr})}{x_{max} - x_{min}} \quad (2)$$

The statistical analysis of the experiment results and the normalization of the original (real) values of the input were carried out using the software *STATISTICA 8 PL*.

2.2. Research on the significance of the factors

The results of the experimental tests were used to determine the real significance of the pre-determined inputs for the engine model (quantitative correlation) [**Błąd! Nie można odnaleźć źródła odwołania.**]. The verification of the correlation among the adopted descriptive variables was carried out using the sensitivity analysis [**Błąd! Nie można odnaleźć źródła odwołania.**]. The influence of the individual factors and their interactions on the initial value in the entire tested area of the entrances' variability was evaluated using the sensitivity indicators. To estimate them, a statistical method of the regression analysis was used. For the elimination tests, aimed at selecting of the factors with the highest impact force on the output variable and eliminating those with negligible impact, the statistical regression model was limited to the linear relationship [**Błąd! Nie można odnaleźć źródła odwołania.**, **Błąd! Nie można odnaleźć źródła odwołania.**]. The model of the total sensitivity for the dependent variable z_i in normalized form, containing components being products of the input, is expressed in the general formula:

$$z_i = \beta_o + \sum_{i=1}^{m} \beta_j \hat{x}_{ij} + \sum_{j=1}^{m} \sum_{k=j+1}^{m} \beta_{jk} \hat{x}_{ij} \hat{x}_{ik} + \sum_{j=1}^{m} \sum_{k=j+1}^{m} \sum_{l=k+1}^{m} \beta_{jkl} \hat{x}_{ij} \hat{x}_{ik} \hat{x}_{il} + \cdots \quad (3)$$

where: β_o – mean value,
β_j – main impact factors,
β_{jk} – interactive factors between two inputs,
β_{jkl} – interactive factors between the three inputs,
$\hat{x}_{ij} \hat{x}_{ik} \hat{x}_{il}$ – standardized input values.

With the determined (linear) form of the sensitivity model (3), the elimination tests were carried out using the so-called factor plan, which assumes the study of the effects of each factor separately and their mutual interactions at two value levels. The two-level composition plan, which assumes a testing of the systems for five input quantities x_1, x_2, x_3, x_4, x_5, on two values levels, was used. It corresponds to the so-called complete plan 2^5, including all possible combinations of the settings (32 systems). In these studies, the former measurement results were used, obtained on the basis of the multi section quasi-uniform plan. For multi section quasi-uniform plan, the layout of the input values in the plan's nucleus (n_k=32) exactly corresponds to the layout of a two-level compositional plan. Thus, it can be seen that the proper planning of the tests allows to limit the number of the measurements which can be repeatedly used to carry out various analyses. Such an experiment setting allowed to analyse the influence of both main factors and internal links between the main parameters. The *STATISTICA* computer program was used to conduct the factor experiment and analyse its results in order to determine the coefficients of the sensitivity model.

The determination of the coefficients for the total sensitivity model was preceded by a verification through the occurrence of gross errors in the results. The measurements of the position (average) and the dispersion (standard deviation, standard error – standard deviation of the mean) were determined. The fulfilment of the homogeneity condition of the variance was also investigated. This homogeneity check of the variance was carried out by Levene and Brown-Forsythe tests. The module *Basic statistics and tables* (*Sections, Straight ANOVA, ANOVA tests*) of the *STATISTICA* program was used. On the basis of the p-significance level values greater than 0.05 in both tests, the homogeneity of the variance (at the significance level of 0.05) was shown for all the parameters tested.

The coefficients β of the sensitivity model (3) in the whole space variability of the input were determined using the least squares method. The module *Industrial statistics* (*Experimental planning, bivalent plans, ANOVA, effects*) of the *STATISTICA* program was used.

For the estimated coefficients β of the sensitivity model (3), the indexes of partial sensitivity S_i and total sensitivity S_{Ti} were determined. The partial sensitivity S_i for each of the five inputs x_i was calculated as a partial derivative of the regression function (3) against this variable:

$$S_i = \frac{\partial z}{\partial x_i} \tag{4}$$

The total impact of the single factors and their combined impacts on the output in a given zone of the variation space was evaluated by the total sensitivity index S_{Ti}, defined for the variable x_i as:

$$S_{Ti} = |\beta_i| + \sum_{i \neq j} |\beta_{ij}| + \sum_{\substack{i \neq j \\ j \neq k}} \sum_{i \neq j} |\beta_{ijk}| + \cdots \tag{5}$$

The obtained functions (3) approximating the empirical data, containing components that are products of three input variables, were subjected to the further statistical analysis in order to determine the final form of the sensitivity model of the dependent variables. The verification of the significance of the structural parameters for the regression model was performed by analysing the inaccuracy of the measurements using the Student's t-test. The value of the empirical statistics $t(\beta) = |\beta|/S(\beta)$ was determined, comparing it with the critical value $t_{p,f}$, which is depending on the number of the degrees of freedom f at the significance level p = 0.05. The stating of the inequalities $t(\beta) > t_{p,f}$ means the significance of the model coefficient. For evaluation of the inaccuracy, the results of repeat measurements for one selected system in the center of the multi section quasi-uniform plan were used. For this system, r = 7 measurements were made, specifying the standard deviation S (z_i) being a measure of the dispersion for the results of the individual measurements. In the quantitative assessment of the changes in the results of measurements for the output z_i, apart from the inaccuracy of the test object, the effect of the inaccuracy of the applied methods and measuring means was taken into account, determining the total measurement uncertainty in u(z_i). Next, the standard deviations of the polynomial coefficients $S(\beta) = u(z_i)/\sqrt{N}$ were calculated. On the basis of a comparison of the statistics values $t = t(\beta)$ and the critical value $t_p = 2{,}4469$ (with the number of degrees of freedom $f = f_1 = r - 1 = 6$), the significance of the main effects of all model factors {n, λ, p_{ba}, T_{ba}, p_{g1}} of the exhaust temperature sensitivity T_t was found. In the model of the sensitivity of the volumetric efficiencyη_v, the impact of λ factor was irrelevant, and in the case of the engine thermal efficiencymodel η_c – the exhaust pressure p_{g1}. In the statistical assessment of the sensitivity model of the considered dependent variables, the significance of the influence of some interaction elements at significance level 0.05 was stated. The final results of the statistical analysis of the model for the sensitivity of the volumetric efficiency η_v, engine thermal efficiency ηc and exhaust gas temperature T_t for standardized values of the factors, containing the relevant components, are presented in tables 2-4.

Table 2. Statistically significant (at p = 0.05) regression's coefficients of the engine thermal efficiency (η_c) for sensitivity model, determined taking into account the measurement uncertainty

	Oceny efektów ; Zmn.: η_c; R^2= ,99364;Popr:,99178 (model wrażliwości etac-dane pomiarowe) 5 wielkości dla 2 wart.; Resztowy MS=,0000027 ZZ η_c									
Wejśc.	Efekt	Błąd std	t(24)	p	-95,% Gran.ufn	+95,% Gran.ufn	Wsp.	Błąd std Wsp.	-95,% Gran.ufn	+95,% Gran.ufn
Średn./Stała	0,474610	0,000293	1619,032	0,000000	0,474005	0,475215	0,474610	0,000293	0,474005	0,475215
(1)n	0,018141	0,000589	30,797	0,000000	0,016926	0,019357	0,009071	0,000295	0,008463	0,009679
(2)λ	0,025643	0,000583	43,992	0,000000	0,024440	0,026846	0,012822	0,000291	0,012220	0,013423
(3)p_{ba}	0,009985	0,000573	17,419	0,000000	0,008802	0,011168	0,004992	0,000287	0,004401	0,005584
(4)T_{ba}	-0,013132	0,000542	-24,210	0,000000	-0,014252	-0,012013	-0,006566	0,000271	-0,007126	-0,006006
1 wz.2	0,003748	0,000586	6,395	0,000001	0,002538	0,004957	0,001874	0,000293	0,001269	0,002479
1 wz.3	0,002430	0,000563	4,313	0,000238	0,001267	0,003593	0,001215	0,000282	0,000634	0,001796
2 wz.4	-0,002703	0,000542	-4,983	0,000043	-0,003823	-0,001584	-0,001352	0,000271	-0,001911	-0,000792

The verification of the adequacy for the regression function of the sensitivity model in relation to the measurement results was carried out using the Fischer-Senecord test, calculating the statistics $F = S^2(z_i)_a / S^2(z_i)$. The variance of the inaccuracy $S^2(z_i)$ was specified based on the calculation of the total measurement uncertainty u(z_i) determined on the basis of the measurement repetitions for the chosen experiment system and the inaccuracy of the measurement technique, assuming $S^2(z_i) \approx u^2(z_i)$. The variance of the adequacy (residual) $S^2(z_i)_a = SQ_a/f_a$ was determined with the degrees of freedom $f_2 = f_a = N - N_b$ (defined by the number of systems N = 32 and the number of polynomial coefficients N_b), calculating for each system the sum squared deviations SQ_a for the values measured and calculated from the regression function. The comparison of the F statistic values with the critical value F_p at p = 0.05 helps to state the occurrence of the relation $F < F_p$, which means the adequacy of the sensitivity model function for the dependent variables.

Table 3. Statistically significant (at p = 0.05) regression's coefficients of the volumetric efficiency (η_v) for sensitivity model, determined taking into account the measurement uncertainty

Oceny efektów ; Zmn.:η_v; R^2= ,99542;Popr:,99408 (model wrażliwości etav-dane pomiar)
5 wielkości dla 2 wart.; Resztowy MS=,0000076
ZZ η_v

Wejśc.	Efekt	Błąd std	t(24)	p	-95,% Gran.ufn	+95,% Gran.ufn	Wsp.	Błąd std Wsp.	-95,% Gran.ufn	+95,% Gran.ufn
Średn./Stała	0,894109	0,000490	1825,685	0,000000	0,893098	0,895120	0,894109	0,000490	0,893098	0,895120
(1)n	-0,023524	0,000985	-23,887	0,000000	-0,025557	-0,021492	-0,011762	0,000492	-0,012778	-0,010746
(3)p_{ba}	0,040368	0,000951	42,431	0,000000	0,038405	0,042332	0,020184	0,000476	0,019202	0,021166
(4)T_{ba}	0,026033	0,000912	28,540	0,000000	0,024150	0,027915	0,013016	0,000456	0,012075	0,013958
(5)p_{g1}	-0,033465	0,000965	-34,690	0,000000	-0,035456	-0,031474	-0,016732	0,000482	-0,017728	-0,015737
1 wz.3	-0,007063	0,000943	-7,487	0,000000	-0,009010	-0,005116	-0,003532	0,000472	-0,004505	-0,002558
1 wz.5	0,007282	0,000976	7,462	0,000000	0,005268	0,009296	0,003641	0,000488	0,002634	0,004648
3 wz.5	-0,007074	0,000920	-7,689	0,000000	-0,008972	-0,005175	-0,003537	0,000460	-0,004486	-0,002587

Table 4. Statistically significant (at p = 0.05) regression's coefficients of the flue gas temperature (T_t) for sensitivity model, determined taking into account the measurement uncertainty

Oceny efektów ; Zmn.:T_t; R^2= ,99922;Popr:,99899 (model wrażliwości Tt-dane pomiarowe)
5 wielkości dla 2 wart.; Resztowy MS=6,704093
ZZ T_t

Wejśc.	Efekt	Błąd std	t(24)	p	-95,% Gran.ufn	+95,% Gran.ufn	Wsp.	Błąd std Wsp.	-95,% Gran.ufn	+95,% Gran.ufn
Średn./Stała	814,103	0,457999	1777,522	0,000000	813,158	815,048	814,1029	0,457999	813,1577	815,0482
(1)n	52,527	0,920518	57,063	0,000000	50,627	54,427	26,2636	0,460259	25,3136	27,2135
(2)λ	-140,205	0,911673	-153,789	0,000000	-142,087	-138,324	-70,1026	0,455836	-71,0434	-69,1618
(3)p_{ba}	-46,557	0,890380	-52,289	0,000000	-48,395	-44,720	-23,2787	0,445190	-24,1975	-22,3598
(4)T_{ba}	29,706	0,855064	34,741	0,000000	27,941	31,471	14,8529	0,427532	13,9705	15,7353
(5)p_{g1}	12,982	0,900379	14,419	0,000000	11,124	14,840	6,4911	0,450189	5,5620	7,4202
1 wz.2	-10,773	0,917191	-11,746	0,000000	-12,666	-8,880	-5,3866	0,458595	-6,3331	-4,4401
2 wz.3	7,168	0,888007	8,073	0,000000	5,336	9,001	3,5842	0,444003	2,6678	4,5006

For the obtained form of the function for the sensitivity model of the dependent variables (tables 2-4), the partial sensitivity indexes were calculated and, after taking into account interactive elements, the total sensitivity indicators were calculated also. The total sensitivity model was evaluated in this way. The results of the calculations for the sensitivity analysis of the dependent variables z_i for statistically significant factor effects are presented in Table 5. In order to compare the impact of each factor more easily on the dependent variable, the partial and total sensitivity indicators were also expressed in relative values in the relation to the calculated maximum absolute value of the indicator. The graphic interpretation of the analysis results obtained with this approach is illustrated by figures 1 and 2.

The substantive analysis of the functions (3) determined for each of the set parameters Z $\{z_1, z_2, z_3\}$ confirmed the conformity of the main effects assessment and the interaction between the tested variables and the actual mechanisms for the influence of the factors on the tested parameters of the motor cycle. Obtaining such compliance confirms the adequacy of the engine model and its suitability for the further research.

Table 5. The values for partial and total sensitivity indicators

	Sensitivity indicators (factors)	exhaust gas temperature T_t		engine thermal efficiency η_c		volumetric efficiency η_v	
		value	%	value	%	value	%
Partial	S_1 (n)	26,2636	0,37	0,0091	0,71	0,0118	0,58
	S_2 (λ)	70,1026	1,00	0,0128	1,00	–	–
	S_3 (p_{ba})	23,2787	0,33	0,0050	0,39	0,0202	1,00
	S_4 (T_{ba})	14,8529	0,21	0,0066	0,51	0,0130	0,64
	S_5 (p_{g1})	6,4911	0,09	–	–	0,0167	0,83
	S_{12} (n – λ)	5,3866	0,08	0,0019	0,15	–	–
	S_{13} (n – p_{ba})	–	–	0,0012	0,09	0,0035	0,17
	S_{15} (n – p_{g1})	–	–	–	–	0,0036	0,18
	S_{23} (λ – p_{ba})	3,5842	0,05	–	–	–	–
	S_{24} (λ – T_{ba})	–	–	0,0014	0,11	–	–
	S_{35} (p_{ba} – p_{g1})	–	–	–	–	0,0035	0,18
Total	S_{T1} (n)	31,6501	0,40	0,0122	0,76	0,0189	0,69
	S_{T2} (λ)	79,0734	1,00	0,0160	1,00	–	–
	S_{T3} (p_{ba})	26,8629	0,34	0,0062	0,39	0,0272	1,00
	S_{T4} (T_{ba})	14,8529	0,19	0,0079	0,49	0,0130	0,48
	S_{T5} (p_{g1})	6,4911	0,08	–	–	0,0239	0,88

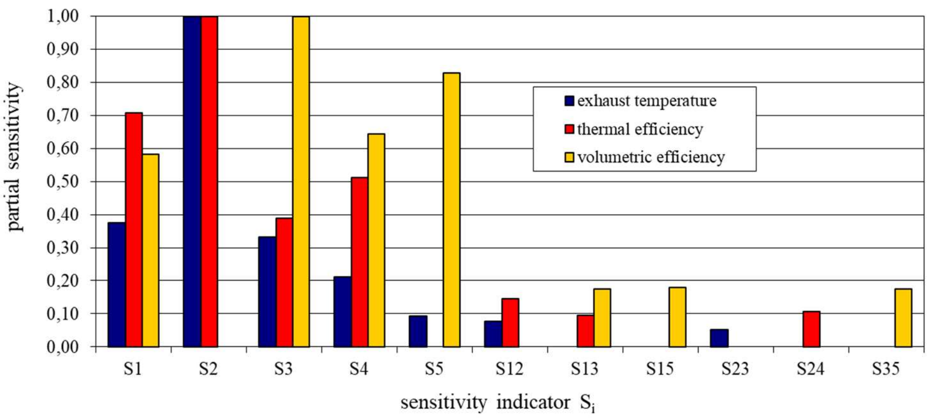

Figure 1. The indicators of the partial sensitivity S_i for all factors including interaction

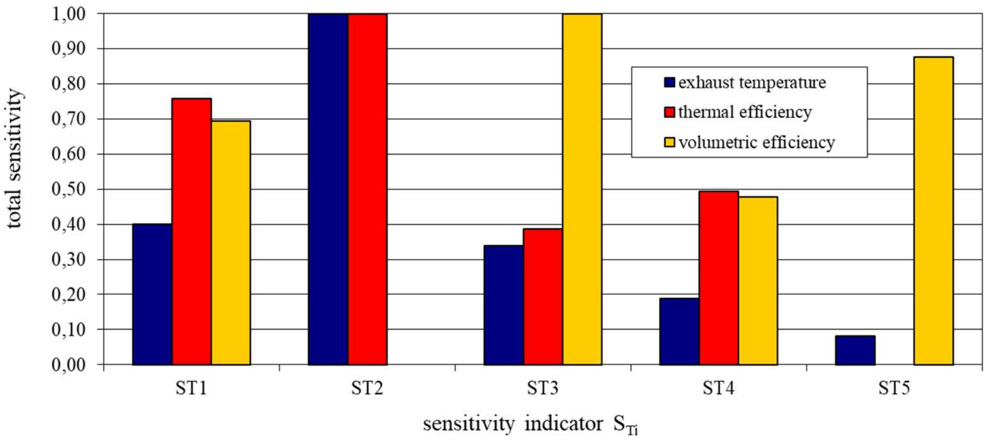

Figure 2. The indicators of the total sensitivity S_{Ti}

2.3. Functions of the engine model

Based on the verification of the correlation among the adopted descriptive variables (see table 5), the engine model was defined through a set of functions for four variables:

$$\eta_c = F_1(n, \lambda, p_{ba}, T_{ba}) \tag{6}$$

$$\eta_v = F_2(n, p_{ba}, T_{ba}, p_{g1}) \tag{7}$$

$$T_t = F_3(n, \lambda, p_{ba}, T_{ba}) \tag{8}$$

The results of the measurements were approximated with the aid of the second degree polynomials with the interactive elements, taking into account the influence of the combined effect of first-order factors, with the general form:

$$\hat{z} = b_o + \sum_{i=1}^{m} b_i x_i + \sum_{i=1}^{m} b_{ii} x_i^2 + \sum_{i=1}^{m} \sum_{j=i+1}^{m} b_{ij} x_i x_j \tag{9}$$

The verification of the regression's function was performed on the normalized values $\hat{x}_i$ for the motor function in the form:

$$z = a_o + \sum_{i=1}^{m} a_i \hat{x}_i + \sum_{i=1}^{m} a_{ii} \hat{x}_i^2 + \sum_{i=1}^{m} \sum_{j=i+1}^{m} a_{ij} \hat{x}_i \hat{x}_j \tag{10}$$

The polynomial coefficients a_o, a_i, a_{ii}, a_{ij} (10) were determined using the multidimensional regression methods. The statistical analysis establishing the searched values for coefficients and the final form of the function approximating the dependencies (6-8) were performed using the *STATISTICA* program. The module *Industrial Statistics* was used (*Experiment planning, Composite central plans (response surface), ANOVA, effects*). The inaccuracy of the engine model functions obtained in this way was specified similar as in the case of the elimination tests, taking into account the total measurement uncertainties determined on the basis of the additional measurements for the selected plan layout. The verification of the coefficients' significance was carried out with the Student's t-test, specifying the appropriate statistic values. The standard deviations of the coefficients for the adopted significance level p = 0.05 and the number of degrees of freedom $f = f_1$ were calculated. The results of the coefficient significance test are presented in tables 6-8.

The adequacy of the obtained model functions to the measurement results was checked in the previously described way, determining the values of F Fisher-Senecord's statistics at the significance level p = 0.05 and the number of degrees of freedom $f = f_2$ defined by the number of the significant coefficients of the dependent variables for the model function. The obtained features are characterized by a very high coefficient of a determination R^2 (match factor) of 0.989 for the engine thermal efficiency function η_c, 0.988 for the volumetric efficiency function η_v and 0.99 for the exhaust gas temperature function T_t, which allows to assume that the functions explain almost 100% of the variability of the dependent variables.

Table 6. The final results of the regression analysis for the engine thermal efficiency function η_c after removal of non-significant coefficient.

	Oceny efektów ; Zmn.: η_c; R^2= ,99442;Popr:,9897 (dane do identyfikacji funkcji etac) 4 wielk. , 1 Bloki , 25 ukła; Resztowy MS=,0000046 ZZ η_c									
Wejśc.	Efekt	Błąd std	t(13)	p	-95,% Gran.ufn	+95,% Gran.ufn	Wsp.	Błąd std Wsp.	-95,% Gran.ufn	+95,% Gran.ufn
Średn./Stała	0,475124	0,001228	386,9180	0,000000	0,472471	0,477776	0,475124	0,001228	0,472471	0,477776
(1)n(L)	0,042758	0,001981	21,5861	0,000000	0,038478	0,047037	0,021379	0,000990	0,019239	0,023518
n(Q)	-0,029894	0,004182	-7,1475	0,000008	-0,038930	-0,020859	-0,014947	0,002091	-0,019465	-0,010429
(2)λ(L)	0,066429	0,001939	34,2534	0,000000	0,062240	0,070619	0,033215	0,000970	0,031120	0,035310
λ(Q)	0,013534	0,004035	3,3542	0,005179	0,004817	0,022251	0,006767	0,002017	0,002408	0,011125
(3)p_{ba}(L)	0,023608	0,002133	11,0659	0,000000	0,018999	0,028218	0,011804	0,001067	0,009500	0,014109
p_{ba}(Q)	0,018201	0,004537	4,0116	0,001479	0,008399	0,028003	0,009101	0,002269	0,004200	0,014002
(4)T_{ba}(L)	-0,031497	0,001831	-17,2027	0,000000	-0,035452	-0,027541	-0,015748	0,000915	-0,017726	-0,013771
T_{ba}(Q)	-0,009902	0,003061	-3,2347	0,006517	-0,016515	-0,003289	-0,004951	0,001531	-0,008257	-0,001644
1L wz.2L	0,020897	0,006127	3,4106	0,004647	0,007660	0,034135	0,010449	0,003064	0,003830	0,017067
1L wz.3L	0,015090	0,005852	2,5787	0,022914	0,002448	0,027733	0,007545	0,002926	0,001224	0,013866
2L wz.4L	-0,014072	0,005242	-2,6846	0,018735	-0,025397	-0,002748	-0,007036	0,002621	-0,012698	-0,001374

Table 7. The final results of the regression analysis for the volumetric efficiency function η_v after removal of non-significant coefficient.

	Oceny efektów ; Zmn.: η_v; R^2= ,99347;Popr:,98881 (dane do identyfikacji funkcji etav) 4 wielk. , 1 Bloki , 25 ukła; Resztowy MS=,0000133 ZZ η_v									
Wejśc.	Efekt	Błąd std	t(14)	p	-95,% Gran.ufn	+95,% Gran.ufn	Wsp.	Błąd std Wsp.	-95,% Gran.ufn	+95,% Gran.ufn
Średn./Stała	0,888854	0,001737	511,8347	0,000000	0,885129	0,892578	0,888854	0,001737	0,885129	0,892578
(1)n(L)	-0,043466	0,003381	-12,8566	0,000000	-0,050718	-0,036215	-0,021733	0,001690	-0,025359	-0,018108
n(Q)	-0,038889	0,006571	-5,9186	0,000037	-0,052981	-0,024796	-0,019444	0,003285	-0,026491	-0,012398
(2)p_{ba}(L)	0,088414	0,003768	23,4621	0,000000	0,080332	0,096496	0,044207	0,001884	0,040166	0,048248
p_{ba}(Q)	0,035861	0,008643	4,1491	0,000983	0,017323	0,054398	0,017930	0,004321	0,008662	0,027199
(3)T_{ba}(L)	0,055510	0,003094	17,9382	0,000000	0,048873	0,062147	0,027755	0,001547	0,024436	0,031073
(4)p_{g1}(L)	-0,073448	0,003729	-19,6969	0,000000	-0,081446	-0,065450	-0,036724	0,001864	-0,040723	-0,032725
p_{g1}(Q)	0,032692	0,007867	4,1555	0,000971	0,015819	0,049566	0,016346	0,003934	0,007909	0,024783
1L wz.2L	-0,028596	0,010074	-2,8387	0,013140	-0,050201	-0,006990	-0,014298	0,005037	-0,025101	-0,003495
1L wz.4L	0,040327	0,010472	3,8509	0,001764	0,017867	0,062788	0,020164	0,005236	0,008933	0,031394
2L wz.4L	-0,048785	0,010037	-4,8603	0,000252	-0,070312	-0,027257	-0,024392	0,005019	-0,035156	-0,013628

Table 8. The final results of the regression analysis for the exhaust gas temperature function T_t after removal of non-significant coefficient.

	Oceny efektów ; Zmn.: T_t; R^2= ,99698;Popr:,99517 (dane do identyfikacji modelu Tt) 4 wielk. , 1 Bloki , 25 ukła; Resztowy MS=33,02374 ZZ T_t									
Wejśc.	Efekt	Błąd std	t(15)	p	-95,% Gran.ufn	+95,% Gran.ufn	Wsp.	Błąd std Wsp.	-95,% Gran.ufn	+95,% Gran.ufn
Średn./Stała	782,201	2,64438	295,7973	0,000000	776,565	787,838	782,201	2,644382	776,565	787,838
(1)n(L)	116,435	5,27625	22,0677	0,000000	105,189	127,681	58,217	2,638125	52,594	63,840
n(Q)	174,279	10,31534	16,8951	0,000000	152,292	196,265	87,139	5,157669	76,146	98,133
(2)λ(L)	-331,172	5,29537	-62,5399	0,000000	-342,459	-319,885	-165,586	2,647685	-171,229	-159,943
λ(Q)	126,538	9,83209	12,8699	0,000000	105,581	147,494	63,269	4,916046	52,791	73,747
(3)p_{ba}(L)	-82,928	5,55940	-14,9166	0,000000	-94,777	-71,078	-41,464	2,779700	-47,389	-35,539
p_{ba}(Q)	53,248	12,03814	4,4232	0,000493	27,589	78,906	26,624	6,019070	13,794	39,453
(4)T_{ba}(L)	62,011	5,12594	12,0974	0,000000	51,085	72,936	31,005	2,562969	25,542	36,468
1L wz.2L	-66,433	16,92917	-3,9242	0,001353	-102,516	-30,349	-33,216	8,464583	-51,258	-15,175
2L wz.3L	49,418	16,11031	3,0675	0,007819	15,080	83,757	24,709	8,055156	7,540	41,878

Finally, the regression equations describing functional relations (6-8) of the SW 680 engine have the form:

- engine thermal efficiency:

$$\eta_c = 0{,}475124 + 0{,}021379\hat{x}_1 + 0{,}033215\hat{x}_2 + 0{,}011804\hat{x}_3 - 0{,}015748\hat{x}_4 - 0{,}014947\hat{x}_1^2 + 0{,}006767\hat{x}_2^2 + 0{,}009101\hat{x}_3^2 - 0{,}004951\hat{x}_4^2 + 0{,}010449\hat{x}_1\hat{x}_2 + 0{,}007545\hat{x}_1\hat{x}_3 - 0{,}007036\hat{x}_2\hat{x}_4 \quad (11)$$

– volumetric efficiency:

$$\eta_v = 0{,}888854 - 0{,}021733\hat{x}_1 + 0{,}044207\hat{x}_3 + 0{,}027755\hat{x}_4 - 0{,}036724\hat{x}_5 - 0{,}019444\hat{x}_1^2 + 0{,}017930\hat{x}_3^2 + 0{,}016346\hat{x}_5^2 - 0{,}014298\hat{x}_1\hat{x}_3 + 0{,}020164\hat{x}_1\hat{x}_5 - 0{,}024392\hat{x}_3\hat{x}_5 \quad (12)$$

– exhaust gas temperature:

$$T_t = 782{,}201 + 58{,}217\hat{x}_1 - 165{,}586\hat{x}_2 - 41{,}464\hat{x}_3 + 31{,}005\hat{x}_4 + 87{,}139\hat{x}_1^2 + 63{,}269\hat{x}_2^2 + 26{,}624\hat{x}_3^2 - 33{,}216\hat{x}_1\hat{x}_2 + 24{,}709\hat{x}_1\hat{x}_2 \quad (13)$$

Taking into account the normalization relations (2), for the determined regression equations (11-13), the following relationships are applicable:

$$\hat{x}_1 = \frac{n-16}{600};\ \hat{x}_2 = \frac{\lambda-2}{0{,}7};\ \hat{x}_3 = \frac{p_{ba}-0{,}14}{0{,}035};\ \hat{x}_4 = \frac{T_{ba}-35}{30};\ \hat{x}_5 = \frac{p_{g1}-0{,}15}{0{,}04} \quad (14)$$

The differences between the measurement results of η_c, η_v and T_t parameters and the values calculated from the approximation functions were determined. However, in the case of the engine thermal efficiency η_c the maximum difference exceeds 5%. In the case of the other two parameters η_v and T_t, the difference does not exceed 2%.

3. Conclusion

The experimental results presented above indicate a high efficiency of the experimental design methods, especially with a large number of the factors examined. It is also important to plan the experiment appropriately. In the presented example, the results of the measurements, made during engine identification tests according to the multi section quasi-uniform plan, were used later in elimination tests to assess the significance of the factors. The verification of the correlation among the adopted descriptive variables was carried out using the sensitivity analysis. The results of the analysis enable to select statistically the significant factors which have the greatest impact on the engine performance parameters. The significance of these factors was verified by means of the appropriate statistics based on the analysis of the inaccuracies' measurement, whose results enable to consider only those factors whose impact is greater than the measurement uncertainties. On the basis of the results of the accuracy test for the motor parameters approximation, a good matching of the characteristics with the use of second degree polynomials with the iterative elements was stated.

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