

Design of a digital controller for linear plant based on model following control system

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Abstract. In this paper, design of a digital controller based on model-following system is proposed for single-input-single-output (SISO) system. The controller needs a design method which could easily be done so the plant responses will follow the model responses. The digital controller could be designed for use when the orders of the model and the plant are same. As a result, a sample of simulation was showed that the proposed method was stable to plant parameter variations.

1. Introduction

A model in a second-order system or more in a control system must have good performance characteristics, determined by the damping factor and the natural frequency of the system.

A plant may not have a time response specification such as the model; therefore it is necessary to do a digital controller design to fulfill the specifications such as the model.

In order to satisfy both statements above, reviewed a control system diagram as shown in figure 1.

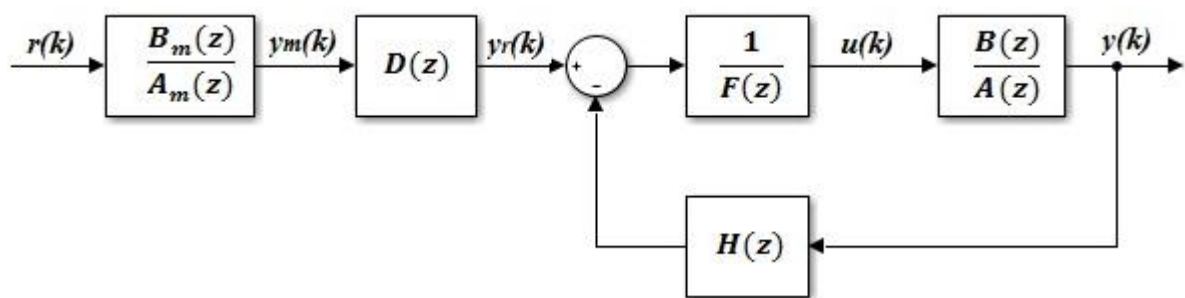


Figure 1. A digital compensation on a plant with model output as input to the system

From Figure 1, was defined a model which has dynamic equations and transfer functions such as equations (1) and (2) is given below:

$$A_m(z)y_m(k) = B_m(z)r(k) \quad (1)$$

$$G_m(z) = \frac{B_m(z)}{A_m(z)} \quad (2)$$

Where $r(k)$ is the input to the model, $y_m(k)$ is output of the model.

It also was defined a plant that having dynamic equations and transfer functions such as equations (3) and (4) given below:

$$A(z)y(k) = B(z)y(k) \quad (3)$$

$$G(z) = \frac{B(z)}{A(z)} \quad (4)$$

If it is desired that the response of the plant $y(k)$ may follow the response behavior of the model $y_m(k)$, then a controller is required to compensate the plant. The controller used two filters of filter-1 and filter-2, with the dynamic equations of both filters as shown below:

$$F(z) u(k) = y_r(k) - H(z)y(k) \quad (5)$$

The filter-1 has a dynamic $1/F(z)$ and filter-2 has a dynamic $H(z)$, where $u(k)$ is the controller, $y_r(k)$ is the input to the controller, and $y(k)$ is output of the plant. Furthermore, if it is desired that $F(z)$ and $H(z)$ in equation (5) are capable of giving control signals to the plant to stabilize the plant, then to determine the dynamics of both filters $F(z)$ and $H(z)$, it was needed the filter-3 with the dynamic $D(z)$ as shown in figure 1.

2. The Controller Design

In order to determine $F(z)$ and $H(z)$ of equation (5), several steps in the design are described as follows:

- a. First, multiply $F(z)$ on both sides of equation (3) and it will find equation (6) as follows:

$$A(z) F(z)y(k) = B(z) F(z)y(k) \quad (6)$$

- b. Then the form $F(z) u(k)$ will be eliminated in both equations (5) and (6), so the equation (7) will be found as follows:

$$A(z) F(z)y(k) = B(z) [y_r(k) - H(z)y(k)] \quad (7)$$

This equation (7) can be simplified in the equation (8) below:

$$[A(z) F(z) + B(z) H(z)] y(k) = B(z) y_r(k) \quad (8)$$

- c. Furthermore the input to the controller is obtained by the following equation (9):

$$y_r(k) = D(z)y_m(k) \quad (9)$$

- d. The next step, the $y_r(k)$ from the equation (9) is substituted into equation (7), then equation (10) has found as the following below:

$$[A(z) F(z) + B(z) H(z)] y(k) = B(z) D(z) y_m \quad (10)$$

- e. As a final result, if it is desired that the response of $y(k)$ approaches equally or follows the response of $y_m(k)$ or $y(k) \approx y_m(k)$ in equation (10), then it can only be realized by making the statement in equation (11) below:

$$A(z) F(z) + B(z) H(z) = B(z) D(z) \quad (11)$$

If the response of the plant closes equally or follows the response of the model, then the system like this is called the model following control system.

- f. The next goal is to determine the parameters $F(z)$ and $H(z)$.

From equation (11), $A(z)$ and $B(z)$ are two known polynomials. $F(z)$ and $H(z)$ are two polynomials that must be found to form compensation on filter-1 and filter-2. Both polynomials can only be obtained analytically only if polynomial $D(z)$ has been determined first.

Assume that $B(z)$ is not included in the determination of $F(z)$ and $H(z)$, but only determined by $D(z)$. This is done to avoid system instability, when $B(z)$ has unstable poles. Therefore, the equation (11) is transformed into the following equation (12):

$$A(z)F(z) + B(z)H(z) = D(z) \quad (12)$$

If $A(z)$ is defined as having a polynomial of degree n , and $B(z)$ has a polynomial of degree $n-1$, and it is also defined that $F(z)$ and $H(z)$ have the same polynomial of degree $n-1$, then the degree of polynomial $D(z)$ is equal to $2n-1$. All the polynomials are written as the following equation (13) to (17).

$$A(z) = z^n + a_{n-1} z^{n-1} + a_{n-2} z^{n-2} + \dots + a_1 z + a_0 \quad (13)$$

$$B(z) = b_{n-1} z^{n-1} + b_{n-2} z^{n-2} + \dots + b_1 z + b_0 \quad (14)$$

$$F(z) = z^{n-1} + f_{n-2} z^{n-2} + \dots + f_1 z + f_0 \quad (15)$$

$$H(z) = h_{n-1} z^{n-1} + h_{n-2} z^{n-2} + \dots + h_1 z + h_0 \quad (16)$$

$$D(z) = z^{2n-1} + d_{2n-2} z^{2n-2} + \dots + d_1 z + d_0 \quad (17)$$

If equation (13) to (17) are substituted to equation (12), then the parameter of the both polynomial $F(z)$ and $H(z)$ can be obtained and its arranged as shown in equation (18).

$$\begin{bmatrix} 1 & 0 & \dots & \dots & 0 & 0 & b_{n-1} & 0 & \dots & \dots & \dots & 0 & 0 \\ a_{n-1} & 1 & \dots & \dots & 0 & 0 & b_{n-2} & b_{n-1} & \dots & \dots & \dots & 0 & 0 \\ a_{n-2} & a_{n-1} & \dots & \dots & 0 & 0 & b_{n-3} & b_{n-2} & \dots & \dots & \dots & 0 & 0 \\ a_{n-3} & a_{n-2} & \dots & \dots & 0 & 0 & b_{n-4} & b_{n-3} & \dots & \dots & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & 1 & 0 & \dots & \dots & \dots & \dots & \dots & 0 & 0 \\ a_2 & 1 & \dots & \dots & a_{n-1} & 1 & b_1 & 0 & \dots & \dots & \dots & b_{n-1} & 0 \\ a_1 & 1 & \dots & \dots & \dots & a_{n-1} & b_0 & b_1 & \dots & \dots & \dots & b_{n-1} & \dots \\ a_0 & 1 & \dots & \dots & \dots & a_{n-2} & 0 & b_0 & \dots & \dots & \dots & \dots & \dots \\ 0 & 1 & \dots & \dots & \dots & 0 & 0 & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 1 & \dots & \dots & a_1 & a_2 & 0 & 0 & \dots & \dots & \dots & b_1 & b_2 \\ 0 & 1 & \dots & \dots & a_0 & a_1 & 0 & 0 & \dots & \dots & \dots & b_0 & b_1 \\ 0 & 1 & \dots & \dots & 0 & a_0 & 0 & 0 & \dots & \dots & \dots & 0 & b_0 \end{bmatrix} \begin{bmatrix} f_{n-2} \\ f_{n-3} \\ \dots \\ f_1 \\ f_0 \\ h_{n-2} \\ h_{n-3} \\ \dots \\ h_2 \\ h_1 \\ h_0 \end{bmatrix} = \begin{bmatrix} d_{2n-2} - a_{n-1} \\ d_{2n-3} - a_{n-2} \\ \dots \\ \dots \\ \dots \\ d_{n-2} \\ \dots \\ d_2 \\ d_1 \\ d_0 \end{bmatrix} \quad (18)$$

3. The Design Example

It is known that a servo position control system has dynamics in equation (19). Plant has degree $n = 3$.

$$\begin{aligned} B(z) &= 0.0018 z^2 + 0.0062 z + 0.0014 \\ A(z) &= z^3 - 2.2291 z^2 + 1.8356 z + 0.6065 \end{aligned} \quad (19)$$

The desired model has the dynamic as described in equation (20) below:

$$\begin{aligned} B_m(z) &= (0.0677 z + 0.0577)(z + 0.8) \\ A_m(z) &= (z^2 - 1.4934 z + 0.6188)(z + 0.8) \end{aligned} \quad (20)$$

The filter-3 has the dynamic as described in equation (21) below:

$$D(z) = (z^2 - 1.4934 z + 0.6188)(z + 0.8)^3 \quad (21)$$

For $n = 3$, by using equation (15) and (16) it chooses $F(z) = z^2 + f_1 z + f_0$ and $H(z) = h_2 z^2 + h_1 z + h_0$. Insert all dynamics from equation (19) and (21) into equation (12), it can be formulated in equation (22).

$$(z^3 + a_2 z^2 + a_1 z + a_0)(z^2 + f_1 z + f_0) + (b_2 z^2 + b_1 z + b_0)(h_2 z^2 + h_1 z + h_0) = z^5 + d_4 z^4 + d_3 z^3 + d_2 z^2 + d_1 z + d_0 \quad (22)$$

By using analytical method, it found equation (23) below:

$$\begin{aligned} f_1 + a_2 + b_2 h_2 &= d_4 \\ a_2 f_1 + f_0 + a_1 + b_2 h_1 + b_1 h_2 &= d_3 \\ a_2 f_0 + a_1 f_1 + a_0 + b_2 h_0 + b_1 h_1 + b_0 h_2 &= d_2 \\ a_1 f_0 + a_0 f_1 + b_1 h_0 + b_0 h_1 &= d_1 \\ a_0 f_0 + b_0 h_0 &= d_0 \end{aligned} \quad (23)$$

Equation (23) also can be solved in matrix form using equation (24) below:

$$\begin{bmatrix} 1 & 0 & b_2 & 0 & 0 \\ a_2 & 1 & b_1 & b_2 & 0 \\ a_1 & a_2 & b_0 & b_1 & b_2 \\ a_0 & a_1 & 0 & b_0 & b_1 \\ 0 & a_0 & 0 & 0 & b_0 \end{bmatrix} \begin{bmatrix} f_1 \\ f_0 \\ h_2 \\ h_1 \\ h_0 \end{bmatrix} = \begin{bmatrix} d_4 - a_4 \\ d_3 - a_3 \\ d_2 - a_2 \\ d_1 \\ d_0 \end{bmatrix} \quad (24)$$

From equation (24) using matrix inverse, it found that $F(z)$ and $H(z)$ has equation (25) below:

$$\begin{aligned} F(z) &= z^2 - 1.6762 z + 0.5189 \\ H(z) &= 6.6163 z^2 - 4.9955 z - 1.5141 \end{aligned} \quad (25)$$

To see the system response using the three filters, the simulation is done using matlab-simulink as shown in Figure 2. There are some additional blocks also used for simulation.

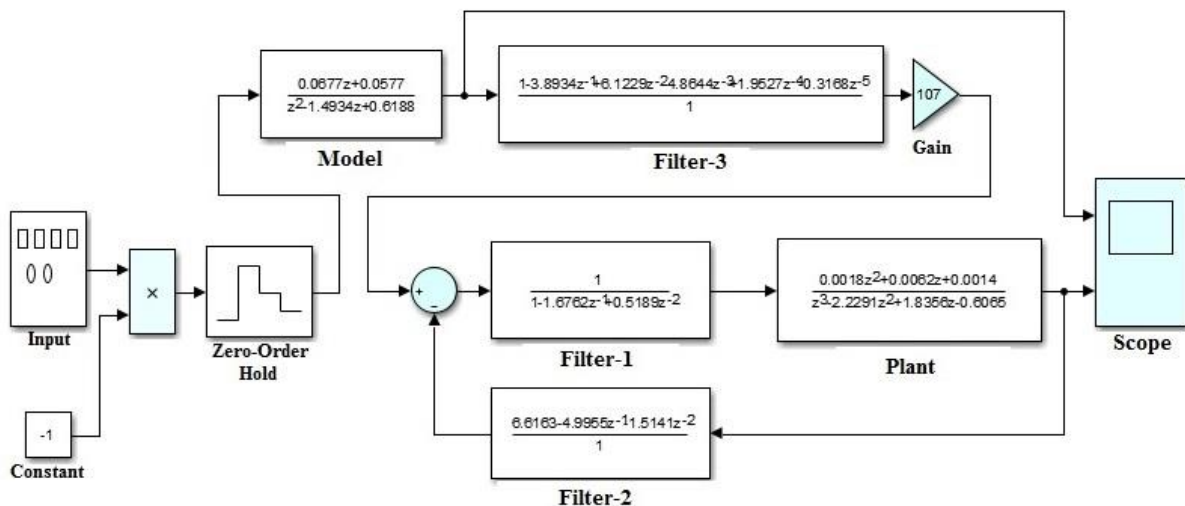


Figure 2. A sample of digital compensation using model following control system



Figure 3. The time responses of the model and plant

After the simulation is executed, the response is obtained as shown in figure 3. The plant response (blue color) follows the model response (yellow color).

4. Conclusion

A digital controller formula based on model-following system has been designed for (SISO) system, as shown on equation (5) to (18). In a sample on simulation on equation (19) to (25) was showed using simulink diagram on figure 2. The response of the plant can be controlled to follow the response of the model. The amplitude of plant can be settled using the gain that placed after filter-3 as shown on figure 2.

5. References

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