

Combining the use of analytical hierarchy process and lexicographic goal programming in selecting project executor

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Abstract. The aim of this study was to determine selected project executor based on multicriteria decision making. Analytic Hierarchy Process (AHP) is applied for selecting chosen project. The global priorities were obtained based on relation criterias respect to alternatives in order to select the best one. Next, a new approach for solving optimization of chosen executor project is developed that is lexicographic goal programming by zero one method. By an illustrative example, that is found the method is efficient in reaching solution. Final result showed the Ltd. A is the best chosen project. For each criteria this model enabled the decision maker to choose the optimal solution.

1. Introduction

Indonesia is a developing country. By the growth of economic, infrastructure for public being a main tool to support economical society activities. The building of road is one of the most important supporting to make economic society works well. In order to get that situation we need project executor to handle.

Goal programming, one class of multiobjective optimization problems that relies the prior articulation of preferences in setting goals, is a framework that allows for an examination of such trade-offs while retaining the other advantages of multiobjective programming. Lexicographic goal programming simply permits an ordinal ranking of goals rather than a mathematical specification of weights [5]. Goal programming is a well-known modification and extension of linear programming, developed in the early 1960s owing to the study of [3]. Linear programming deals with only one single objective to be minimized or maximized, and subject to some constraint; it, therefore, has limitations in solving a problem with multiple objectives. Goal programming, instead, can be used as an effective approach to handle a decision concerning multiple and conflicting goals. Also, the objective function of a goal programming model may consist in non-homogeneous units of measure.

Lexicographic goal programming is actually one of the most significant devices in tackling multicriteria decision problems: the different goals can be ranked according to different priority levels that reflect the target allocated to them by the decision maker. The lexicographic approach defines different priority levels P_j for the goals of the analysis. The different priority levels reflect the hierarchical relationship between the targets in the objective function where they are arranged in order of decreasing priority ($P_1 > P_2 > \dots > P_m$) [2].



The AHP is a technique used for dealing with complex, unstructured problems that involve the consideration of multiple criteria simultaneously. It is based on three principles: hierarchization, priority-setting and logical consistency.

Hierarchization: this involves breaking down the problem into a homogenous set of components, and organizing them according to a hierarchy in order to incorporate significant quantities of information and present a more comprehensive portrait of the problem.

Priority setting: this involves pairing elements at each level of the hierarchy by assigning a weight to each element as described [11].

Logical consistency: since the various elements are paired using subjective means, such as judgment or opinion, the comparisons are not necessarily consistent. To remedy this, the AHP method uses a Consistency Index that enables to test the consistency of judgments systematically. For example: if A is considered five times preferable to B, and B is twice as preferable as C, then A must be considered 10 times preferable to C, otherwise the judgments are inconsistent.

The objective of this paper is to illustrate by application the use of lexicographic goal programming combine with AHP to examine selected project executor especially to build road. The specific context of the study is how to select the best project executor with its criteria.

2. Project Executor Selection by AHP method

In this paper, we will determine a goal or first level that is select an executor project, the second level is criteria. There are ten criterias such as Letter of Intent (LI), Quarantee of Offering (QO), Execution Method (EM), Due date of Execution (DE), Kind of Equipment (KE), Job Specification (JS), List of Staff Executif (LSE), Offering Budget (OB), Unbalance of Unit Price (UUP) and Price of Preference (PP) [based on quitionaring by the expert]. The third level is alternative, they are Company Ltd A, Ltd B, Ltd C, Ltd D, Ltd E.

The first step in AHP is to calculate the relative importance of different criteria. From the pair wise comparison matrix of criteria, then eigenvector method is applied to get the criteria weights.

Tabel 1. The criteria priority values

Criteria	Weights
Letter of Intent (LI)	0.039
Quarantee of Offering (QO)	0.027
Execution Method (EM)	0.141
Due date of Execution (DE)	0.106
Kind of Equipment (KE)	0.224
Job Specification (JS)	0.114
List of Staff Executif (LSE)	0.051
Offering Budget (OB)	0.024
Unbalance of Unit Price (UUP)	0.106
Price of Preference (PP)	0.167

After that, at this level the consistency index should be calculated. Perfect consistency rarely occurs in practice. Ratings should be consistent in two ways. First, ratings should be transitive. Second, ratings should be numerically consistent. To calculate the consistency ratio we must solve

$$Aw = \lambda_{max} \cdot w \text{ by solving } \det(\lambda I - A) = 0$$

and we get $\lambda_{max} = 11.039$. then the CI index is found by

$$CI = \frac{(\lambda_{maks}-n)}{n-1} = \frac{11.039-10}{9} = 0.115$$

The final step is to calculate the CR by using the table derived from Saaty's book $CR = CI/RI = 0.115/1.49 = 0.077$. CR value is less than 0.1 so the evaluation are consistent.

After computing the importance of criteria, the same methodology is applied to find the respective values for alternatives. But now, the alternatives should be pair-wised compared with respect to each criterion particularly.

Table 2. Pair-wise compared weights alternatives with respect to criteria

Alternatives	Criteria									
	LI	QO	EM	DE	KE	JS	LSE	OB	UUP	PP
Ltd.A	0.081	0.186	0.091	0.139	0.094	0.053	0.268	0.307	0.115	0.359
Ltd.B	0.114	0.301	0.305	0.214	0.203	0.472	0.263	0.428	0.261	0.181
Ltd.C	0.255	0.339	0.451	0.193	0.203	0.276	0.356	0.125	0.349	0.190
Ltd.D	0.249	0.130	0.074	0.164	0.288	0.089	0.059	0.071	0.195	0.212
Ltd.E	0.301	0.044	0.080	0.289	0.213	0.110	0.053	0.069	0.080	0.059

Similar to criterion calculation methodology, the eigen vector method is computed to get the alternatives scores. Hence the relative priority matrix given in table 3.

Table 3. Alternatives weight priority matrix

Alternatives	Weights
Ltd. A	0.141
Ltd. B	0.264
Ltd. C	0.280
Ltd. D	0.176
Ltd. E	0.139

The next step is to calculate the global priorities to obtain the final ranking of alternatives and to select the best one. So, to determine these final scores we will multiply the criteria weights' by ratings for the decision alternatives for each criterion and summing the respective products.

Table 4. The global priority matrix

Criteria	Weights' criteria	Alternatives	Weights' alternatives respect to criteria	Priority/ Final Weights
LI	0.039	Ltd. A	0.081	0.003
		Ltd. B	0.114	0.004
		Ltd. C	0.255	0.01
		Ltd. D	0.249	0.01
		Ltd. E	0.301	0.012
QO	0.027	Ltd. A	0.186	0.005
		Ltd. B	0.301	0.008
		Ltd. C	0.339	0.009
		Ltd. D	0.130	0.004
		Ltd. E	0.044	0.001

EM	0.141	Ltd. A	0.091	0.013
		Ltd. B	0.305	0.043
		Ltd. C	0.451	0.064
		Ltd. D	0.074	0.01
		Ltd. E	0.080	0.011
DE	0.106	Ltd. A	0.139	0.015
		Ltd. B	0.214	0.023
		Ltd. C	0.193	0.02
		Ltd. D	0.164	0.017
		Ltd. E	0.289	0.031
KE	0.224	Ltd. A	0.094	0.021
		Ltd. B	0.203	0.045
		Ltd. C	0.203	0.045
		Ltd. D	0.288	0.065
		Ltd. E	0.213	0.048
JS	0.114	Ltd. A	0.053	0.006
		Ltd. B	0.472	0.054
		Ltd. C	0.276	0.031
		Ltd. D	0.089	0.01
		Ltd. E	0.110	0.013
LSE	0.051	Ltd. A	0.268	0.014
		Ltd. B	0.263	0.013
		Ltd. C	0.356	0.018
		Ltd. D	0.059	0.003
		Ltd. E	0.053	0.003
OB	0.024	Ltd. A	0.307	0.007
		Ltd. B	0.428	0.01
		Ltd. C	0.125	0.003
		Ltd. D	0.071	0.002
		Ltd. E	0.069	0.002
UUP	0.106	Ltd. A	0.115	0.019
		Ltd. B	0.261	0.044
		Ltd. C	0.349	0.058
		Ltd. D	0.195	0.033
		Ltd. E	0.080	0.013
PP	0.167	Ltd. A	0.359	0.038
		Ltd. B	0.181	0.019
		Ltd. C	0.190	0.02
		Ltd. D	0.212	0.022
		Ltd. E	0.059	0.006

3. Lexicographic goal programming technique

In order to identify the solution to the problem, the highest priority goals and constraints are considered first; if more than one solution is found in the first step, another goal programming problem is formulated which takes into account the second priority level targets. The procedure is repeated until a unique solution is found, gradually considering decreasing priority levels. The lexicographic optimisation can then avoid the estimate of the different deviation weights, but the results of the analysis may be biased by the analyst's personal opinion.

In this paper, the LGP model is applied defining a binary structural variable (Zero-One Programming) and the objective function shows that the goal of the problem consists in the minimization of the unwanted deviations from the target.

Taking into account m objectives we have

$$\min Z = \sum_{j=1}^m P_j (d_j^-, d_j^+) \quad (1)$$

The deviations are mutually exclusive and this lead to the condition expressed in

$$d_j^- d_j^+ = 0 \quad (2)$$

The P_j factors reflect the problem hierarchy : P_1 represent the highest level, P_2 the second priority level, and so on. The objective function is subject to m constraint equations, as shown:

$$\left[\sum_{i=1}^n (a_{ji} x_i) \right] + d_j^- - d_j^+ = B_j \quad (3)$$

With $j = 1, 2, \dots, m$

B_j : represent the objective target of the j th resource

a_{ji} is the usage of j th resource of every possible alternative i th decision

Here is the formulation of the model :

$$\begin{aligned} \min Z = & 0.224d_5^- + 0.167d_9^+ + 0.141d_3^- + 0.114d_6^- + 0.106d_4^- + 0.106d_{10}^- + 0.051d_7^- \\ & + 0.039d_1^- + 0.027d_2^- + 0.024d_8^+ + 0.280d_{13}^- + 0.264d_{12}^- + 0.176d_{14}^- \\ & + 0.141d_{11}^- + 0.139d_{15}^- \end{aligned}$$

Subject to :

- LI constraint : $0.003x_1 + 0.004x_2 + 0.010x_3 + 0.010x_4 + 0.012x_5 - d_1^+ + d_1^- = 1$
- OB constraint : $7947x_1 + 8098x_2 + 8140x_3 + 8349x_4 + 8141x_5 - d_8^+ + d_8^- = 8400$
- UUP constraint : $1x_1 + 3x_2 + 5x_3 + 2x_4 + 4x_5 - d_9^+ + d_9^- = 0$

For QO, EM, DE, KE, JS, LSE, and PP constraints is have the same constraints like LI, OB, and UUP. With alternative constraint is

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1$$

The AHP-GP model described in the above was solved using LINDO software, finding out the optimal selected project executor. The analysis of the results point out that in most cases the goals of the model are reached by choosing the predictive project executor. And based on software showed us that the chosen alternatives is Ltd. A.

4. Conclusion

This paper proposes a goal programming approach to the selection of project executor for building a road. The combined AHP-GP model was applied in two subsequent stages : the first part of the analysis provided the priority levels for the different criteria and alternatives. The second step with formulation of the goal programming with zero one model has led to the identification of priority the best set of project executor, the optimal results show the best executor project is Ltd.A. Decision model proposed compares five alternatives of Ltd and respect to ten criterias. The application of the GP technique combine with AHP methodology proved to be flexible tool to optimally select the best goal of AHP.

5. References

- [1] Bayazit, O. 2006. "Use of Analytic Network Process in Vendor Selection Decision". *An International Journal* Vol.13 No.5 hal 566-579.
- [2] Bertolini M., Bevilacqua M. 2006." A combined goal programming AHP approach to maintenance selection problem". *A Reliability Engineering and System Safety* 91. Pp. 839-848.
- [3] Charnes A, Cooper WW. 1961. *Management models and industrial applications of linear programming*. New York, NY: Wiley
- [4] Lee JW, Kim SH. 2000. Using analytic network process and goal programming for interdependent information system project selection. *Comput Oper Res* 27:367–82.
- [5] Lonergan S.C., Cocklin C. 1988. The use of lexicographic goal programming in economic/ecological conflict analysis. *Socio-Econ. Plann.Sci.* Vol.22, No.2, pp.83-92.
- [6] Orumie, U.C., Ebong, D.W. 2013. An efficient method of solving lexicographic linear goal programming problem. *International Journal of Scientific and Research Publication*. Volume 3, Issue 10. ISSN 2250-3153
- [7] Ostadhashemi R., et al. 2014. Goal programming and analytical hierarchy process approaches for sustainable plantation. *Caspian Journal of Environmental Sciences*. Vol.12. pp. 233-244.
- [8] Romero C. 1991. *Handbook of critical issues in goal programming*. Oxford: Pergamon Press.
- [9] Saaty, T.L. 1980. "The Analytic Hierarchy Process," McGraw- Hill Book Company, New York.
- [10] Saaty TL. 1980. *The analytic hierarchy process*. New York, NY: McGraw-Hill.
- [11] Sedroz, K., Marouane, A. and Assogbavi T. 2012. Analytical hierarchy process and goal programming approach for asset allocation. *Journal of Mathematical Finance*. Vol 2. 96-104