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Crowning Analysis of Involute Helical Pairs Shaved by CNC Gear Shaving Machine with Two Synchronous Axes

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Abstract. Gear Shaving is one of the most efficient and economical ways to finish gears after gear hobbing or gear shaping. In tradition, the shaved gear can be crowning by shaving machine with the rocking mechanism. In this paper, we propose a mathematical model of a CNC shaving machine with two synchronous axes. The proposed mathematical model can be used to calculate the crowning tooth surface of the gear in parallel-shaving process.

1. Introduction

Gear shaving today is diffusely used in the automotive industry, truck and bus industry gears. The shaving cutter is meshed with the gear is similar to that in a crossed helical gear set. Gear shaving is a free cutting process that removes small amounts of metal from the working surface of gear teeth. The shaved tooth of the pinion and gear will be in line contact in an aligned gear drive. The new tendencies design for crowning gears are used to reduce the sensitivity of mating gears to misalignment and reduce the noise. In this paper, we propose a mathematical model to simulate the shaving process with crowning motion.

Basic mesh conditions for the involute helical gear set have been outlined in many textbooks [1]. In 1990, both Endoy [2] and Dugas [3] discussed important aspects of the shaving process, including the approximate life cycle and conditions for even contact. Moriwaki [4] and Moriwaki and Fujita [5] then proposed a stochastic model to predict the effect of shaving cutter performance on the finished tooth form. Hsu and Fong[6] proposed a mathematical model of pitch cylinder operation for the tooth profile of a gear finished by parallel shaving with an auxiliary crowning mechanism. Hsu[7] proposed a shaving method for double crowning that has no natural twist in the tooth flanks on the work gear surfaces is proposed, which uses a variable pressure angle shaving cutter in a parallel gear shaving process.

In this paper, we derive a mathematical models for the shaving cutter and coordinate systems for the CNC shaving machine and shaving gear. The parallel-shaving process is only need a CNC shaving machine with two synchronous axes.

2. Mathematical model of the parallel-shaving gears

The gear shaving process can be simplified as shown in Fig. 1. The coordinate systems and are rigidly connected to the shaving cutter and the gear, respectively. The coordinate system S_a is rigidly connected to the frame of the CNC shaving machine. The machine settings of the gear shaving machine include the traverse movement of the shaving cutter along the axis of the gear z_t, the crossed angle between the cutter's and the gear's axes γ , and the center distance between the gear and the shaving cutter E_o-$a_1 z_t$-$a_2 z_t^2$.The parameters ϕ_1 and $\phi_2(\phi_1)$ are the rotating angle of the shaving cutter and the gear, respectively.

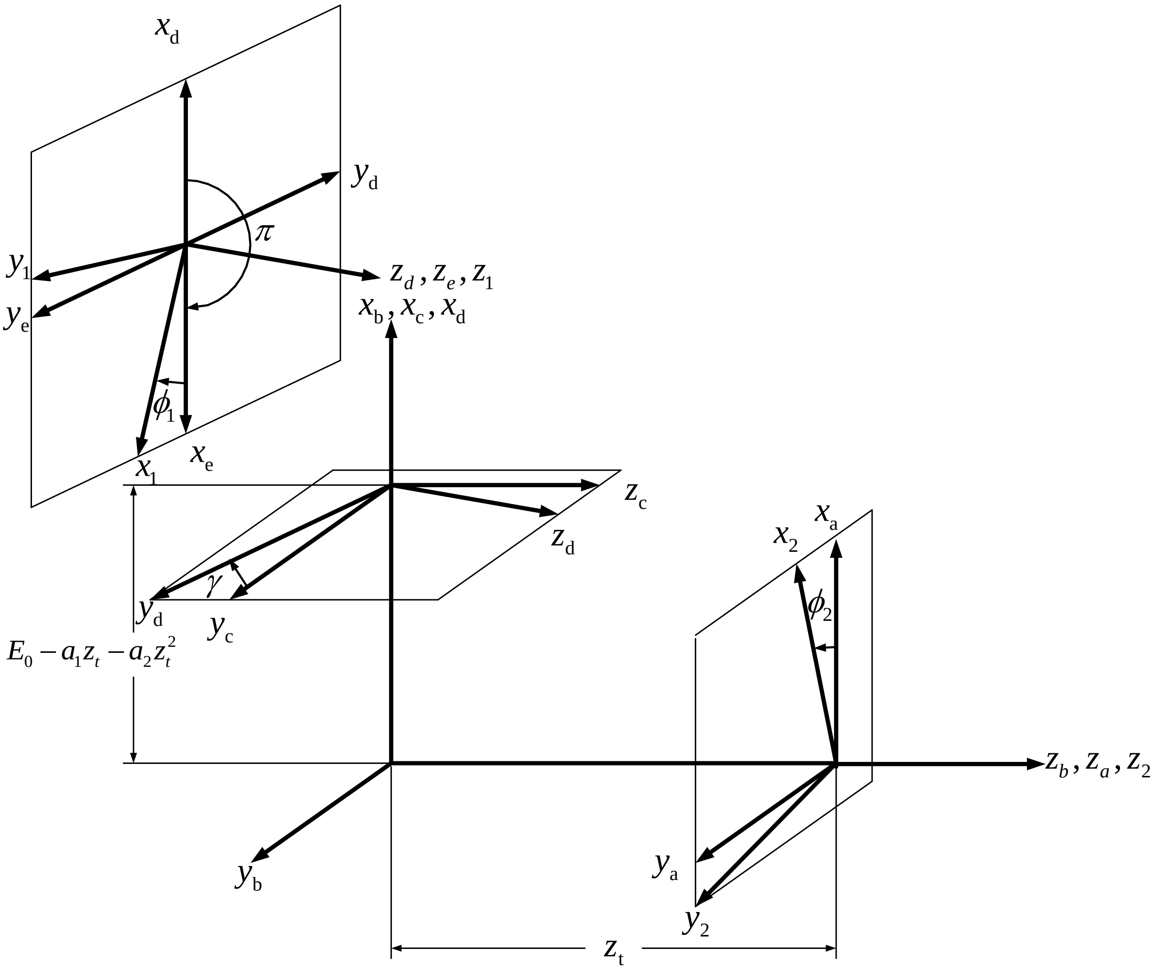

Figure 1. Coordinate systems of shaving machine.

The shaving cutter is made in the form of standard helical gear, the mathematical model of standard helical gear has been derived by Litvin [1]. Assume that the position vector $\mathbf{r}_1[x_1(u,v), y_1(u,v), z_1(u,v), 1]^T$ and the unit normal vector $\mathbf{n}_1[n_{x1}(u,v), n_{y1}(u,v), n_{z1}(u,v)]$ of shaving cutter are given and represented in coordinate system S_1. The locus of the gear $\mathbf{r}_2$ represented in coordinate system S_2 is shown in Eq. (1).

$$\mathbf{r}_2(u,v,\phi_1,\theta) = \mathbf{M}_{21}(\phi_1, z_t)\cdot \mathbf{r}_1(u,v) \tag{1}$$

where

$$\mathbf{M}_{21} = \mathbf{M}_{2c}\cdot \mathbf{M}_{c1} \tag{2}$$

$$\mathbf{M}_{2c}=\begin{bmatrix}\cos\phi_2 & \sin\phi_2 & 0 & sin\phi_2(\mathrm{E}_o-az_t^2E_o\text{-}a_1z_t\text{-}a_2z_t^2)\\ -\sin\phi_2 & \cos\phi_2 & 0 & \cos\phi_2(E_o\text{-}a_1z_t\text{-}a_2z_t^2)\\ 0 & 0 & 1 & -z_t\\ 0 & 0 & 0 & 1\end{bmatrix} \tag{3}$$

$$\mathbf{M}_{c1}=\begin{bmatrix}-\cos\phi_1 & \sin\phi_1 & 0 & 0\\ -\cos\gamma\sin\phi_1 & -\cos\gamma\cos\phi_1 & \sin\gamma & 0\\ \sin\gamma\sin\phi_1 & \cos\phi_1\sin\gamma & \cos\gamma & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \tag{4}$$

In the parallel-shaving process, the cutter is fed into the gear with table traverse in the direction of the gear longitudinal. If the gear needed to crown, the cutter is radial feed-in $a_2z_t^2$ along the center distance between the gear and the cutter. If the gear tooth surface requires taper modification, it needs to add a correction factor a_1z_t
Therefore, the locus of the gear $\mathbf{r}_2$ is simplified as shown in Eq. (5):

$$\mathbf{r}_2(u,\theta,\phi_1,z_t)=[x_2(u,\theta,\phi_1,z_t),y_2(u,\theta,\phi_1,z_t),z_2(u,\theta,\phi_1,z_t),1]^T \tag{5}$$

where

$$x_2=y_1\cos\phi_2\sin\phi_1+(E_o\text{-}a_1z_t\text{-}a_2z_t^2+z_1\sin\gamma-\mathrm{x}_1\cos\gamma\sin\phi_1)\sin\phi_2-\cos\phi_1(\mathrm{x}_1\cos\phi_2+\mathrm{y}_1\cos\gamma\sin\phi_2) \tag{6}$$

$$y_2=(E_o\text{-}a_1z_t\text{-}a_2z_t^2+z_1\sin\gamma-\cos\gamma(\mathrm{y}_1\cos\phi_1+\mathrm{x}_1\cos\phi_1))\cos\phi_2+\sin\phi_2(\mathrm{x}_1\cos\phi_1-\mathrm{y}_1\sin\phi_1) \tag{7}$$

$$z_2=-z_t+z_1\cos\gamma+\mathrm{y}_1\cos\phi_1\sin\gamma+\mathrm{x}_1\sin\phi_1\sin\gamma \tag{8}$$

The equation of meshing between the gear and the shaving cutter is shown in Eq. (9) and (10):

$$f_1(u,v,z_t,\phi_1)=\mathbf{n}_2\cdot\mathbf{v}_2^{(12)}=\frac{\mathrm{d}\phi_1}{\mathrm{d}t}(\frac{\partial}{\partial\phi_1}\left[x_2(u,\theta)\quad y_2(u,\theta)\quad z_2(u,\theta)\right]^T)=0 \tag{9}$$

$$f_2(u,v,z_t,\phi_1)=\mathbf{n}_2\cdot\mathbf{v}_2^{(12)}=\frac{\mathrm{d}z_t}{\mathrm{d}t}(\frac{\partial}{\partial z_t}\left[x_2(u,\theta)\quad y_2(u,\theta)\quad z_2(u,\theta)\right]^T)=0 \tag{10}$$

where

$$\mathbf{n}_2=\mathbf{L}_{21}\cdot\begin{bmatrix}n_{x1}(u,\theta)\\ n_{y1}(u,\theta)\\ n_{z1}(u,\theta)\end{bmatrix} \tag{11}$$

$\mathbf{L}_{21}$ is the up-submatrix of the order(3×3) of the matrix $\mathbf{M}_{21}$.
Gear shaving is a free cutting process that removes small amounts of metal from the working surface of gear teeth, therefore the operating pitch helical angle will be constant. According to literature the rotating angle between the shaving cutter and the gear is as shown in following equation:

$$\phi_1=\frac{N_2}{N_1}\phi_2\pm\frac{\tan\beta_{o2}}{r_{o2}}z_t\qquad(+\text{ for R.H. },-\text{ for L.H.}) \tag{12}$$

Where N_1 and N_2 are teeth number of the shaving cutter and the gear. The parallel-shaving shaved gear be solved by using Eqs. (5), (9), (10) and (12) simultaneously.

3. Numerical examples

The basic gear data is shown in Table 1. The basic shaving cutter setup on the CNC shaving machine, we adopt the basic meshing condition for a crossed helical gear set derived by Litvin [1]. The simulation crowning amount of the shaved gear in the pith circle as shown in Fig. 2 and Fig. 3. The vertical axis is the measured face width of the gear and the unit is mm, and the horizontal axis is the crowning amount of the shaved gear and the unit is μm in Fig. 2 and Fig. 3.

Table 1. Basic data for gear, shaving cutter and machine settings.

Work gear data	
Number of teeth	3
Normal module	2.65 mm
Normal circular-tooth thickness	5.174 mm
Normal pressure angle	20
Helix angle	10° L.H.
Outer diameter	102.717 mm
Form diameter	92.43865 mm
Face width	30 mm
Shaving cutter data	
Number of teeth	73
Helix angle	22° R.H.
Normal circular-tooth thickness	2.502 mm
Machine settings	
Operating center distance (E_o)	151.846 mm
Operating crossed angle (γ_o)	11.931°

Table 2. Coefficient value of tooth surface modification in the axial direction

	a_1 (for taper)	a_2 (for crown)
Case1	0	1.0×10^{-4}
Case2	5.0×10^{-4}	1.0×10^{-4}

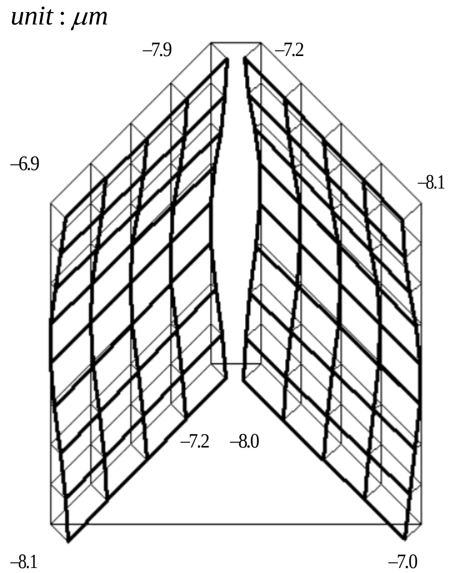

Figure 2. The topography of simulation of the crowned tooth form for Case1

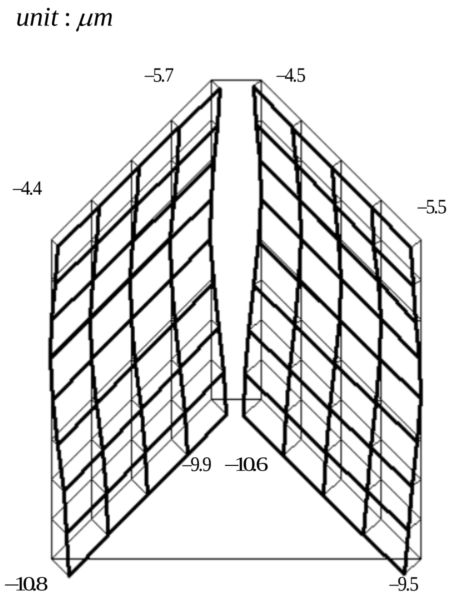

Figure 3. The topography of simulation of the crowned plus tapered tooth form for Case2

Coefficients a1 and a2 in Table 2 represent the first-order and second-order of polynomial function for center distance between shaving cutter and the gear, to achieve the effect of taper and crown modification. The results show that the maximum amount of crowning in Figure 2 is close to 8 μm. Figure 3 shows that the amount of crowing is changed due to the effect of tapering. According to the numeral examples, the proposed crowning and tapering method can shave a crowned or tapered helical gear by CNC gear shaving machine with two synchronous axes.

4. Conclusions

In this paper, we propose a mathematical model of tooth profile modification by using the two-axis synchronous motion of the gear shaving machine. The use of this processing method can replace the traditional shaving mechanism to do crowning tooth processing, simplifying the machine structure, and can make the tooth shaving process more stable.

Acknowledgments

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