Heating lines Arrangement Method of Hull Complex Surface Based on Interpolation Principle[1]

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Abstract. The technology of hull complex surface is an important part in the ship-building industry. It has been widely concerned by the researchers about how to make the complex surface forming better and faster. In this study, an interpolation principle is proposed and utilized to substitute for the conventional manual process. In the proposed process, the new heating line arrangement methods are adopted to conFigure an equivalent forming arc plate, S plate, sail plate and saddle plate according to the interpolation principle. And they are transferred to a numerical simulation model using COMSOL Multiphysics, and finite element analysis is conducted to check the formability of the process. Further, experimental investigations are carried out to verify the feasibility of the process using the new heating lines. The results obtained from the finite element analysis are in good agreement with results of experiments. Consequently, it is confirmed that four kinds curved thick plate could be obtained by interpolation principle heating lines arrangement methods.

1. Introduction

Large complex curved plate is widely used to construct hull structures in the ship-building industry. Complex curved surface forming technology is a hard issue in ship-building for a long period. On the one hand, the ship manufacturing technology is becoming more and more advanced, such as the demand of luxury cruise ships are increasing, the hull surface shape is more complex than before, the research on the hull surface rapid precision forming method and manufacturing equipment also has attracted much attention. On the other hand, this procedure, has low productivity, is ineffective, and is economically taxing because all the processes involved in the procedure are done by experts based on their experience, and the traditional forming method cannot meet the precision requirement of ship manufacturing.

Therefore, accurate hull surface forming technology can greatly improve the efficiency and precision of the manufacturing process, and it has a positive impact on improving the competitiveness of ship manufacturing industry. As for hull complex curved surface forming technology, researchers

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have made some positive exploration in hull surface forming method, optimization on machining route arrangement and the automation of hull surface forming system.

Chen J predicted the hull plate bending deformation by the temperature field of FE simulation, and used FE model to calculate the shape of steel plate deformation, the simulation results was consistent with the experimental results [1]. Ishiyma M etc. in Japan IHI heavy industry co., LTD used finite element method to simulate the plate elastic-plastic deformation under static induction heating, and compared the simulation results with the experimental results, finally they confirmed that the finite element modeling was reliable [2]. Imatani S has studied and discussed the thermodynamic mathematical model of plate bending deformation under high frequency induction heating, and proved that the mathematical model was feasible combined FE simulation and the experiments [3]. A study on a continuous thermal processing program applied contour processing methods was developed by Ishiyma M [4]. Reutzel E W used finite element method to simulate the deformation of metal plate at different geometric sizes. The deformation of metal plate was obtained by the method of progressive approximation and the computation time was optimized greatly [5]. Jong G S and his partner set up three-dimensional finite element simulation analyze modeling to study the high frequency induction heating plate temperature field and stress field of the thermal stress bending forming, and put forward the processing algorithm of saddle plate high frequency induction heating thermal stress forming, then the artificial neural network method was applied to parameter prediction [6].

A simplified thermodynamic model is used to study the forming process of metal plate under high frequency induction heating with finite element simulation method, and the angular deformation of sheet metal has been predicted [7]. A set of system IHI-Alpha which can automatically design and process the program was developed by Yu G [8]. Professor Jang C D used electromagnetic - thermal - plastic large deformation structure coupling analyze method to simulate of the plate bending process under high frequency induction heating, and established a high frequency induction heating plate forming finite element model and computer integrated manufacturing system [9]. Nguyen T T et. put forward a kind of artificial neural network to establish a method which can simplify the relation between deformation and heating parameters, the results showed that this method can easily to calculate the heating parameters in the deformation of the hull plate in the heating process [10]. Jan C D proposed an iterative method of the hot-line position under machining control mode through intelligent robot [11]. The system of flame heating lines heating plate computer auxiliary design developed by Ueda Y etc., and the relationship between induction heating conditions, inner heat source distribution by the high frequency heating planking forming experiments is also studied [12]

Jan C D proposed an iterative method of the hot-line position under machining control mode through intelligent robot [10]. Heo S C In this study proposed a flexible forming process, and it is confirmed that a double-curved thick plate could be obtained by the flexible forming process [13]. Seong W J etc. developed an automation system to increase not only the plate forming productivity, but also product quality and establish the relationship between the bending angle and the radius of the curvature of the deformed curve shape[14]. Pankaj B found initial plate templates to be applied in the desired shape of the ship plate and decide the line heating parameters[15]. Lee I K proposed a hot multi-point forming process. The hot multi-point forming die can form the various curved surface by controlling the height of each punch. In addition, the cooling system was installed inside each punch and can be selectively used as a punch or a cooling nozzle [16].

Since then, the hull plate curved surface forming has been extensively explored and it is still under investigation as well in methodological aspects as in concrete applications. In this study, a relatively simple heating lines of interpolation algorithm is developed, and the heating lines of arc plate, S curved plate, sail plate and saddle plate is obtained in this method. And the nonlinear finite element simulation prediction of plate forming is carried out by COMSOL Multiphysics software. Furthermore, experimental investigations are taken to verify the feasibility of the hull complex curved surfaces forming process. From the results, displacements of finite element simulation and experimental results are compared.

2. Complex hull surface craft design

2.1. Interpolation principle

Interpolation is a method of constructing new data points within the range of a discrete set of known data points. One of the simplest methods is linear interpolation. That is the method of calculating the intermediate point between the known points according to some algorithm, also known as "the density of data points".
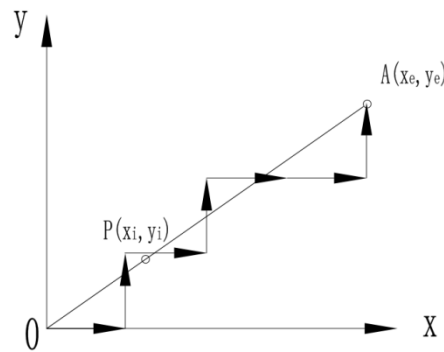

Figure 1. Linear interpolation principle.

Figure 1 is the line in first quadrant. The end point coordinate is *A (Xe, Ye)*, starting coordinate is origin point (*0, 0*), random point coordinate is *P (Xi, Yi)*, which plots around the line. There are three possibilities: *P* is above the line, *P* is on the line, *P* is below the line. In order to determine the relationship between *P* point and the line, give the following judgment:

$$\frac{Y_i}{X_i} = \frac{Y_e}{X_e} \,, \tag{1}$$

$$\text{then,} \quad Y_i \square X_e - Y_e \square X_i = 0 \tag{2}$$

Here we definite deviation discriminant function is *Fi*, and its equation is as follow:

$$F_i = Y_i \square X_e - Y_e \square X_i \,, \tag{3}$$

the relationship between *P* and the line is determined by F_i:

- When $F_i = Y_i \square X_e - Y_e \square X_i > 0$, *P* is above the line;
- When $F_i = Y_i \square X_e - Y_e \square X_i = 0$, *P* is on the line;
- When $F_i = Y_i \square X_e - Y_e \square X_i < 0$, *P* is below the line;

  If $F_i \geqslant 0$, move one step along +*X*;

  If $F_i < 0$, move one step along +*Y*, in a recursive form, until the end.

In the interpolation principle, the heating lines of sail plate and the saddle plate is redesigned. However, the arc and S curved plate are designed according to the method of line heating. In the design process, the heating lines are obtained based on conventional heating lines of sail plate and saddle plate in the literature [17]. The conventional heating lines (arcs in the Figures) of sail plate and saddle plate are shown in Figure 2 and Figure 3. The number "1" in the Figures expresses heating lines on the top surface, and the number "0" is the under surface.
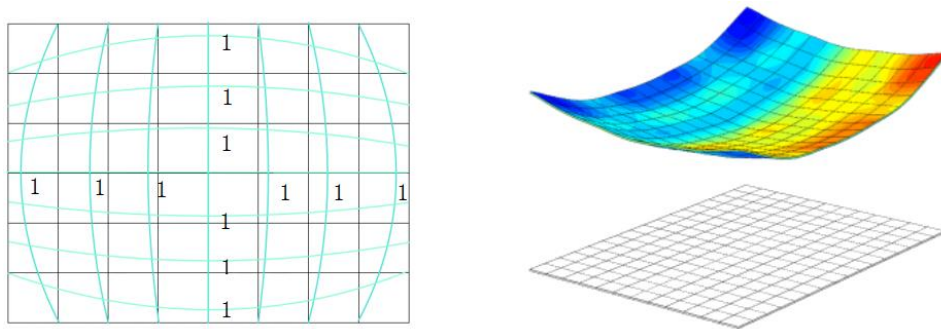
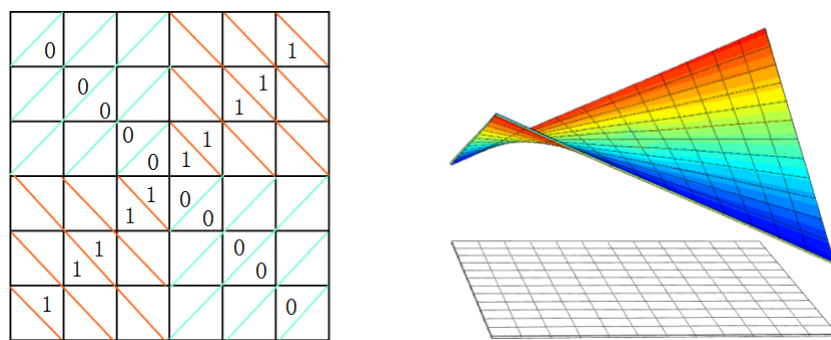**Figure 2.** Sail plate heating lines and approximate implementation.

Figure 3. Saddle plate heating lines and approximate implementation.

2.2. The heating lines plan of hull surface

The heating lines of the sail plate and saddle plate, which marked a number in Figure 2 and Figure 3, are rearranged by using the interpolation algorithm. The size of the hull plate is 1000mm * 600mm* 12mm in designing process. The steps are as follow: firstly, the hull plate is divided into squares, and the size of each squares is 80mm*80mm, then the last one at the end is 100mm*80mm. Secondly, the heating lines in each square are replaced to the horizontal and vertical straight lines. All of the Numbers "1" in Figure 4 represents heating on the upper surface of plate, and the number "0" represents heating on the lower surface of plate in Figure 5.
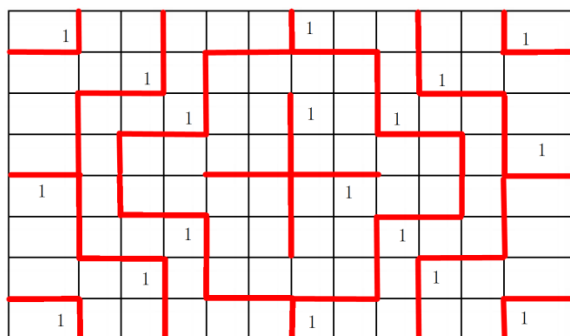

Figure 4. Sail plate heating lines arrangement   **Figure 5.** Saddle plate heating lines arrangement
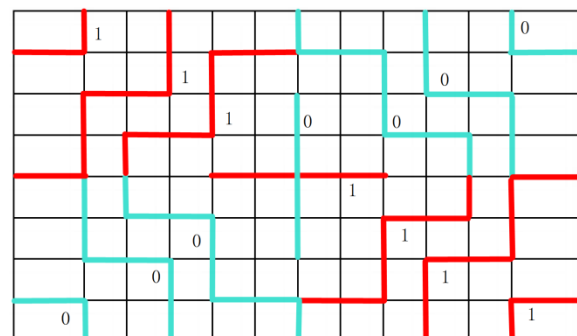
Heating lines of arc plates and S curved plates is designed according to the thermal stress forming rule of hull plate, which is rearranged in Figure 6 and Figure 7. The heating sequence of the arc plate is appeared in Figure 6 and the numbers represents the heating sequence, and the number "1 "represents the plate is heated in the first time, the number "2" represents the second time, and the

number "3" represents the last heating. And the number "1" is middle line of the plate, and the number "2" located at 1/4 of half plate. The number "3" located at 1/2 position after first and second times. The S cured plate heating lines are shown in Figure7. The numbers "1" and "2" represent heating sequence. The solid line represents the upper surface heating lines, and dash line represents the lower surface heating lines. The number "1" and "2" are located at 1/4 and 3/4 of the hull plate.
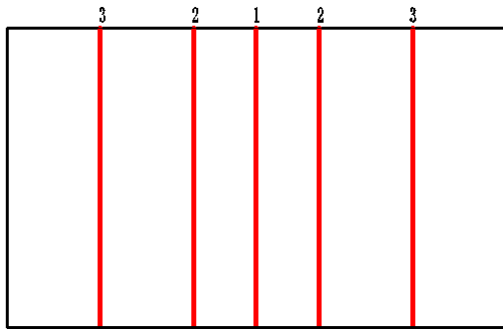

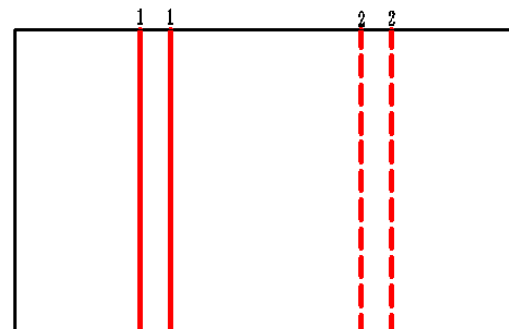**Figure 6.** Arc plate heating lines arrangement   **Figure 7.** S urved plate heating ling arrangement

For the interpolation principle is the data intensive process, the length of sail plate and saddle plate heating lines is larger than the length of conventional heating lines. However, the conventional heating lines of hull plate are intensive, and the interpolation form heating lines are optimized. Heating lines actual distance along the lengthwise and widthwise directions were investigated to confirm the forming accuracy. The interpolation heating lines actual distance in Figure 4 was measured as 5,740mm, which has about 58.48% error with regard to the conventional heating lines. In Figure 5, the interpolation heating lines actual distance was also measured as 5,740mm, with about 75.65% error. These errors could be considered as an ineviTable characteristic of the "interpolation form" forming process. The results are shown in the Table 1.

Table 1. The comparison of heating lines and actual distance.

	Heating lines actual distance of conventional form (mm)	Heating lines actual distance of interpolation form (mm)	Percentage took by the interpolation form (mm)
Sail plate	9815	5740	58.48%
Saddle plate	7588	5740	75.65%

In Table1, sail plate and saddle plate interpolation forms heating lines distance are shorter than conventional forms obviously. It means that interpolation heating lines are able to shorten the processing time greatly, and improves machining efficiency.

3. Finite element prediction of hull surface forming

3.1.Finite element simulation scheme
To predict the deformation of large hull plate, the heating lines based on interpolation method is calculated by finite element (FE) simulation. The finite element prediction model of arc plate, S curved plate, sail plate and saddle plate is established by using the COMSOL Multiphysics software. The current is 3200A in heating process, the power is 35kw, and the heating time is 10s. Air cooling method is used in the cooling process and the cooling time is 9000s. The finite element simulation scheme is shown in Table 2.

Table 2.   Finite element simulation scheme

Arc plate	S curved plate	Sail plate	Saddle plate
Heat 10s，cool 9000s	Heat 10s，cool 9000s	Heat 10s，cool 9000s	Heat 10s，cool 9000s

3.2. Finite element simulation of hull surface

3.2.1. Finite element simulation process
The arc and s curved plates are simulated according to the heating lines in Figure 4 and Figure 5. The 2 dimensional plane modes are used for the simulation of the sail plate and the saddle plate. Firstly, the plate is divided into 2D planes along the width direction and takes 1/4 of plate as the object. Secondly, the arc lines are replaced by the straight line, and the electromagnetic induction heating and cooling calculation is carried out. Thirdly, each displacement data of hull plate simulation is recorded, and the whole plate forming data can be got. Finally, the 3 dimensional curved surface of whole plate can be drawn by the FE simulation results. The finite element simulation process is shown in Figure 8.
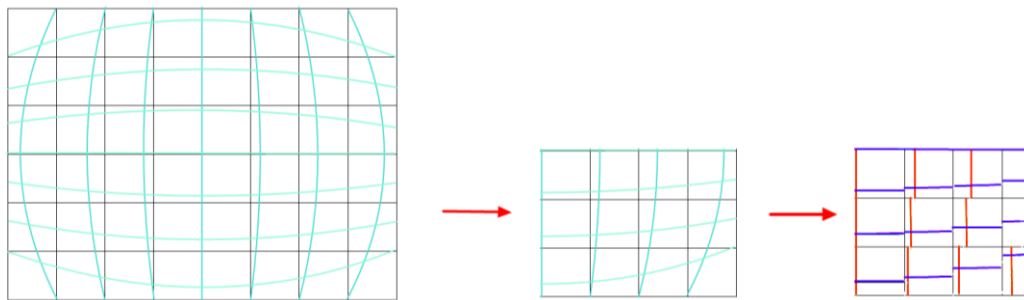

Figure 8. Finite element simulation method.

In the simulation process of high frequency induction heating, the FE geometry model size of hull plate is 1000mm * 600mm * 12mm, and the material is steel plate. The heating coil material is copper, The clearance between the hull plate and the induction coil is 2mm. The hull plates is around by air , which is used for cooling. Because of the symmetry of the model, 1/2 plate is used as the object, and the 2D model is shown in Figure 9.
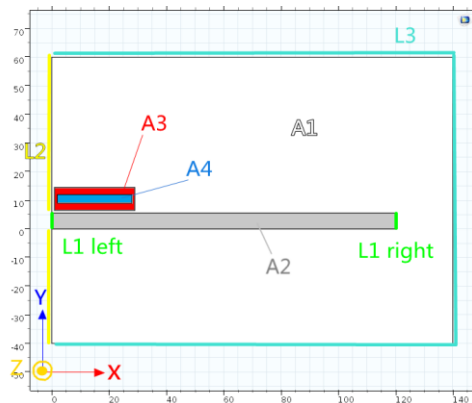
Figure 9. Geometrical model of finite elements.   **Figure 10.** The model of finite element mesh
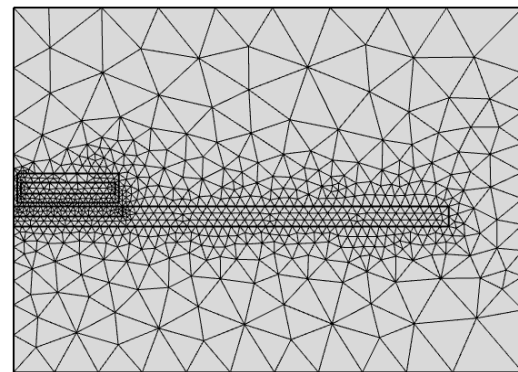
In Figure 9, where: *A1* is air, *A2* is steel plate, *A3* is copper, and *A4* is cooling water.

The finite element boundary conditions are given in the Figure 9. Where: the constraint of *L1* left is $Ux = 0$. The constraint of *L1* right is $Uy = 0$, and *L2* is the symmetry plane .*L3* is initial boundary temperature whose value is 20 ℃. All material initial temperature is 20 ℃.

3.2.2. Material parameters
The physical parameters of steel plate are nonlinear with temperature variation. In the COMSOL Multiphysics software, the interpolation function is used in the parameters definition. The high

frequency induction coil is pure copper material, and the cooling water in it. Therefore, the coil temperature is 20 ℃ during the heating process.

3.2.3. Mesh

The model is divided into uneven meshes, which is benefit to reduce the operation time. Due to skin effect, the heat is mainly focus on upper surface of hull plate. So the grids of the upper surface are dense, and the grids of the lower surface are loose. The mesh results are shown in Figure 10.

3.2.4. Multi-physical field coupling calculation

The physical field of magnetic field, solid heat transfer and solid mechanics were used in the analysis. The electromagnetic heat source, boundary magnetic - thermal coupling, electromagnetic heat source, thermal expansion and temperature coupling is used in multi-physical field coupling calculation.

3.2.5. Finite element prediction results of hull surface

The Z direction displacement of hull plate is obtained through the simulation of induction heating and cooling. For the arc plate and the S curved plate are 2D flat, whose y direction displacement is recorded in Table 3 and Table 4. Then the Z direction displacement results of sail plate and saddle plate is got. The simulation results are shown in Table 5 and Table 6.

Table 3. Simulation results of arc plate (unit/cm)

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10
Y	4.86	3.30	2.25	1.64	0.00	0.00	1.54	2.49	3.51	4.83

Table 4. Simulation results of S curved plate (unit/cm)

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10
Y	0.00	1.41	3.23	1.76	1.07	0.68	0.00	0.73	1.85	3.11

Table 5. Simulation coordinate results of sail plate (unit/cm)

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13
Y1	0.000	0.915	1.390	1.840	2.135	2.220	2.260	2.220	2.135	1.840	1.390	0.915	0.000
Y2	0.260	1.260	1.740	2.125	2.325	2.510	2.620	2.510	2.325	2.125	1.740	1.260	0.260
Y3	0.480	1.510	2.055	2.555	2.815	2.925	2.985	2.925	2.815	2.555	2.055	1.510	0.480
Y4	0.510	1.845	2.390	2.735	2.970	3.190	3.290	3.190	2.970	2.735	2.390	1.845	0.510
Y5	0.520	1.995	2.540	2.915	3.115	3.320	3.550	3.320	3.115	2.915	2.540	1.995	0.520
Y6	0.510	1.845	2.390	2.735	2.970	3.190	3.290	3.190	2.970	2.735	2.390	1.845	0.510
Y7	0.480	1.510	2.055	2.555	2.815	2.925	2.985	2.925	2.815	2.555	2.055	1.510	0.480
Y8	0.260	1.260	1.740	2.125	2.325	2.510	2.620	2.510	2.325	2.125	1.740	1.260	0.260
Y9	0.000	0.915	1.390	1.840	2.135	2.220	2.260	2.220	2.135	1.840	1.390	0.915	0.000

Table 6. Simulation coordinate results of saddle plate (unit/cm)

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13
Y1	-2.1070	-1.5470	-1.2250	-0.9030	-0.5957	-0.2947	-0.2889	0.2947	0.5957	0.9030	1.2250	1.5470	2.1070
Y2	-1.5680	-1.1620	-0.9170	-0.6755	-0.4410	-0.2170	-0.1708	0.2170	0.4410	0.6755	0.9170	1.1620	1.5680
Y3	-1.0430	-0.7700	-0.6076	-0.4452	-0.2863	-0.1358	-0.0098	0.1358	0.2863	0.4452	0.6076	0.7700	1.0430
Y4	0.5229	-0.3857	-0.3031	-0.2198	-0.1365	-0.0574	-0.0014	0.0574	0.1365	0.2198	0.3031	0.3857	0.5229
Y5	0.0392	0.0308	0.0245	0.0175	0.0105	0.0028	0.0000	0.0028	0.0105	0.0175	0.0245	0.0308	0.0392
Y6	0.5229	0.3857	0.3031	0.2198	0.1365	0.0574	0.0014	-0.0574	-0.1365	-0.2198	-0.3031	-0.3857	-0.5229
Y7	1.0430	0.7700	0.6076	0.4452	0.2863	0.1358	0.0098	-0.1358	-0.2863	-0.4452	-0.6076	-0.7700	-1.0430
Y8	1.5680	1.1620	0.9170	0.6755	0.4410	0.2170	0.1708	-0.2170	-0.441	-0.6755	-0.9170	-1.1620	-1.5680
Y9	2.1070	1.5470	1.2250	0.9030	0.5957	0.2947	0.2889	-0.2947	-0.5957	-0.9030	-1.2250	-1.5470	-2.1070

In Table 5, the maximum Z direction displacement of the sail plate is 3.55 cm, which is located at the center of the hull plate. The maximum displacement of the saddle plate is 4.214cm in Table 6, which appears in the diagonal position. To check the deformation tendency of sail plate and saddle plate, the finite element simulation data is drawn by MATLAB surface fitting algorithm. The fitted space surfaces of the sail plates and saddle plates are shown in Figure 11 and 12. In the two Figures, the surface shape is consistent with the space shape in Figure 2 and 3. The heating lines of sail plate and saddle plate which is arranged by interpolation method is feasible and reliable.
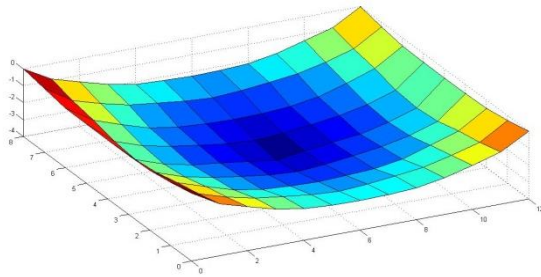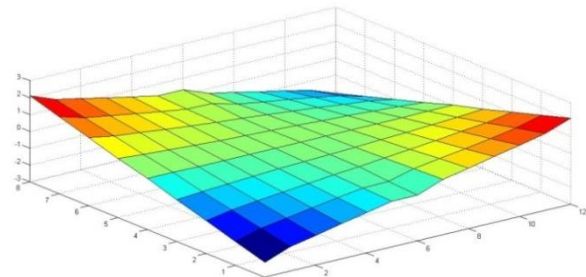

Figure 11. Data fitting shape of sail plate.   **Figure 12.** Data fitting shape of saddle plate

4. Experiment of hull surface forming

4.1. Test materials and parameters
A forming experiment using high frequency induction heating was carried out to manufacture a large steel plate. The test material is Q345. The processing parameters are shown in Table 7.

Table7. Heating parameters

Power (kW)	Frequency (kHz)	Heating time (s)	Heating mode	Cooling way
35	20	10s	Static heating	Water cool

4.2. Experiment process
In the experiment, the surface temperature of the plate is heated to 500 ℃. When it is heating, the coil moves along the prefabricated heating lines. In the heating process, the steel plate is heated the upper surface firstly, then it is cooled by water. After the temperature drops to room temperature, the other surface starts to heating and repeat the process，until all the heating lines are completed. The experiment process is shown in Figures 13 and 14.
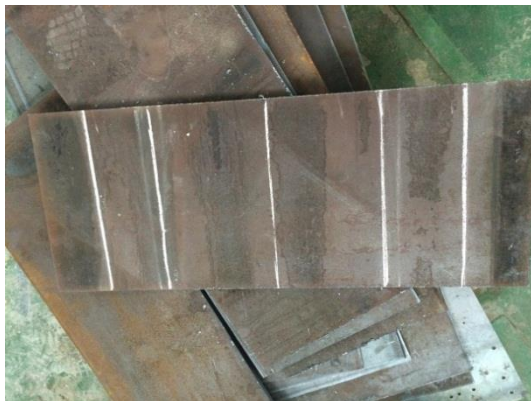

Figure 13. Heating lines of arc plate.   **Figure 14.** Heating lines of sail plate

The arc plate, s curved plate, sail plate and saddle plate are completed according to the interpolation heating lines, and the deformation of plates is shown in Figure 15 、Figure 16 、Figure 17 and Figure 18. Thus, it was verified that the simulation results matched those that obtained from the experiment
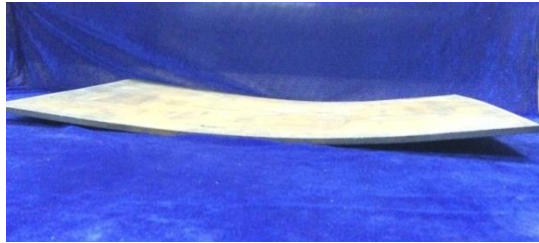

Figure 15. Forming test for arc plate.

Figure 16. Forming test for S curved plate.

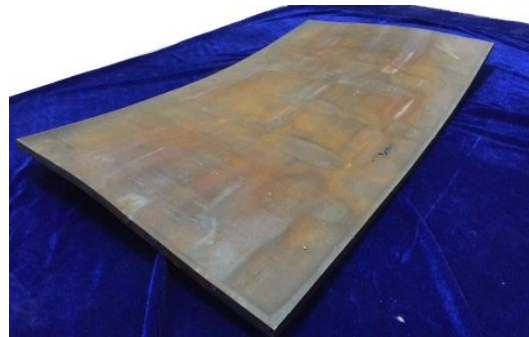**Figure 17.** Forming test for sail plate**.**

Figure 18. Forming test for paddle plate.

To compare forming test results of the sail plate and saddle plate with the simulation results, the Z direction displacement of the forming test were also measured, and the error of experiment process is calculated, which is shown in Table 8. The results from Finite element simulation are compared with those from the experiments, the average relative percentages of deformation errors are most within 5%.

The result reveals that the interpolation heating lines arrangement method can well be applied for the manufacture of the hull curved plate in ship-building.

Table 8. Comparison of prediction displacement and experimental displacement of sail plate

Number	Finite element displacement (cm)	Experimental displacement (cm)	Error	Number	Finite element displacement (cm)	Experimental displacement (cm)	Error
1	0.915	1.000	8.50%	10	1.390	1.400	0.70%
2	1.390	1.400	0.70%	11	0.915	0.900	1.70%
3	1.840	1.900	3.20%	12	0.260	0.300	13.30%
4	2.315	2.400	3.55%	13	0.480	0.500	4.00%
5	2.220	2.200	1.00%	14	0.510	0.500	2.00%
6	2.260	2.300	1.70%	15	0.520	0.600	13.30%
7	2.220	2.300	3.40%	16	0.510	0.500	2.00%
8	2.135	2.200	3.00%	17	0.480	0.600	2.00%
9	1.840	1.800	2.20%	18	0.260	0.3	13.30%

Table 9. Comparison of predicted displacement and experimental displacement of saddle plate

Number	Finite element displacement (cm)	Experimental displacement (cm)	Error	Number	Finite element displacement (cm)	Experimental displacement (cm)	Error
1	-2.1070	-2.000	9.00%	12	1.547	1.600	3.30%
2	-1.5470	-1.700	12.50%	13	2.1070	2.200	4.20%
3	-1.2250	-1.400	9.70%	14	-2.1070	-2.000	5.30%
4	-0.9030	-1.000	0.70%	15	-1.5680	-1.600	2.00%
5	-0.5957	-0.600	1.70%	16	-1.0430	-1.000	4.30%
6	-0.2947	-0.300	3.70%	17	-0.5229	-0.600	12.50%
7	-0.2889	-0.300	1.70%	18	0.0392	0.040	2.00%
8	0.2947	0.300	1.70%	19	0.5229	0.600	12.80%
9	0.5957	0.700	14.90%	20	1.0430	1.000	4.300%
10	0.9030	0.900	0.30%	21	1.5680	1.600	2.00%
11	1.2250	1.300	5.80%	22	2.1070	2.200	4.20%

5. Conclusions

In this study, a method of interpolation principle is applied for the manufacture of curved surfaces thick plates used in hull structures in ship-building. With this method, the heating lines of the arc plate, the S curved plate, the sail plate and the saddle plate are rearranged. The induction heating is simulated by the multiphysics coupling software. Finally, experiment tests are carried out using a 12mm thick steel plate. The conclusions are as follows:

(1) Based on the interpolation principle, the heating lines of sail plate and the saddle plate are designed by this method, then the distance of conventional heating lines and interpolation heating lines distance are compared, the results show that the interpolation heating lines reduce distance greatly, which save time and improve efficiency.

(2) Multiphysical fields' software COMSOL Multiphysics is used to simulate the steel plate deformation under the interpolation heating lines. Displacements of induction heating after cooling are obtained by finite element analysis, then whole hull surface is obtained using the simulation point displacement by MATLAB surfaces fitting algorithm. And simulation hull surface is consistent with the theoretical sail plate and saddle plate curved surface, so interpolation heating lines for the hull plate curved surfaces forming are proved to be reasonable.

(3) The experiment tests of the hull plate are carried out with the interpolation heating lines. The experimental results are in accord with the finite element simulation results, and the error is within 5%. It shows that the finite element simulation is reliable, and proves that the interpolation heating lines method is feasible. This method can not only contribute to the hull surface forming, but also simplify the numerical control program and facilitate learning.

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