The stress-strain state of thin-walled bar of variable cross section with different variants of fastening of the ends that is used in the energy construction

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Abstract. The authors proposed equations for the calculation of the stability and strength of thin-walled beams with swivel bearing ends can be generalized to the problem with boundary conditions corresponding to the elastic interaction with neighboring elements of the lattice pylons.

1. The basic equations of the problem. Various energy facilities, supports, trestles, various thin-walled rods of variable cross-section, working on compression are widely used. In articles [1, 2, 3], the problem of the stability and strength of hinged supported thin-walled rods of variable cross-section under longitudinal compression was solved. In work [4], a generalization of the calculation technique for rods with fastening of the ends by types "free end + sealing", "hinge + sealing", "sealing + sealing" is given. Here we continue to consider the range of problems associated with the various conditions for fixing the ends of the rod, and we consider a more general case of their elastic fastening.

Variants of elastic fastening of rods in a structure can be various variants, but two main classes can be distinguished: a connection with adjacent thin-walled structural elements and sealing of the ends of a rod or other parts thereof into an elastic array. The second class of problems is more complicated, since it leads to the determination of the elastic interaction coefficients of the elements to the contact problem of the theory of elasticity or to other, also complex, ways of solving the problem. But if these coefficients are defined, the following calculation method is fully applicable to both classes of problems.

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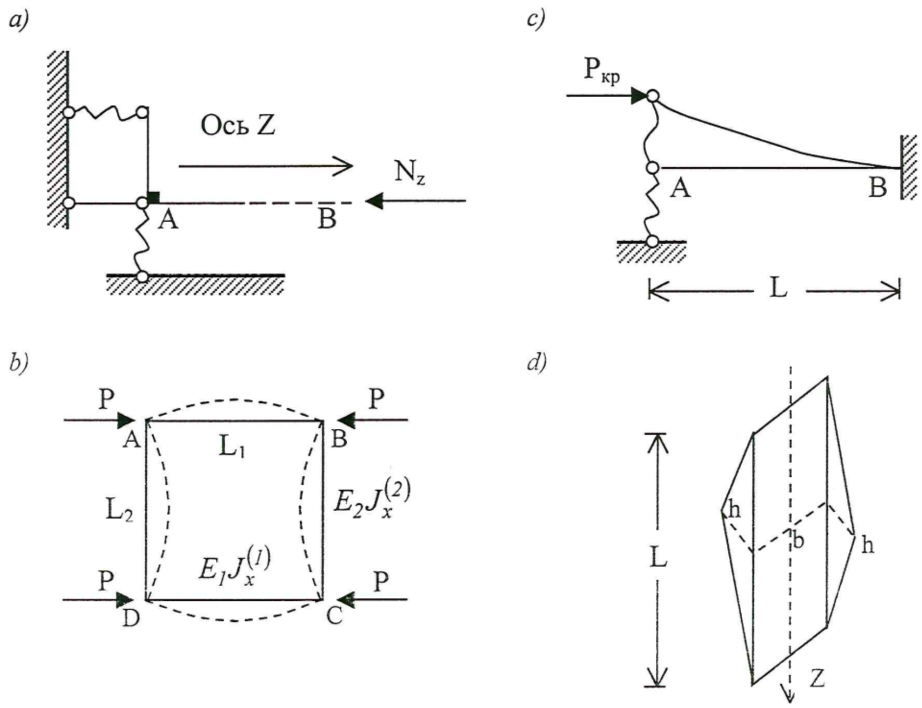

Fig.1 Schemes of elastic fastening of rods and a rod of variable cross-section

As shown in [4], the resolving equations of the problem under different conditions for fixing the rod have the form:

$$EJ_{\omega}\varphi^{IV} + EJ_{\omega}^{\prime}\varphi^{\prime\prime\prime} + k_1\varphi^{\prime\prime} + k_2\varphi^{\prime} + k_3\varphi - p(x_0 - e_x)v^{\prime\prime} + p(y_0 - e_y)u^{\prime\prime} = 0, (1)$$

$$EJ_y(z)\cdot u^{\prime\prime}(z) + pu(z) + p[y_o(z) - e_y(z)]\varphi(z) = pe_x(z) + C_1 z + C_2; (2)$$

$$EJ_x(z)\cdot v^{\prime\prime}(z) + pv(z) + pv(z) + p[-x_0(z) + e_x(z)]\varphi(z) = pe_y(z) + C_3 z + C_2; (3)$$

$$k_1 = -C - 2p\left[e_x(x_0 + R_y) + e_y(y_0 + R_x) - r_0^2/2\right], \quad r_0^2 = x_0^2 + y_0^2 + (J_x + J_y)/F;$$

$$k_2 = -C^{\prime} + 2p\left[x^{\prime}{}_0(x_0 - e_x) + y_0^{\prime}(y_0 - e_y)\right], k_3 = p\left[x^{\prime\prime}{}_0(x_0 - e_x) + y_0^{\prime\prime}(y_0 - e_y)\right]$$

$$-2p(\alpha^{\prime})^2[e_x(x_0 - R_y) + e_y(y_0 - R_y) + e_y(y_0 - R_x) + r_0^2/2];$$

$$R_x = \frac{1}{2J_x}\cdot\iint_F y(x^2 + y^2)dF, R_y = \frac{1}{2J_y}\cdot\iint_F x(x^2 + y^2)dF, \quad r_0^2 = x_0^2 + y_0^2 + (J_x + J_y)/F;$$

parameters u,v,φ – the required displacements; x0, y0, – the coordinates of the bending center of the section Z; ex, ey – coordinates of the trace of the loading line in the given section Z; C – torsional stiffness of the rod in section Z; C1, C2, C3 , C4 – are the integration constants; α(z) – is the angle of rotation of the main central axes of the section in naturally twisted rods; for medium- and strongly-structurally twisted rods the flexural stiffnesses EJx(z) and EJy(z) must be calculated in a special way [5]. All the coefficients of the equations, except for the modules E,G and the load P, depend on the longitudinal coordinate Z.

2. The constant of the cross section. As in [4], we first explain the method of solving the problem by the example of a non-thin-walled rod of constant cross section. Let the rod be fixed so that its left

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end (Fig. 1a) elastically interacts with the neighboring elements of the structure (Rasco + support belt).

Section A can be vertically intermixed and rotated in a vertical plane with an elastic support resistance. With the central compression of the rod and the conservative direction of the compressive force, the stability equation of the rod can be written in the form [4]:

$$EJ_x \cdot v''(z) + pv(z) = +C_1 z + C_2,$$

where C1 and C2 – constants of integration.

When the point A moves vertically, an external vertical transverse force (reaction of the support) QA(1), appears in it, proportional to the vertical displacement vA :

$$M_A = k_{1A} v'_A.$$

Taking into account the known differential material resistance relations for the internal shear force and the internal bending moment, we obtain two boundary conditions for equation (4) at the end A:

$$EJ_x \cdot v''_A = \alpha_{1A} \cdot k_{1A} \cdot v'_A, (6)$$

$$EJ_x \cdot v'''_A = \alpha_{2A} \cdot k_{2A} \cdot v'_A + P \cdot v'_A \cdot \beta_A . (7)$$

In (6) and (7), for convenience, the coefficients αiA, βA are introduced. The point is that the directions of displacements when the rod is unstable are not known to us in advance, and by changing the signs of the indicated coefficients, we can consider the various forms of stability loss. Secondly, by changing the value of αiA from zero to large values, we can consider various conditions for the elastic fastening of the rod ends - from the case of the free end to the case of rigid embedding, without changing the elasticity coefficients kiA and k2A in the program of the computer. In all cases | βA | =1.. Two other similar boundary conditions must be written for the end face B.

Equation (4) can be solved, for example, by the method of finite differences [4]. To calculate the third derivative in (7) one-sided differences have proved to be well-established:

$$v'''_A \approx \frac{1}{h}(v''_{A+1} - v''_A) = \frac{1}{h^3}(-v_{A-1} + 3v_A - 3v''_{A+1} + v_A)$$

or

$$v'''_B \approx \frac{1}{h}(v''_B - v''_{B-1}) = \frac{1}{h^3}(-v_{B-2} + 3v_{B-1} - 3v''_B + v_{B+1})$$

When the signs of the coefficients αiA, βA are varied in (6) and (7) and the analogous end-to-end coefficients B, the solution of equation (4) gives several different values of the critical load. To choose the true value, you must follow the following rules: First - with increasing rigidity of the support, the critical load must increase; the second - if the first criterion is met by several critical loads, choose the smallest.

Test problem 1. Consider the problem of S.P. Tymoshenko [6]. In the core system - Fig.1b - all the rods are rigidly connected and, consequently, their mechanical action under the load will be elastic. The plane of the least flexural rigidity of each rod coincides with the plane of the bar rectangle. The rods AB and BC are L2, E2Jx(2) and the cross-section area F2. Determine the force P at which the bars AB and CD have a length L1 and bending stiffness E2Jx(1); rods AD and BC - L2, E2Jx(2) and the cross-section area F2. Determine the force P at which the bars AB and CD lose their stability.

The solution of the bending equation for the beam E2Jx(2)*v''(z)=MA for the rod AD under the boundary conditions Z=±L2/2; v'(z)= ± vA' gives MA=2E2Jx(2) vA'/L2.

But since MA=k1A·vA', we obtain the stiffness coefficient k1A =2E2Jx(2)/L2.

If a transverse reaction QA(1)= k1A·vA appears in the cross section A of the rod AB with loss of stability, then the longitudinal deformation of the rod AD is ΔL2=·vA=QA(1)L2/(E2F2). Consequently, the second stiffness coefficient is k2A=E2F2/L2.

It is obvious that in the section B the stiffness coefficients are the same.

Let all the rods of the system be the same, have a rectangular constant section b * t = 4 * 2 (cm), a length L = 100 cm and an elastic modulus E = 2,1 * 105 MPa.

The decision of S.P. Timoshenko: P cr =16,46*EJx/L2=92,18kN.

Solving the finite difference method [4], equation (4) with boundary conditions (6), (7) for sections A and B for 84 segments of the partition, we obtain Pcr = 92.17kN. This result also agrees well with the graphical representation of the solution in [6]. Our solution is obtained for the following values of the additional coefficients in (6) and (7) for the cross sections A and B:

α1A = -α1B = 1; | α2A | = | βA | = | α2B | = | βB | = 1 (the solution does not depend on the sign - transverse forces do not arise or are very small).

Test problem 2. Figure 1c shows a rod of length L, the right end of which is rigidly clamped, and the left one has an elastic transverse support. If the stability is lost, the cross section A can freely rotate and the reactive moment in this section does not arise.

To determine the critical load, we solve equation (4).

At the left end, in the boundary conditions (6) and (7), we set α1A = 0, βA = -1, the value α2A in the calculations is changed to study the influence of rigidity of the support on the critical load. We assume for definiteness, just as in the previous example, k2A =E2F2/L2=1.68*104MPa.

At the right end, we can also use conditions (6) and (7), assuming α1B=10+50, α2B=10+50, β=0 (or any small number). The number of segments of the partition of the rod is accepted for a numerical solution - 84.

The critical loads obtained by us were compared with solutions of an analogous problem, given in [8] - see Table 1 (force values - in kN). The last value of the table Pcr [8] = 13.82 kN corresponds to the free end A and is calculated by the Euler formula.

Table 1. Critical forces (kN)

α2B	Pкр	Pкр[8]
1.0	113.03	113.06
0.1	112.96	112.99
0.01	112.24	112.27
0.001	98.30	98.32
0.0002	39.98	39.98
0.0001	27.19	27.19
0.0	13.82	13.82

The results of solving test problems 1 and 2 confirm the correctness and good accuracy of the proposed solution technique. We have considered these problems in sufficient detail, which makes it possible to dispense with unnecessary explanations in the next, main problem.

3. A thin-walled rod of variable cross-section AB, shown in Fig. 2a and the same - in Fig. 1d, elastically interacts with loss of stability with a rod of a constant circular cross section of radius R; resistance of the rod AC to the force P before the loss of stability of the rod AB is not taken into account. The loading of the rod AB can be either central or eccentric; its end B is rigidly clamped, as is the end C of the rod AC. We determine the critical value of the compressive force P, assuming that the dimensions of the rods and the elastic constants of their materials are given.

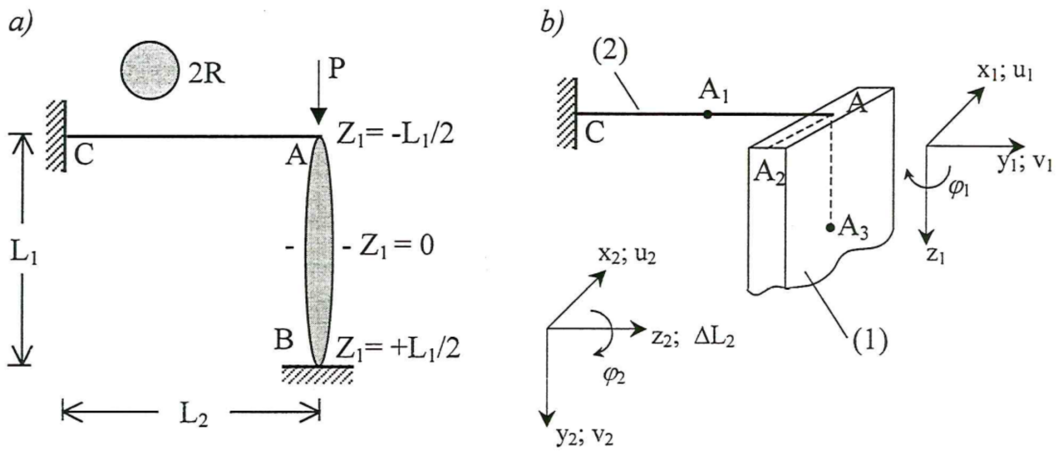

Fig.2 Fastening and loading of a rod of variable cross-section

The deformation of the rod AB under load is described by a system of equations (1) - (3) having the eighth differential order and 4 additional integration constants. For each of the displacements u1, v1, φ1, two boundary conditions at each end (12 in total) must be formulated.

The conditions for pinching the end of B are obvious:

v1= v1'=u1= u1'=φ1= φ1'=0.

Formulate for the rod AB six boundary conditions at the end of A, where the rod is expressed in a plate.

1) The rotation of the section A around the axis AA2 (Fig. 2b) at an angle v'A creates in the rod AS a bending moment MA = k1A • v'A. Consequently, the first boundary condition has the form (6):

$$E_1 J_{xA}^{(1)} \cdot v_A''' = \alpha_{1A} \cdot k_{1A} \cdot v_A', (9)$$

Where k1A =E2Jx(2)/L2.

2) The displacement vA of the cross section A of the rod AB in the direction AA1 causes the appearance of a transverse force Qy1= k2A·vA. Then the second condition has the form (7)

$$E_1 J_{xA}^{(1)} \cdot v_A''' = \alpha_{2A} \cdot k_{2A} \cdot v_A + P \cdot v_A' \cdot \beta_{1A}, (10)$$

where k2A =E2F2 /L2.

3) The displacement uA of the rod AB causes the appearance of a transverse force Qx1 = k3A • uA, proportional to the flexural rigidity of the rod AC. Then the third boundary condition is written as

$$E_1 J_{yA}^{(1)} \cdot u_A''' = \alpha_{3A} \cdot k_{3A} \cdot u_A' + P \cdot u_A' \cdot \beta_{2A}, (11)$$

Where k3A =3E2Jy(2)/L2 3.

4) For the rod AB, the quantity uA 'is connected with rotation at the point A around the axis AA1 - that is, with twisting of the rod AC. The torque Mcr = k4A * uA 'is bending for the rod AB in the plane A2AA3. Then the fourth boundary boundary condition is

$$E_1 J_{yA}^{(1)} \cdot u_A'' = \alpha_{4A} \cdot k_{4A} \cdot u_A' (12)$$

where k4A=G2Jp(2)/L2, G2 – the material shear module of the second rod Jp(2) – polar moment of inertia of its cross section.

5) The adopted design scheme allows us to assume that there is no normal voltage from deplanation at the end A of the rod AB. Then the fifth condition for the end A is written as

φA''=0 (13)

6) When the end A of the rod AB is twisted about the axis AA3, the rod AC bends in the plane A1AA2 by the moment M2=k5AφA(1), where k5A=E2Jy(2)/L2.

The relative angle of twisting of the rod AB in section A (the end of the plate) is $\varphi'_{1A} = \dfrac{M_{кр}^{1A}}{G_1 \cdot J_t^{(1)}}$, where for a thin plate $\varphi_t^{(1)} = \dfrac{1}{3} bt^3.$ at $M_2 = M_{кр}^{1A}$, we have G1Jt(1)φ1A'=k5AφA(1) and the sixth boundary condition of the problem:

$$G_1 J_t^{(1)} \cdot \varphi'_A = \alpha_{5A} k_{5A} \cdot \varphi_A (14)$$

As in the previous test problems, under the conditions (9) - (14), the coefficients αiA, βjA; the coefficients βjA can take values ± 1; the coefficients αiA can not only have different signs, but also take different numerical values to change the stiffness of the rod AS in the calculations. Since different critical loads are obtained for different signs of the coefficients αiA, βjA, we use the same two rules for the final choice:

First, with increasing rigidity of the support, the critical load must increase; the second - if the first criterion is met by several critical loads, choose the smallest.

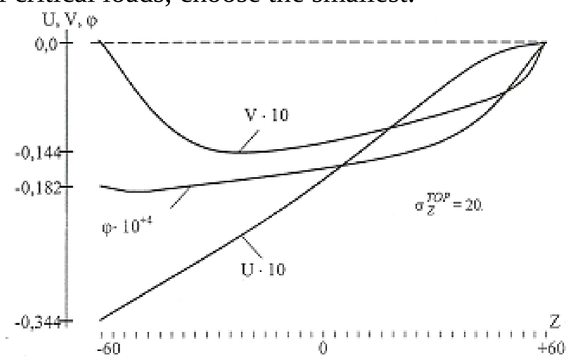

Fig.3 Subcritical displacements in a rod of variable cross-section with an elastic support

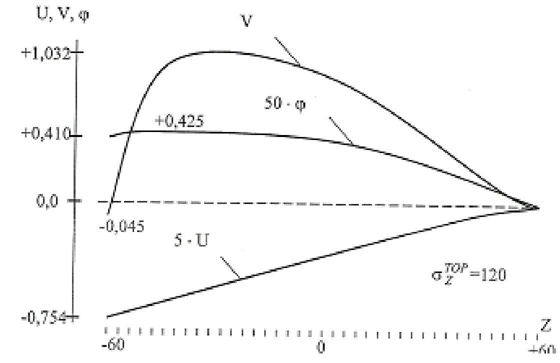

Fig. 4 Subcritical displacements in the rod under a load close to critical

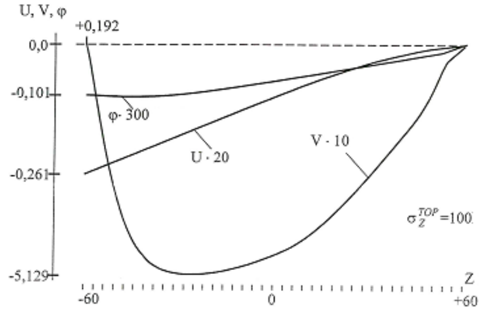

Fig.5 Supercritical displacements in the rod

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A numerical example. Table 2 shows the values of the critical end compressive stresses σor = P / Ftor for the rod AB (Figures 1d, 2a, 2b) with the following numerical data: L1 = L2 = 120cm; b = h = 5 cm; t = 0.5cm (sheet thickness); R = 0.5 cm; E1 = E2 = 2.1 * 105 MPa; G1 = G2 = 8.1 * 104MPa. Equations (1) - (3) with boundary conditions (9) - (14) were solved by the method of finite differences [4] when the rod was divided into N = 50 equal segments (the convergence of the solution with respect to the number N was controlled). The rigidity of the AS support varied in calculations using the coefficients αiA. The problem was solved in a linear formulation, a very small eccentricity of the compressive force at the ends of ≈10-5 cm was introduced into the calculations.

Table 2. Critical stresses (MPa)

αiA	*φ*кр	αiA	*φ*кр	αiA	*φ*кр	αiA	*φ*кр
10+50	212.38	2	116.88	0.05	107.22	0.0005	22.26
100	200.43	1	112.68	0.01	101.30	0.0001	10.42
10	143.13	0.5	110.45	0.005	93.60	0.00005	7.24
5	128.10	0.1	108.13	0.001	43.41	0.0	4.05

Let us follow the change in the displacements as the load increases.

The following results were obtained for the stiffness variant αiA = 1.0 for calculations with one-sided normative eccentricity [2] at the ends en=imin/20+L/750=0.1672cm. The critical value of the compressive stress at the ends in this case in this case is □торкр=112.59MPa.

Figure 3 shows the displacement curves for a small subcritical load **σ**торz=100MPa, which is close to critical. With the growth of the load, the deflection v (z) and φ (z) increase most signically and sharply increase; The displacement u (z) continues to increase monotonically with increasing load, and if the stability of the sign does not change, it does not change.

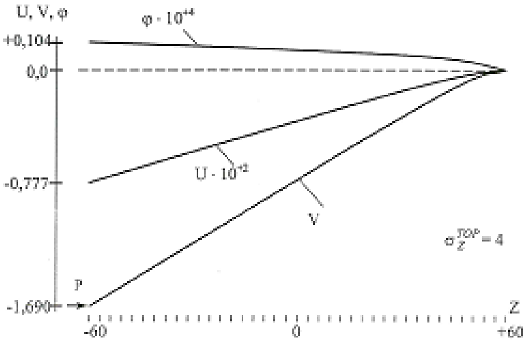

Fig.6 Subcritical displacements in a rod with a free end

With almost zero reference stiffness at the left end (αiA = 2 * 10-5, the other parameters are the same), as follows from Fig. 6 and 7, the rod almost everywhere remains straight both before and after the loss of stability - except for a small zone of strong bending in the vicinity of a blind seal on the right. The critical value of the compressive stress at the end of this version is **σ**торz=5.33MPa.

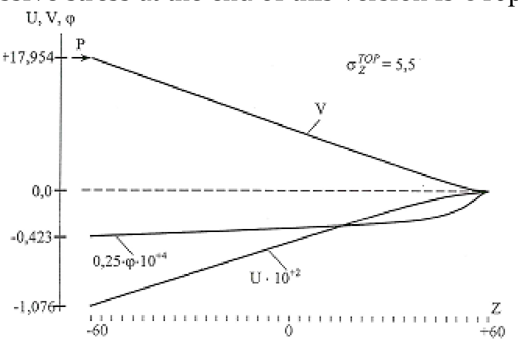

Fig.7 Supercritical displacements in a rod with a free end

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Conclusions. The system of equations (1) - (3) with a reduced differential order of equations (2), (3) is well suited for solving the class of problems considered. The boundary conditions for the elastic sealing of the ends of a thin-walled rod of variable cross-section are proposed to be written in the form (9) - (14). The imposition of elastic bonds on the free end of the rod, even a very small stiffness, sharply increases its stability under longitudinal compression (Table 2). The proposed equations and the method for calculating stability and strength can also be used to solve complex problems related to thin-walled bars of constant cross-section, fixed and loaded at the ends in various ways.

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