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The way to improvement of the method of calculation of rigidity of bending reinforced concrete elements from conventional ferro-concrete

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Abstract. The aim of the research was to improve the physical model of deformation of a bent ferro-concrete element with cracks on the basis of modern achievements in the theory of damage accumulation and fracture mechanics. The presented model allows to determine the rigidity of the element in the operation stage taking into account the damage, the work of the stretched concrete over the macrocrack, the concentration of stresses at its apex, and the presence of a pre-destruction zone in it. The analysis of the results of calculating the rigidity of elements with different percentages of reinforcement by the proposed method and the diagram method using different diagrams of concrete deformation is performed. It has been established that the work of stretched concrete over the macrocrack has a greater effect on the rigidity at a load exceeding the cracking time by no more than 30%, which is especially pronounced in strongly reinforced elements (up to 48%).

As a rule, when designing bent structures from ordinary reinforced concrete (without prestressing of the armature), the computational requirements of the second group of limiting states, in particular, the requirements for limiting the deflections by limiting values, determine the safety and reliability of their operation. To calculate the deflections of the structures under consideration, it is necessary to determine their stiffness, which in turn is significantly affected by the presence of cracks in the stretched concrete. At the same time, it should be noted that in the operational phase, the presence of cracks in the bend is allowed, only the width of their opening is limited.

The numerical value of the rigidity of the reinforced concrete section, as is known, is included in the calculation of not only deflections, but is also used in determining the bending moments in statically indeterminate structures, taking into account the redistribution of forces. In both cases, allowance for cracks allows to reflect the actual operation of the structure and thus ensure its rational design and efficient material consumption.

In this regard, research aimed at developing and improving the methodology for calculating the rigidity of reinforced concrete bending elements with cracks in operation is relevant.

Normative calculation by the method of limiting states (MPS) considers crack formation, as achievement of limiting value of this or that quantity chosen as criterion of durability [1]. Moreover, the physical processes that take place in the operational stage and precede the final destruction of the element, are either not included here at all, or are indirectly taken into account through empirical coefficients. In particular, the work of the stretched concrete over the macrocrack is neglected, the stress concentration at its apex is not taken into account, as well as the effect of scattered small cracks on the behavior of reinforced concrete under load. Not taking into account these factors can lead to a significant underestimation of the rigidity of the cross sections and, as a consequence, to the overexpenditure of steel and concrete in the design of bent structures and elements.

In the development of the Ministry of Railways in the 1960s. Ya.M. Nemirovsky first considered the problems of calculating the deformations of bent barrels, taking into account the work of stretched concrete over the macrocrack [2]. To this end, an additional linear relation was added to the system of resolving equations describing the model of the reinforced concrete section, which relates the height of the macrocrack l to the bending moment M. This was done on the basis of a small number of experimental data. The latter circumstance, in itself, is the complexity of this approach, as noted in the work of A.S. Zalesova and V.V. Figaro [3], did not allow him at that time to find wide practical application. In addition, the later experiments of K.A. Piradova [4] showed a nonlinear relationship between l and M.

Eliminate these inconsistencies of the Ministry of Railways to the real work of the construction to a certain extent is capable of a diagram method of calculating the SCR [1, 5]. A positive feature of this method is that it makes it possible to calculate the bent RCFs for the I and II groups of limiting states on the basis of a unified approach. Although the diagram method has recently been intensively introduced into the regulatory framework for the design of the concrete plant, its wide distribution in practice has been hampered by the fact that the question of how to approximate the experimental curves for the deformation of concrete and steel reinforcement remains still debatable. In addition, there is another unresolved problem, this is how to take into account the stress concentration at the top of the macrocrack within the framework of this method?

In this connection, both in our country and abroad, research is conducted and methods are developed for a relatively new young science - the mechanics of destruction [6]. In general, the use of its methods makes it possible to supplement the MPS and the diagram method with new practical results in estimating the rigidity of the cross sections. However, they are also unable to take into account the decrease in the resistance of concrete due to accumulation in it of scattered small cracks. The theory of damage accumulation is able to refine the results obtained in this question [7]. It assumes the existence of a continuous isotropic medium in which individual structural defects are distributed-scattered small cracks that obey statistical laws and determine the actual strength and deformability of the material of the structure.

Thus, in work two calculation schemes of the cross-section of the element are taken separately:

- **before the formation of a macrocrack** in the normal section of the element, its stretched zone is represented by a model in the form of a bundle of coupled fibers of H. Daniels [8] (Fig. 1, a). They have different tensile strengths R_i, which is due to the development in the material of initial shrinkage mesotrends of different lengths. In this case, the uneven distribution of the strength of the stretched concrete along the beam section is described by the normal law. The concrete of the compressed zone is considered to be a linearly elastic, undamaged material. The condition of continuity at this stage, according to the assumptions of the theory of accumulation of damages, is provided by the introduction of the Kachanov-Rabotnov damping parameter in the calculation formulas;
- **after the formation of the macrocrack**, the equilibrium of the reinforced concrete section with the mathematical sharp Irvine-Orovan cut modeling the macrocrack (Fig. 1, b) is considered. Thus, the work of stretched concrete over the macrocrack and the concentration of stresses at its apex are taken into account. In addition, microcracking at the top of the cut is taken into account, using the model of the fictitious crack, modified by the author, for this purpose. Sha [6]. This in turn assumes 1) the elongation of the Irvin-Orovan cut (the actual macrocrack) by an amount equal to the size of the prefracture zone d_f, and 2) the

application to the newly formed cracked surfaces of the cohesion forces p distributed in a triangular manner; and the size d_f is taken as the material constant, which is determined from the experiments.

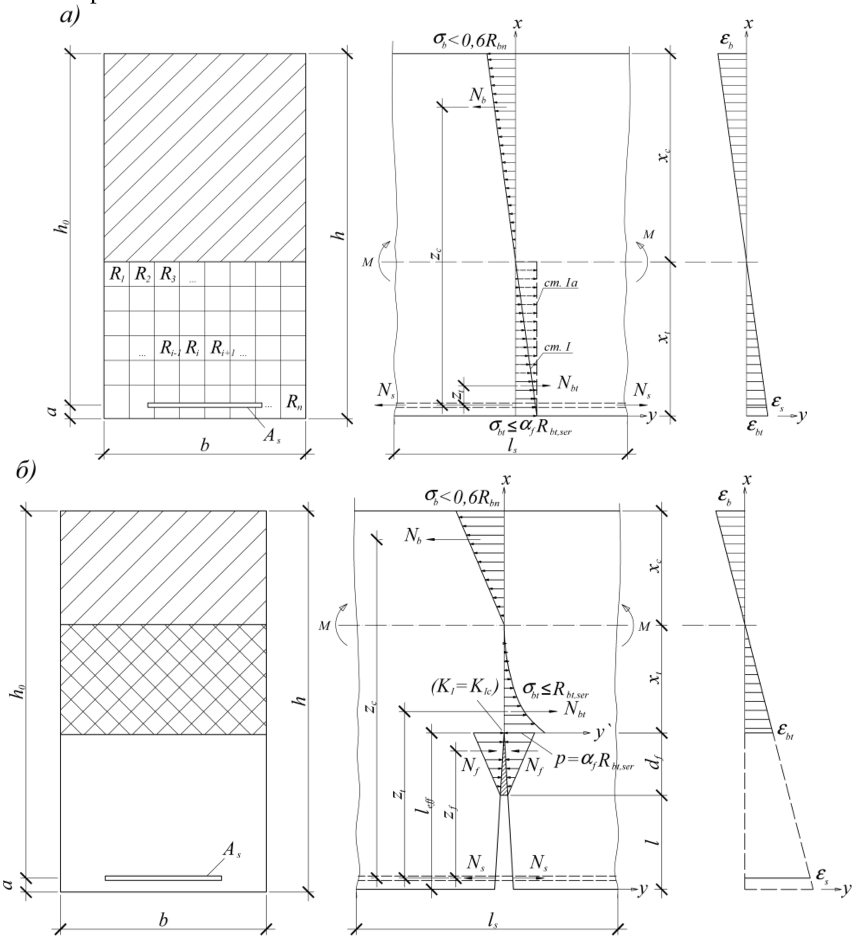

Fig. 1. The calculation scheme of the section:
a) before and b) after the emergence of macrocrack

In general, the formula for determining the stiffness of a section with respect to its center of gravity before and after the appearance of the macrocrack has the form:

$$B = I_{bc}E_b + I_{bt}E_b\, f\left[D(M)\right] + I_s \frac{E_s}{\psi_s}, \tag{1}$$

Where I_{bc} I_{bt} I_s – the moment of inertia, respectively, of the compressed concrete zone, the stretched zone of concrete and the stretched reinforcement relative to the neutral axis; E_b, E_s - modulus of deformation, respectively, of concrete and steel reinforcement; ψ_s is a coefficient that takes into account the work of stretched concrete between adjacent macrocracks (before their appearance ψ_s=1,0); $f\left[D(M)\right]$ - function of reducing the modulus of deformation of concrete in a stretched zone due to accumulation of damage:

$$f\left[D(M)\right] = 1 - \left(D_f M\right) / \left(\alpha_f M_{crc}\right) \tag{2}$$

where α_f – coefficient that takes into account that in experiments macrocracks appears at stresses somewhat smaller than $R_{bt,ser}$, D_f – the damaging parameter in this case (α_f=0,88÷0,95, D_f=0,16÷0,32), M_{crc} – normative cracking moment.

In order to find the values of the parameters α_f and D_f, it was required to express LM Kachanov's equation for the effective stresses through statistical parameters and to write it in the following transformed form:

$$1 - sV_m = \frac{\alpha_f}{1 - D_f} \tag{3}$$

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where s – reliability indicator; V_m – coefficient of variation in the strength of concrete for tension, installed in accordance with GOST 18105-86. The damping parameter in equation (3) has a probabilistic nature:

$$D(\sigma_{bt}) = \int_{R_{bt,ser}}^{\sigma_{bt}} f(R)dR + q \tag{4}$$

where σ_{bt} – nominal stresses in stretched concrete; $f(R)$ – density distribution of tensile strength by its cross-section, $R_{bt,ser}$ – standard tensile strength of concrete, q – confidence probability.

In order to determine the unknown quantities in (1), the equations of equilibrium and compatibility of deformations for the cross sections in Fig. 1:

- until macrocrack formation:

$$\sum M_{N_s} = 0 \quad => \quad M\text{-}N_b z_c + N_{bt} z_t = 0, \tag{5}$$

$$\sum Y = 0 \quad => \quad N_b\text{-}N_{bt}\text{-}N_s = 0, \tag{6}$$

$$\varepsilon_b / \varepsilon_{bt} = x_c / x_t, \tag{7}$$

where resultant efforts $N_b = \sigma_b b x_c/2$, $N_{bt} = \sigma_{bt} b x_t/2$, $N_s = \sigma_s A_s$ and the corresponding distances between them $z_c = h_0 - x_c/3$, $z_t = x_t/3 - a$ – for stage I, $z_t = x_t/2 - a$ – foe stage Ia; the height of the compressed and stretched zone of concrete is determined by the geometric characteristics of the cross section:

$$x_t = \frac{S_{red}}{A_{red}} = \frac{bh^2/2 + E_s/E_b\, A_s a}{bh + E_s/E_b\, A_s},\ x_c = h - x_t \tag{8}$$

- after the formation of the macrocrack:

$$\sum M_{N_s} = 0 \quad => \quad M\text{-}N_b z_c + N_{bt} z_t\text{-}N_f z_f = 0, \tag{9}$$

$$\sum Y = 0 \quad => \quad N_b\text{-}N_{bt} + N_s\text{-}N_f = 0 \tag{10}$$

$$\varepsilon_b / \varepsilon_{bt} = x_c / x_t, \tag{11}$$

where resultant efforts $N_b = \sigma_b x_c b/2$, $N_{bt} = b \int_l^{l+x_t} \sigma_{bt} dx$, $N_f = \alpha_f R_{bt,ser} b d_f/2$, $N_s = \sigma_s A_s$ and corresponding to the figure of the distance between them $z_c = h_0 - x_c/3$, $z_t = l - a + \int_l^{l+x_t} \sigma_{bt} x dx \Big/ \int_l^{l+x_t} \sigma_{bt} dx$, $z_f = l + 2d_f/3$. The height of the compressed and stretched zone of concrete is determined by the geometric characteristics of the reduced section from equation:

$$bx_c^2/2 - bx_t^2/2 - E_s/E_b\, A_s (h_0 - x_c) = 0 \tag{12}$$

where $x_c + x_t + l + d_f = h$, x – the current coordinate of the point along the vertical axis with the origin on the lower extended face of the element.

The tensile stresses over the macrocrack (Fig. 1, b) are distributed according to the curvilinear law and are described by the dependence:

$$\sigma_{bt} = \frac{K_I}{\sqrt{2\pi(x-l)}} + \frac{ME_b}{B}(h - x_c - x) \le R_{bt,ser},\quad l \le x \le l + x_t, \tag{13}$$

where K_I – stress intensity factor, x – the current coordinate along the vertical axis with the origin on the lower border of the section.

The macrocrack start condition is written in the form:

$$K_I = K_{Ic},\ \sigma_{bt}\big|_{x=l} = R_{bt,ser}, \tag{14}$$

where K_{Ic} – the critical stress intensity factor, determined experimentally [9].

The joint solution of the equations (5 ÷ 8) allows to estimate the VAT of the reinforced concrete section before the appearance of the macrocrack and, in particular, to determine the height of the compressed and stretched zone of concrete x_c, x_t, the values of which are required for calculating the rigidity by formula (1).

The simultaneous solution of the equations (9 ÷ 14) does not allow to do the same after the emergence of the macrocrack, since the number of unknowns in this system is greater than the number of equations. As an additional equation, the dependence of the macrocrack height on the bending moment is taken as:

$$M(l) = M_0 \; \Phi_I(l)/\Phi_I(l_{ult}), \tag{15}$$

where M_0 is the bending moment when the stretched concrete is completely turned off from work (determined by the well-known formula of A.S. Zalesov [10]); Φ_I - calibration function, constructed based on the results of finite element modeling of cracks with the use of E.M. techniques. Morozova and V.Z. Parton [11]; l_{ult} - the length of the macrocrack is bounded from above by the compressed zone of concrete with complete switching off of the stretched concrete above it from work:

$$l_{ult} = h - \frac{\sqrt{(E_s A_s)^2 + 4E_b b h_0 E_s A_s} - E_s A_s}{2E_b b} \tag{16}$$

The table presents the results of the numerical determination of the rigidity of a bent reinforced concrete element with a cross section of $b \times h = 200 \times 400$ mm from heavy concrete of class B25 at different percentages of reinforcement: μ = 0,63% (2Ø18 A400), μ = 2,31% (3Ø28 A400) , μ = 3.08% (4Ø28 A400). The table allows you to compare the data obtained by four different methods (see notes to the table). The first three of them are based on the diagrammatic calculation method and the nonlinear deformation model using various diagrams of concrete deformation. In all of the cases considered, in order to evaluate the effect on the rigidity of the stretched concrete only on the macrocrack, the coefficient ψ_s was assumed to be 1.0, that is, its work on the section between macrocracks was not taken into account. The diagram of deformation of compressed concrete and stretched reinforcement in the considered techniques is adopted linear. In Fig. 2 shows the schemes of qualitative distribution of stresses and forces in the cross section of the element (with μ = 2.31%), constructed according to different methods with the same value of the bending moment (M = 23 kN·m).

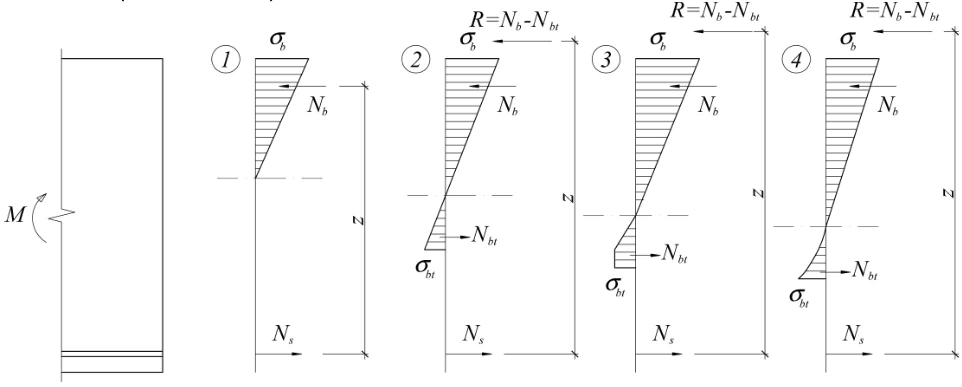

Fig. 2. Diagrams for the distribution of stresses and forces in the section of a reinforced concrete element:

The numbers of the positions to the left of the diagrams correspond to those in the table 1.

Table 1. Results of calculation of rigidity by different methods

μ,	Methodology	Moment, кН·м	Model design parameters						
			l, m	x_t, m	x_c, m	σ_{bt}, MPa	σ_b, MPa	σ_s, MPa	B, 10^3·kN·m^2
0,63	1	M_1=17	0,304	0	0,096	0	-4,91	96,98	9,534
		M_2=50	0,304	0	0,096	0	-14,44	285,24	9,534
	2	M_1=17	0,272	0,032	0,096	1,279	-5,065	95,23	9,647
		M_2=50	0,296	0,008	0,096	0,524	-14,447	284,97	9,594
	3	M_1=17	0,224	0,064	0,112	1,6	-5,33	87,482	10,115
		M_2=50	0,280	0,024	0,096	1,6	-14,663	283,11	9,609
	4	M_1=17	0,008	0,184	0,208	1,6	-2,944	14,841	-
		M_2=50	0,288	0,016	0,096	1,6	-14,617	283,72	9,602
	1	M_1=23	0,24	0	0,16	0	-4,328	38,59	24,961

		M_2=95	0,24	0	0,16	0	-17,878	159,4	24,961
	2	M_1=23	0,168	0,064	0,168	1,585	-4,46	36,115	25,566
		M_2=95	0,224	0,016	0,168	1,245	-17,931	158,97	24,968
	3	M_1=23	0,064	0,144	0,192	1,6	-4,267	26,738	30,509
		M_2=95	0,208	0,032	0,160	1,6	-18,16	158,05	24,999
	4	M_1=23	0,072	0,128	0,2	1,6	-4,232	24,415	31,990
		M_2=95	0,216	0,024	0,16	1,6	-17,974	157,86	24,990
3,08	1	M_1=25	0,224	0	0,176	0	-4,341	32,035	30,013
		M_2=100	0,224	0	0,176	0	-17,362	128,14	30,013
	2	M_1=25	0,144	0,072	0,184	1,588	-4,448	29,727	30,891
		M_2=100	0,208	0,016	0,176	0,992	-17,396	124,87	30,018
	3	M_1=25	0,008	0,176	0,216	1,6	-4,01	19,032	39,982
		M_2=100	0,192	0,032	0,176	1,6	-17,47	127,11	30,052
	4	M_1=25	0,008	0,160	0,232	1,6	-3,831	15,618	44,468
		M_2=100	0,192	0,032	0,176	1,6	-17,509	126,7	30,070

Notes:

1 - calculation of the deformation model without taking into account the work of stretched concrete over the macrocrack;

2 - the same taking into account the work of stretched concrete over the macro crack and the use of the linear diagram " σ_{bt}-ε_{bt} ";

3 - the same with the use of the two-line diagram " σ_{bt}-ε_{bt} " according to [2] (E_{b1}=E_b, ε_{bt0}=$R_{bt,ser}$/E_b, ε_{bt2}=2ε_{bt0});

4 - according to the proposed method;

M_1 - the moment exceeding the cracking time by no more than 30%;

M_2 - the moment from the operational load ($\sigma b \leq 0{,}6R_{bn}$).

It was found that the maximum estimate of the effect of stretched concrete over the macrocrack on the stiffness of the element is obtained by methods 3 and 4, which have close results (the divergence in the values of the stiffness of the cross-section is 5 ÷ 12%). In Fig. 2, these methods also correspond to the largest value of the shoulder of the internal pair of forces z. In this case, the use of the diagram method 3 requires calculations by successive approximations, which practically excludes the possibility of calculating the constructions by a "manual" method. The technique proposed by the author allows us to do this without significantly increasing the complexity of computations in comparison with the Normative Approach [1].

It can be seen from the table that the greatest discrepancy in determining the value of B between methods 1 and 4 is observed at a load close to the moment of crack formation (28 ÷ 48%), which is especially pronounced in strongly reinforced elements (up to 48%). Under the action of the operational load, the difference in the stiffness values by all methods is not so significant (less than 5%).

It should be noted that, since the curvature of the longitudinal axis of the reinforced concrete element working with cracks in the stretched zone varies disproportionately to the change in the external torque, in some cases it is necessary to introduce integration of the stiffness over its individual sections in order to accurately determine the deflection of the element. In this case, some sections of the element, in which $M > M_{crc}$, may be in the stage close to the stage of crack formation and account for the work of stretched concrete over the macrocrack in them, as shown above, will lead to an increase in the calculated rigidity of the section in question. This, in turn, will reduce the design deflection of the structure and will give an opportunity to obtain reinforcement savings in the design up to 17%, depending on the percentage of reinforcement and concrete strength class.

Conclusions

1. An improved model of deformation of a bent ferro-concrete element with cracks is proposed, which, unlike the normative approach [1], can be used to determine its rigidity in the exploitation stage, taking into account the damage, the work of stretched concrete over the macrocrack, the stress concentration at its apex, and the presence of a pre-destruction zone in her.

IOP Conf. Series: Materials Science and Engineering **412** (2018) 012058 doi:10.1088/1757-899X/412/1/012058

2. Analysis of the results of calculating the rigidity of bent elements with different percentages of reinforcement by the proposed method and the diagram method using different diagrams of concrete deformation. It has been established that the work of stretched concrete over the macrocrack has a greater effect on the rigidity at a load exceeding the cracking time by no more than 30%, which is especially pronounced in strongly reinforced elements (up to 48%).

References

[1] *Manual for the design of concrete and reinforced concrete structures of the heavy concrete without prestressing* (to SP 52-102-2003): date of typing 2004-03-01. – Moscow: TsNIIPromzdany, 2005. – 214 p

[2] Borishansky M S, Nemirovsky J M, Nikolaev J K, Petrova K V et al 1968 *Investigation of the stress-strain state of reinforced concrete elements, with the work of the cracked concrete in tension and revision of this calculation based on the theory of deformation and crack opening* Strength and stiffness of reinforced concrete structures: Coll. of sc. art.– Moscow: NIIZhB p 232

[3] Zalesov A S, Figarovsky V V 1976 *A practical method for calculating the deformations of reinforced concrete structures* (Moscow.: Stroiizdat) p 104

[4] Piradov K A 1998 *Theoretical and experimental foundations of fracture mechanics of concrete and reinforced concrete* (Tbilisi) p 355

[5] Karpenko N I 1996 *General models of the mechanics of reinforced concrete* (Moscow: Stroiizdat) p 416

[6] Zaitsev Y V 1991 *Fracture Mechanics for builders* (Moscow: Vysshaja Shkola) p 288

[7] Lemaitre J A 1992 *Course on Damage Mechanics* (Springer-Verlag) p 210

[8] Daniels H 1945 *The Statistical Theory of the Strength of Bundles of Threads* (I in Proc. R. Soc London A 183) pp 405-35

[9] GOST 29167-91 1992 *Concrete. Methods for determining the characteristics of crack resistance (fracture toughness) under static loading: date of typing 1992-07-01* (Moscow.: State Committee for Construction of the USSR) 38 p

[10] Zalesov A S, Kodysh E N, Lemesh P L, Nikitin I K 1988 *Calculation of reinforced concrete for strength, fracture toughness and deformation* (Moscow: Stroiizdat) p 320

[11] Morozov E M, Nikishkov G P 1980 *The finite element method in fracture mechanics* (Moscow: Nauka) p 256