

# Empirical heat transfer correlations of tube bundles under pulsating flows

A I Haibullina<sup>1</sup>, L S Sabitov<sup>1,2</sup>, A R Hayrullin<sup>1</sup>, N H Zinnatullin<sup>3</sup>

<sup>1</sup>Kazan State Power Engineering University, 51 Krasnoselskaya street, Kazan, 420066, Russian Federation

<sup>2</sup>Kazan Federal University, 18 Kremlyovskaya street, Kazan, 420008, Russian Federation

<sup>3</sup>Kazan National Research Technological University, 68 Karla Marks, street, Kazan, 420015, Russian Federation

kharullin@yandex.ru

**Abstract.** In this paper, the influence of the parameters of pulsations on the heat transfer in tube bundles was considered. An analysis of the influence of the parameters of pulsations on heat transfer was carried out by empirical correlation. A new empirical correlation is proposed for calculation of external heat transfer in under of pulsating flow in tube bundles with different configurations. The maximum deviation of the empirical correlation with the data of the numerical experiment was 16.9%.

## 1. Introduction

Unsteady flows can be used to enhance the heat transfer of various heat exchange equipment. Therefore, many authors are engaged in the study of pulsating flows. Studies of heat transfer in pulsating flow are mainly devoted to individual pulsation regimes [1-4]. Therefore, for the frequently obtained data, it is not enough to obtain an empirical correlation, which can be used to calculate heat transfer in pulsating flows. Or empirical correlations are obtained for a narrow range of parameters [5]. Obtaining empirical dependence complicates a large number of designs of heat exchange equipment. Also, the complexity is complemented by the fact that the determining parameters of heat transfer for pulsating currents are greater than for steady-state heat transfer. However, in engineering practice, it is convenient to use the empirical correlation to calculate the heat transfer, which must be obtained for a given geometry and flow regime. To simplify the production of empirical correlation, one can use a numerical experiment instead of a real experiment. In the previous work [6], an external heat transfer in tube bundles of various configurations under pulsating currents was investigated by a numerical method. As a result, an empirical correlation was obtained for a wide range of parameters. However, the maximum deviation of the empirical correlation was  $\pm 35,7\%$ . The aim of this work is to obtain an empirical correlation with a smaller error for the data of the paper [6]. In addition, in this work, an analysis will be made of the empirical correlations obtained, which will allow us to estimate the influence of the regime parameters on the heat transfer for different bundles of tubes in the presence of flow pulsations.

## 2. Methods



Content from this work may be used under the terms of the [Creative Commons Attribution 3.0 licence](https://creativecommons.org/licenses/by/3.0/). Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

To find the empirical correlation was used regression analysis based on the method of least squares. In particular, multiple linear regression and its special case polynomial regression were used [7, 8].

### 3. Results and Discussion

In [6], an empirical correlation of heat transfer was obtained for pulsating flows in tube bundles (1).

$$\frac{Nu_p}{Nu_{st}} = 0,954 Re^{-0,201} \cdot Pr^{-0,211} \cdot \beta^{0,184} \cdot Fo^{-0,230} \cdot \psi^{-0,053} \cdot \varphi^{0,085} \cdot s_1/D^{0,287} \quad (1)$$

where  $Nu_{st}$  – Nusselt number for stationary flow;  $Nu_p$  – Nusselt number for pulsating flow;  $Re$  – Reynolds number;  $Pr$  – Prandtl number;  $\beta$  – relative amplitude of pulsations;  $Fo$  – Fourier number;  $\psi$  – pulsating ratio;  $\varphi$  – tube layout angle;  $s_1$  – tube pitch, m;  $D$  – tube bundle, m. Equation (1) is obtained for the ranges  $215 \leq Pr \leq 363$ ,  $0.25 \leq \psi \leq 0,5$ ,  $100 \leq Re \leq 1000$ ,  $15 \leq \beta \leq 35$ ,  $5.81 \cdot 10^{-4} \leq Fo \leq 14.53 \cdot 10^{-4}$ . Coefficient of determination  $R^2 = 0.906$ , maximum error  $\delta = \pm 35.7\%$ . The definition of the parameters entering into equation (1) is given in [6].

To reduce the error of equation (1), geometric parameters of the tube bundles were removed  $\varphi$  and  $s_1/D$  (2). Thus, the place of one equation (1) yields 12 equations (Table 1).

$$\frac{Nu_p}{Nu_{st}} = A Re^m \cdot Pr^n \cdot \beta^b \cdot Fo^c \cdot \psi^d \quad (2)$$

**Table 1.** The exponents of equation (2)

№	$\varphi$	$s_1/D$	A	m	n	b	c	d	$R^2$	Deviation $\delta, \%$	
										max	mean
Staggered tube bundle											
1	30°	1.25	1.020	-0.154	-0.203	0.198	-0.213	-0.092	0.910	±27.3	±4.7
2		1.50	1.673	-0.232	-0.214	0.199	-0.227	-0.082	0.944	±22.9	±5.3
3		1.75	1.800	-0.224	-0.215	0.181	-0.225	-0.069	0.949	±22.8	±4.8
4	45°	1.25	1.081	-0.170	-0.194	0.195	-0.210	-0.099	0.917	±30.5	±4.9
5		1.50	1.576	-0.249	-0.233	0.232	-0.252	-0.079	0.916	±29.0	±7.3
6		1.75	1.641	-0.230	-0.194	0.170	-0.237	-0.037	0.951	±36.3	±6.8
7	60°	1.25	1.101	-0.129	-0.211	0.193	-0.213	-0.018	0.903	±21.7	±4.6
8		1.50	1.638	-0.206	-0.224	0.192	-0.238	-0.022	0.937	±27.1	±5.1
9		1.75	1.957	-0.218	-0.193	0.142	-0.226	-0.006	0.959	±21.4	±3.9
In-line tube bundle											
10	90°	1.25	1.161	-0.179	-0.194	0.186	-0.228	-0.063	0.929	±21.4	±4.9
11		1.50	1.914	-0.213	-0.228	0.167	-0.234	-0.046	0.950	±21.8	±4.5
12		1.75	1.876	-0.212	-0.232	0.154	-0.255	-0.033	0.950	±22.0	±4.5

Consider the influence of numbers  $Re$  and  $Pr$  on  $Nu_p/Nu_{st}$  according to their degrees, depending on the configuration of the bundles of tubes. Since the degrees  $m$  and  $n$  are negative, then with increasing  $Re$  and  $Pr$ ,  $Nu_p/Nu_{st}$  decreases for all  $\varphi$  и  $s_1/D$ . The values of the degree for  $Re$  in lie in the range  $-0.129 \leq m \leq -0.249$ . Minimal impact  $Re$  on  $Nu_p/Nu_{st}$  is observed in denser beams ( $s_1/D = 1.25$ ),  $m$  are in the range  $-0.129 \leq m \leq -0.179$ . When  $s_1/D = 1.5$  and  $1.75$ ,  $m$  is in the range  $-0.206 \leq m \leq -0.249$ . Influence of  $Pr$  on  $Nu_p/Nu_{st}$  less depends on  $\varphi$  and  $s_1/D$  (compare with the numbers  $Re$ ),  $n$  is the range  $-0.194 \leq n \leq -0.232$ .

Influence  $\beta$  on  $Nu_p/Nu_{st}$  contrary to  $Re$  and  $Pr$ . If with the increase of  $Re$  and  $Pr$  there is a decrease in heat transfer, then with increasing  $\beta$  there is an increase in heat transfer for all  $\varphi$  and  $s_1/D$ ,  $b$  is in the

range  $0.142 \leq b \leq 0.232$ . When  $\varphi = 60^\circ$ ,  $s_1/D = 1.75$   $\beta$  has the minimal influence on  $Nu_p/Nu_{st}$  ( $b = 0.142$ ). When  $\varphi = 45^\circ$ ,  $s_1/D = 1.5$   $\beta$  has the maximum influence on  $Nu_p/Nu_{st}$  ( $b = 0.232$ ).

With increasing  $Fo$ , an increase in heat transfer occurs for all  $\varphi$  and  $s_1/D$ . Influence  $Fo$  on  $Nu_p/Nu_{st}$  less when  $s_1/D = 1.25$ ,  $c$  is the range  $-0.210 \leq c \leq -0.228$ . When  $s_1/D = 1.5$  and  $1.75$  ( $-0.225 \leq c \leq -0.255$ )

With increasing  $\psi$  there is a decrease in heat transfer for all  $\varphi$  and  $s_1/D$ . When  $\varphi = 45^\circ$ ,  $s_1/D = 1.25$   $\psi$  has the maximum influence on  $Nu_p/Nu_{st}$  ( $d = -0.099$ ).  $\psi$  has almost no influence on heat transfer when  $\varphi = 60^\circ$ ,  $s_1/D = 1.75$  ( $d = -0.006$ ).

Table 1 shows that for some  $\varphi$ ,  $s_1/D$ , the error  $\delta$  has decreased to almost 20%. However, for some  $\varphi$ ,  $s_1/D$ , the error  $\delta$  remains about 30% or higher. To further reduce the maximum error  $\delta$ , we remove  $\psi$  from equation (2), and the numbers  $Re$ ,  $Pr$ ,  $Fo$  and the relative amplitude  $\beta$  are grouped into the dimensionless complex  $\beta/(Re \cdot Pr \cdot Fo)$  (3).

$$\frac{Nu_p}{Nu_{st}} = A \left( \frac{\beta}{Re \cdot Pr \cdot Fo} \right)^2 + m \left( \frac{\beta}{Re \cdot Pr \cdot Fo} \right) + n \quad (3)$$

Where ( $A$ ,  $m$  and  $n$ ) are given in Table. 2. As we can see from Table. 2. The maximum error  $\delta$  decreased to 16.9%. The dimensionless complex equation (4) characterizes the ratio of the pulsation velocity to the stationary one, the increase of which leads to the growth of  $Nu_p/Nu_{st}$  irrespective of  $\varphi$ ,  $s_1/D$  and  $\psi$ .

$$\beta/(Re \cdot Pr \cdot Fo) = Af / u_{st} \quad (4)$$

Where  $A$  – pulsation amplitude,  $m$ ;  $f$  – ripple frequency, Hz;  $u_{st}$  – speed in steady flow, m/s.

In Fig. 1-4 shows the curves constructed from equation (2) (solid line) and numerical experiment data for all  $\varphi$  and  $s_1/D$  and  $\psi$ . According to Fig. 1-4 that equation (3) describes the data of a numerical experiment well. The coefficient of determination averaged  $R^2 = 0.97$ .

**Table 2.** The exponents of equation (3)

№	$\psi$	$\varphi$	$s_1/D$	$A$	$m$	$n$	$R^2$	Deviation $\delta$ , %	
								max	mean
1	0.25	30°	1.25	-0.0075	0.473	1.033	0.950	±14.7	±4.0
2			1.50	-0.1333	0.949	0.981	0.998	±3.4	±1.1
3			1.75	-0.1288	0.932	1.011	0.997	±4.4	±1.4
4		45°	1.25	-0.0279	0.544	1.004	0.978	±9.3	±2.8
5			1.50	-0.0893	0.974	0.958	0.993	±6.5	±1.9
6			1.75	-0.0540	0.863	1.000	0.995	±6.8	±1.4
7		60°	1.25	-0.0364	0.545	1.082	0.957	±14.3	±3.4
8			1.5	-0.0781	0.861	1.058	0.992	±8.1	±2.1
9			1.75	-0.1170	0.914	1.071	0.989	±5.6	±2.3
10		90°	1.25	-0.0933	0.754	1.027	0.987	±8.4	±2.4
11			1.50	-0.1359	0.946	1.062	0.979	±11.2	±3.2
12			1.75	-0.1403	1.003	1.113	0.970	±10.5	±4.2
1	0.4	30°	1.25	-0.0221	0.532	0.976	0.956	±15.6	±4.1
2			1.50	-0.1510	1.017	0.928	0.997	±5.3	±1.5
3			1.75	-0.1422	0.983	0.966	0.997	±5.6	±1.8
4		45°	1.25	-0.0296	0.581	0.947	0.980	±10.4	±3.0
5			1.5	-0.1066	1.038	0.904	0.994	±6.2	±2.0

6	0.5	60°	1.75	-0.0424	0.862	0.976	0.993	±7.3	±2.1
7			1.25	-0.0400	0.543	1.083	0.950	±14.6	±3.7
8			1.50	-0.1251	0.937	1.040	0.990	±8.9	±2.4
9			1.75	-0.1621	0.997	1.058	0.980	±10.1	±3.2
10		90°	1.25	-0.0867	0.738	0.999	0.980	±10.6	±3.0
11			1.50	-0.1643	1.007	1.027	0.976	±12.6	±3.2
12			1.75	-0.1765	1.076	1.082	0.977	±10.8	±3.5
1			30°	1.25	-0.0468	0.602	0.925	0.965	±14.6
2		1.5		-0.1585	1.042	0.887	0.995	±6.8	±2.0
3		1.75		-0.1550	1.025	0.928	0.994	±6.5	±2.2
4		45°		1.25	-0.0612	0.674	0.891	0.986	±8.9
5			1.50	-0.1370	1.113	0.861	0.996	±6.0	±1.8
6	1.75		-0.0707	0.928	0.951	0.990	±11.2	±2.9	
7	60°	1.25	-0.0238	0.504	1.081	0.927	±16.2	±4.5	
8		1.50	-0.1241	0.946	1.023	0.985	±10.8	±3.0	
9		1.75	-0.1787	1.071	1.034	0.977	±10.4	±3.6	
10	90°	1.25	-0.0888	0.742	0.976	0.977	±16.9	±4.1	
11		1.50	-0.1405	0.978	1.010	0.981	±11.0	±3.2	
12		1.75	-0.1301	0.982	1.083	0.979	±10.6	±3.6	

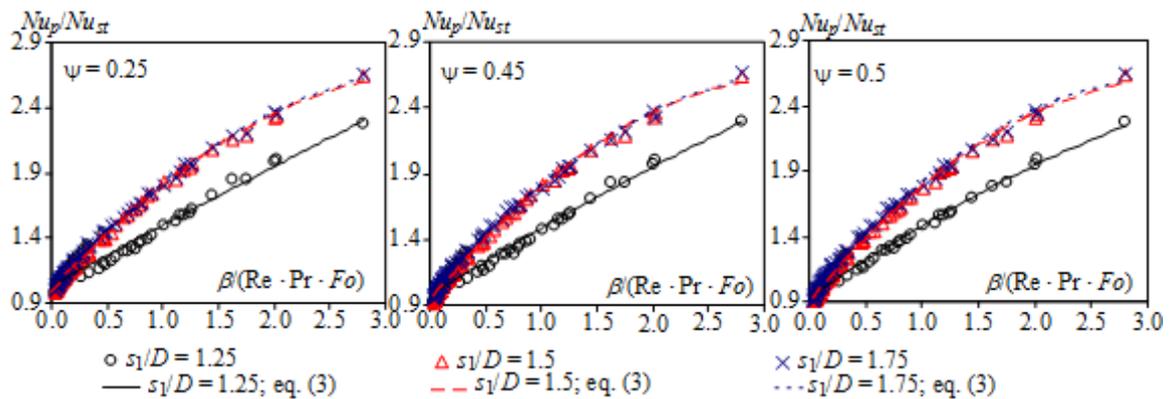


Fig. 1. Dependence of  $Nu_p/Nu_{st}$  from  $\beta/(Re \cdot Pr \cdot Fo)$  for  $\varphi = 30^\circ$

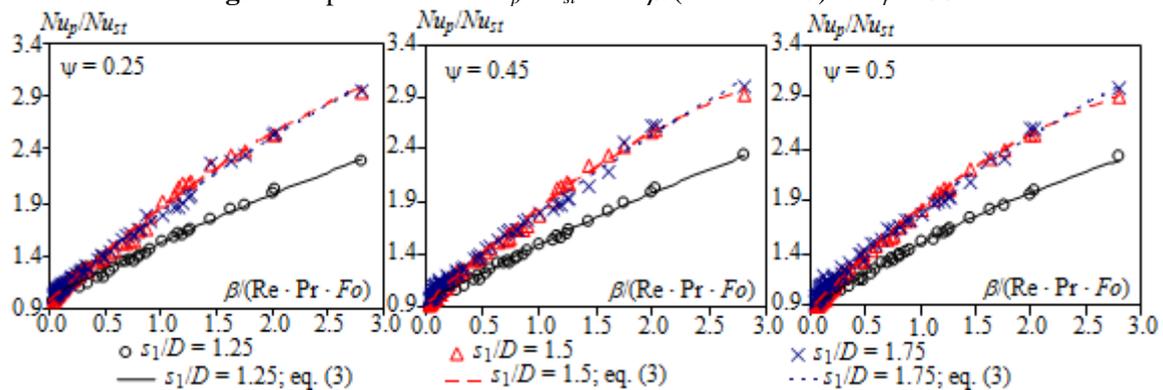
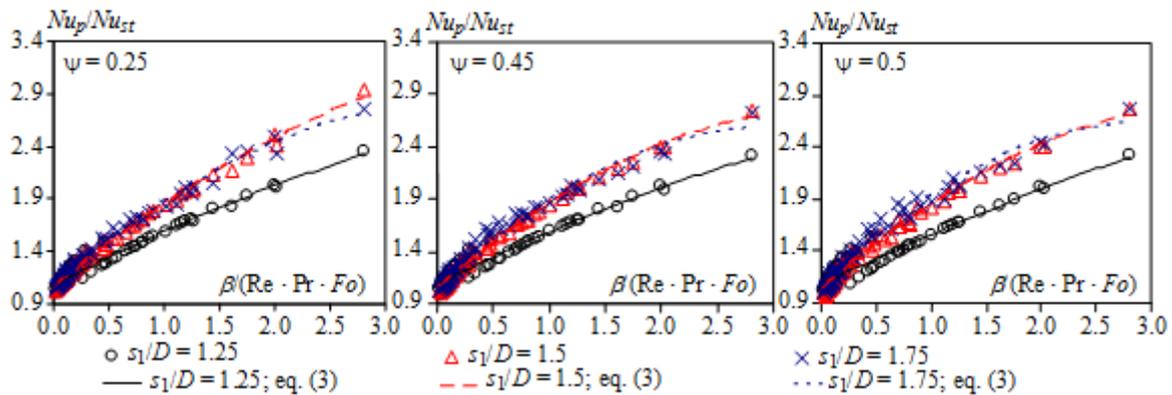
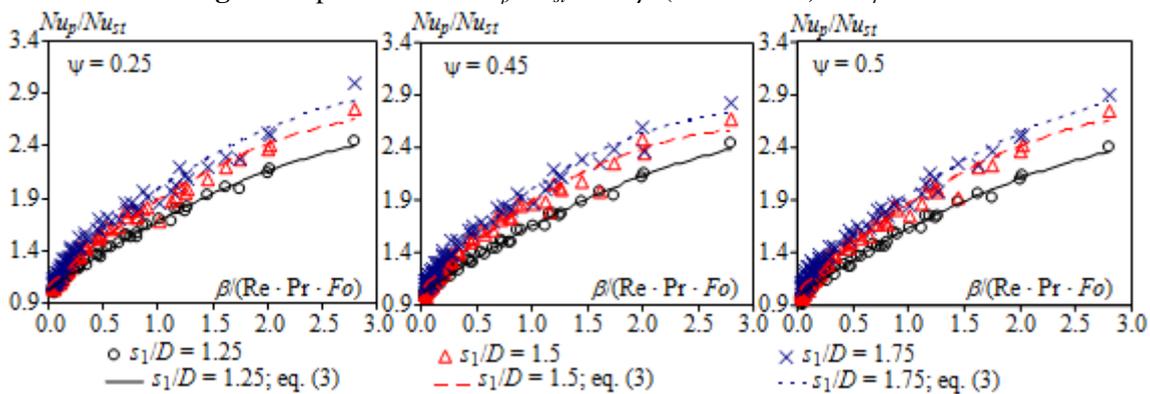


Fig. 2. Dependence of  $Nu_p/Nu_{st}$  from  $\beta/(Re \cdot Pr \cdot Fo)$  for  $\varphi = 45^\circ$



**Fig. 3.** Dependence of  $Nu_p/Nu_{st}$  from  $\beta/(Re \cdot Pr \cdot Fo)$  for  $\varphi = 60^\circ$



**Fig. 4.** Dependence of  $Nu_p/Nu_{st}$  from  $\beta/(Re \cdot Pr \cdot Fo)$  for  $\varphi = 90^\circ$

Analysis of the obtained empirical correlation (2) made it possible to estimate the influence of the pulsation parameters on the heat exchange for different tube bundles in the presence of flow pulsations. Relative amplitude  $\beta$  has minimal effect on  $Nu_p/Nu_{st}$  when  $\varphi = 60^\circ$ ,  $s_1/D = 1.75$ , and maximum at  $\varphi = 45^\circ$ ,  $s_1/D = 1.5$ . The Fourier  $Fo$  number has a minimal effect on  $Nu_p/Nu_{st}$  when  $\varphi = 45^\circ$ ,  $s_1/D = 1.25$ , and the maximum at  $\varphi = 90^\circ$ ,  $s_1/D = 1.75$ . When  $\varphi = 45^\circ$ ,  $s_1/D = 1.25$ , the pulse ripple  $\psi$  has the maximum effect on  $Nu_p/Nu_{st}$  and has practically no effect on heat transfer when  $\varphi = 60^\circ$ ,  $s_1/D = 1.75$ .

### 3. Conclusion

As a result of the study, a new empirical correlation is proposed for calculating the external heat transfer in conditions of pulsating currents in tube bundles. The new empirical correlation (3) has a maximum error of  $\pm 16.9\%$ , which is less than the maximum error of the equation presented in [6], which was  $\pm 35.7\%$ . A new empirical correlation can be used to calculate heat transfer in pulsating flows in heat exchanging equipment at Reynolds numbers  $\leq 1000$ . For example, shell and tube oil coolers, which are widely used in industry.

Analysis of the obtained empirical correlation (2) made it possible to estimate the influence of the pulsation parameters on the heat exchange for different tube bundles in the presence of flow pulsations. Relative amplitude  $\beta$  has minimal effect on  $Nu_p/Nu_{st}$  when  $\varphi = 60^\circ$ ,  $s_1/D = 1.75$ , and maximum at  $\varphi = 45^\circ$ ,  $s_1/D = 1.5$ . The Fourier  $Fo$  number has a minimal effect on  $Nu_p/Nu_{st}$  when  $\varphi = 45^\circ$ ,  $s_1/D = 1.25$ , and the maximum at  $\varphi = 90^\circ$ ,  $s_1/D = 1.75$ . When  $\varphi = 45^\circ$ ,  $s_1/D = 1.25$ , the ripple  $\psi$  has the maximum effect on  $Nu_p/Nu_{st}$  and has practically no effect on heat transfer when  $\varphi = 60^\circ$ ,  $s_1/D = 1.75$ .

The article is executed within the framework of the scientific project 18-79-10136 «Theoretical methods for modeling and developing energy-efficient import-substituting cleaners and deep processing of hydrocarbon raw materials at enterprises of the fuel and energy complex».

**References**

- [1] Tae Ho Ji *et al* 2008 *Int. J. Heat Mass Transfer* **51** 1130–38
- [2] Siddhanath V *at al* 2015 *Int. Research J. of Eng. and Tech.* **2** 487–92
- [3] Khaibullina A, Khairullin A *et al* 2014 *The European Phys. J. Conf.* **76** 01004
- [4] Ilyin V K, Sabitov L S, Haibullina A I and Hayrullin A R 2017 *IOP Conf. Ser.: Mater. Sci. Eng.* **240** 012026
- [5] Ilyin V K, Sabitov L S, Haibullina A I and Hayrullin A R 2017 *IOP Conf. Ser.: Mater. Sci. Eng.* **240** 012025
- [6] Ilyin V K, Sabitov L S, Haibullina A I and Hayrullin A R 2017 *IOP Conf. Ser.: Mater. Sci. Eng.* **240** 012027
- [7] Rouaud M 2013 *Probability, Statistics and Estimation* p 60.
- [8] Chiang C L 2003 *Statistical methods of analysis* (World Scientific Publishing) p 617