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Synthesis of the automatic control system with time-delays as in the case of internal combustion engine

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Abstract. There was conducted the synthesis of the automatic control system with time-delays for multivariable plants in terms of input terminal and the state based on the system embedding technology. To reproduce the plant state vector the compensatory-observation scheme was used, compensating temporary time-delays in controlling. Illustrated by multivariable model of the internal combustion engine the law of control was realized.

Time-delays, concentrated in the ducting of a plant, has a negative impact on controlling processes and can lead not only to the deterioration of the quality of the controlling processes but to the stability of the control system as a whole. Consequently, it's vital to take them into account while designing an automation system. The problem of controlling by such like plants is complicated because of their multivariable nature.

The article offers a precise analytical solution of the task of the synthesis of the automatic control system with time-delays (ACS) for multivariable plants in terms of controlling and the state based on the system embedding technology [1, 2]. It is suggested that the phase plant state vector is unavailable for direct observation. The matrix transmittance function of the regulator or the states determining it is identified whereby the functioning of the control system will be described by the required matrix transfer function (MTF). The synthesis is conducted through the forced element of the closed-loop dynamic system. Illustrated by multivariable model of the internal combustion engine the law of control is realized, which compensates the input time-delays and the achievement of the desired effect in the control system.

Let the observed and the controlled line permanently installed plant with concentrated time-delays in controlling and state be represented as difference-difference equation.

$$\dot{x}(t) = \sum_{i=0}^{l} A_i x(t-\tau_i) + \sum_{j=0}^{r} B_j u(t-\theta_j), \quad y(t) = Cx(t), \tag{1}$$

where $\tau_0 = 0, \quad 0 < \tau_1, \tau_2, ..., \tau_l$ – the permanent time-delays with matrixes A_i, $i = 0,1...,l$, $\theta_0 = 0$, $0<\theta_1, \theta_2, \ldots, \theta_r$ – the permanent time-delays with matrixes B_j, $j = 0,1,\ldots,r, u(t) \in R^s$ – the

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vector of input variables, $y(t) \in R^m$ – the vector of output variables, $x(t) \in R^n$ – the phase plant state vector. The matrixes A_i have a size of $n \times n$, B_j – $n \times s$, C – $m \times n$.

The initial data are set taking into account the time-delays of signals in controlled plants (CO) – as a matter of form we will consider the negative time span $t < 0$, implying that the plant involved dynamic processes before the start time.

$$x(t) = \varphi_x(t),\ t_0 - \tau \le t \le t_0,$$

where τ – the maximum time-delay.

Let the compensatory-observation control scheme, calculating the current phase plant vector $x(t)$ be represented by the equations:

$$\dot{x}_м(t) = \sum_{i=0}^{l} A_i x_м(t-\tau_i) + \sum_{j=0}^{r} B_j u(t-\theta_j),\quad y_м(t) = LCx_м(t),$$
$$\dot{\hat{x}}(t) = \sum_{i=0}^{l} A_i \hat{x}(t) + Ly(t) - y_м(t) + \sum_{j=0}^{r} B_j u(t), \qquad (2)$$

where $\hat{x}(t)$ – the phase vector of the observer state, $x_м(t)$ – the phase vector of the plant model state, $y_м(t)$ – the vector of output of plant model, L – the matrix of the observer.

The analysis of the performance of the compensatory-observation scheme described by equations (2) showed that it enabled to completely compensate time-delays in controlling and reproduce phase plant $x(t)$ state vector.

The application of Laplace transformation to the equations (1) gives the operator form of the description of a plant:

$$px(p) = \sum_{i=0}^{l} A_i e^{-\tau_i p} x(p) + \sum_{j=0}^{r} B_j e^{-\theta_j p} u(p),\quad y(p) = Cx(p). \qquad (3)$$

Let the control law in the general case be described by matrix equation:

$$\nu(p) = K(p)\hat{x}(p) + u(p), \qquad (4)$$

where $K(p)$ – matrix transfer function of the size regulator $n \times s$, $\nu(p) \in R^s$ – the control vector at the input of the system.

Let the required performance of automatic control system (ACS) be set by matrix transfer function $E_y^{\nu}(p)$ from control input v to output y.

The task is: it's necessary to find MTF of the regulator $K(p)$ or states defining it for CO (1), where the functioning of ACS will be described by MTF $E_y^{\nu}(p)$.

Furthermore during the synthesis of the control system we will use the system embedding technology [1, 2]. Considering the equations (1), (2), (4) and the procedure performance of the system enclosing method the matrix in question (promatrix) of the task under consideration will have the following view:

$$\Omega(p)=\begin{bmatrix} (pI_n-\sum_{i=0}^{l}A_i e^{-\tau_i p}) & 0 & 0 & 0 & 0 & -\sum_{j=0}^{r}B_j e^{-\theta_j p} & 0 \\ 0 & (pI_n-\sum_{i=0}^{l}A_i e^{-\tau_i p}) & 0 & 0 & 0 & -\sum_{j=0}^{r}B_j e^{-\theta_j p} & 0 \\ 0 & 0 & (pI_n-\sum_{i=0}^{l}A_i) & -L & I_m & -\sum_{j=0}^{r}B_j & 0 \\ -C & 0 & 0 & I_m & 0 & 0 & 0 \\ 0 & -LC & 0 & 0 & I_m & 0 & 0 \\ 0 & 0 & K(p) & 0 & 0 & I_s & -I_s \\ 0 & 0 & 0 & 0 & 0 & 0 & I_s \end{bmatrix}.$$

The input matrixes α и β, used during the inputs systems have the following view: $\alpha=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0 & I_s\end{bmatrix}^{\mathrm{T}}$, $\beta=\begin{bmatrix}0 & 0 & 0 & I_m & 0 & 0 & 0\end{bmatrix}$ while $\omega=E_y^v(p)$, where ω – the image of synthesized system – the required transfer function of the system.

After the completion of the procedures of inputs technique – the consecutive matrixes factorization $\Omega=\Sigma\Xi$, $\alpha=\Sigma\delta$, $\beta=\pi\Xi$, $\omega=\pi\delta$, one can receive equations which should be satisfactory for MTF of the regulator $K(p)$ during the synthesis in forced element $E_y^v(p)$ of the closed-loop dynamic system.

To solve matrix equations which appear as a result of application of inputs procedures it is possible to apply the device of matrix canonization [3].

The application of the system embedding technology during the synthesis in forced element produces the following equation as for the transmission matrix in question $K(p)$:

$$C(pI_n-\sum_{i=0}^{l}A_i e^{-\tau_i p})^{-1}\sum_{j=0}^{r}B_j e^{-\theta_j p}-E_y^v(p)=E_y^v(p)K(p)(pI_n-\sum_{i=0}^{l}A_i)^{-1}\sum_{j=0}^{r}B_j.$$

A great number of regulators in this case can be described by the formula:

$$\begin{aligned}\left\{K(p)\right\}_{\mu,\eta}&=(E_y^v(p))^{\%}[C(pI_n-\sum_{i=0}^{l}A_i e^{-\tau_i p})^{-1}\sum_{j=0}^{r}B_j e^{-\theta_j p}-E_y^v(p)]\times\\ &\times((pI_n-\sum_{i=0}^{l}A_i)^{-1}\sum_{j=0}^{r}B_j)^{\%}+\overline{E_y^v(p)}^{R}\mu(p)+\eta(p)\overline{(pI_n-\sum_{i=0}^{l}A_i)^{-1}\sum_{j=0}^{r}B_j)}^{L},\end{aligned}\qquad(5)$$

where $\eta(p)$, $\mu(p)$ – arbitrary fractionary polynomial matrixes of the corresponding sizes. The terms of the equation solution (5), and consequently the existence of abundance of solutions (6) have the following view:

$$\begin{aligned}&\overline{E_y^v(p)}^{L}[C(pI_n-\sum_{i=0}^{l}A_i e^{-\tau_i p})^{-1}\sum_{j=0}^{r}B_j e^{-\theta_j p}-E_y^v(p)]=0,\\ &[C(pI_n-\sum_{i=0}^{l}A_i e^{-\tau_i p})^{-1}\sum_{j=0}^{r}B_j e^{-\theta_j p}-E_y^v(p)]\overline{((pI_n-\sum_{i=0}^{l}A_i)^{-1}\sum_{j=0}^{r}B_j)}^{R}=0.\end{aligned}\qquad(6)$$

Thus, we get the ratio (5) with the terms of existence of this solution (6), which allows to find a great number of regulators, satisfying the control law (4).

As in the case of synthesis of the control system of the internal combustion engine [4] the efficiency of the problem solution under consideration was exemplified.

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