The Ballistic Design of Intercepting the Ultra-low Altitude Target

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Abstract. According to the multipath effect problem that the missile intercepts the ultra-low altitude target, when the grazing angle near the Brewster Angle, the reflection coefficient of clutter reach the minimum, and the precision of guidance is higher. To ensure that the missile can intercept the target effectively under the vertical launch condition, a super-low-altitude intercept trajectory model is established. Then an improved proportional guidance law satisfying the Brewster Angle constraint is developed, and the overall trajectory of the missile from launch to strike target finally is designed. The Simulation show that the trajectory designed is relatively straight and meets the requirements. From the perspective of trajectory design, a way to solve the problem of the seeker detection have been explored.

1. Introduction

When the air defense missile intercepts the ultra-low altitude target, it will be affected by the specular reflection signal due to the influence of terrain or the density of ground and sea clutter environment, then causing a multi-path effect[1-2]. It will result in the radar seeker effectively failing to track and recognize the target so that it seriously affects the operational performance of the missile. With the rapid development of the ultra-low-level flying technology, it will be particularly important to study anti-aircraft missile which has the anti-ultra-low altitude penetration capabilities.

The angle between the connecting line of the missile and the mirror target and the horizontal plane is called the grazing angle [3]. The study shows that when the grazing angle reaches a certain angle, the density reflection coefficient of clutter environment is the smallest, and the angle is called the Brewster Angle [4-5]. Therefore, it can reduce the influence of ground image interference that using a vertically polarized radar illumination signal and making the rubbing angle is close to the Brewster Angle by ballistic design and guidance law optimization during the missile attack target and is feasible for intercepting ultra-low altitude targets. Based on this idea, the paper designs a general ballistic scheme including the initial stage (the program turning) and the middle stage (the guided law compensation) and the terminal (proportional guidance law), then carries out Matlab simulation verification and analysis and makes a conclusion.

2. Multipath effect problem description and the ultra-low altitude interception model

2.1. Multipath Effect Problem Description

Figure 1 shows: F is the ground clutter reflection point; β is the grazing angle; P_1 is the path which target signal return, and P_2 is the path which the density of clutter environment reflects. When the

target is flying at low altitude, the ground clutter reflection intensity is very high (sometimes even beyond the target body), so the seeker may attack the mirror target as real target.
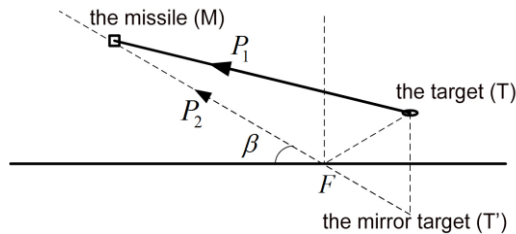

Figure 1. The geometry of missile, target and mirror

The reflection coefficient of the ground clutter varies with the change of the grazing angle. When the grazing angle reaches a certain angle, the reflection coefficient is the minimum. When this reflection coefficient is the minimum, the corresponding grazing angle is called the Brewster Angle. But the size of the Brewster angle is not the same in different environments (grassland or sea). Taking a constant environment as an example [6], the law of the reflection coefficient with the change of the grazing angle is shown in Figure 2.
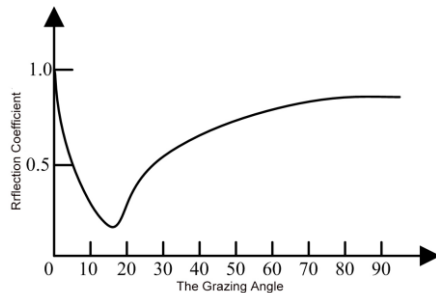

Figure 2. The relation between reflection coefficient and erasure angle

2.2. Approximate calculation of the grazing angle

From the point of the intercepting need for the ultra-low altitude target, we must ensure that the grazing angle of the middle trajectory is constrained near the Brewster Angle. However, actually it is difficult for the missile to measure the size of the grazing angle in real time, and it is more easily realized through the angle of sight between the missile and the target from the perspective of the guidance law. Therefore, the difference between the sightline angle and the grazing angle is analyzed at first [7].

For example, when the missile is flying the flight altitude of 150 meters, the values of the sight line angle are calculated with different distance at the same time the grazing angle is 10 degrees.

Table 1. The error between the erasure angle and the line of sight angle

distance/km	LOS/°	erasure angle /°	error/°
5	6.64	10	3.36
10	8.32	10	1.68
15	8.88	10	1.12
20	9.16	10	0.84
30	9.44	10	0.56
40	9.58	10	0.42
50	9.67	10	0.33
60	9.72	10	0.28

From Table 1, it can be seen that the difference between the sight angle of the true target and the sight angle of the mirror target relative to the missile intercepting an ultra-low altitude target is very small for the flight trajectory, when the distance between the missile and the target is far away. When the target approaches, the difference increases. It is easier to implement the correction for the angle of sight in the design process of the guidance law although there is a certain error between the two angles.

Therefore, we can implement the correction of the sight angle or the line of sight angular velocity and constrain it near the Brewster Angle to improve missile guidance accuracy when designing the mid-trajectory guidance law.

2.3. The ultra-low altitude interception model
The establishment of the ultra-low altitude interception model is as follows. The three-dimensional space can be divided into the vertical pitch plane and the transverse turning plane. For the convenience of analysis, the vertical pitch plane can only be studied. As is shown in Figure 3: M is the missile; v_m is the speed of the missile; θ_m is the trajectory inclination angle of the missile; a_m is the normal acceleration of the missile; T is the target; v_t is the speed of the target; θ_t is the dead reckoning angle of the target; a_t is the normal acceleration of the target; R is the distance from the missile to the target; q is the angle of sight between the missile and the target. According to the Flight Mechanics [8], the counter clockwise rotation is positive and the clockwise rotation is negative for all angles.
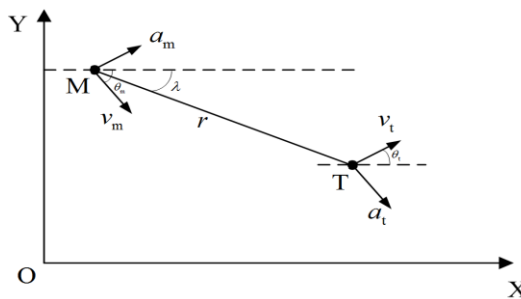

Figure 3. The motion relation between missile and target

As can be seen from Figure 3, the relative equation of motion between the missile and the target [9] is

$$\begin{cases} \dot{R} = -v_m \cos(q-\theta_m) + v_t \cos(q-\theta_t) \\ R\dot{q} = v_m \sin(q-\theta_m) - v_t \sin(q-\theta_t) \end{cases} \tag{1}$$

According to formula (1) can be drawn:

$$\ddot{q} = \frac{-2\dot{R}\dot{q}}{R} - \frac{a_m \cos(q-\theta_m)}{R} + \frac{a_t \cos(q-\theta_t)}{R} \tag{2}$$
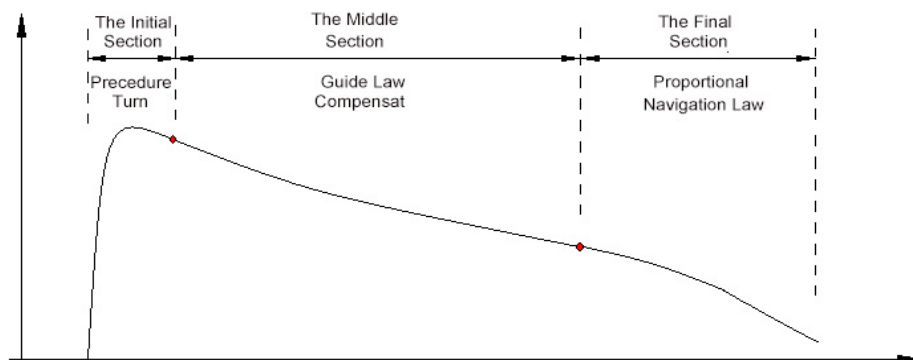
3. Ballistic design

Figure 4 The ballistic scheme

The missile flies close to the ground or the surface of the water in the process of intercepting the target, and the missile can fly at a higher altitude and track the target by the radar seeker. The characteristics of the target movement determine the trajectory of interception for the ultra-low altitude targets.

The missile is launched vertically from the ground, and turns toward the target after reaches a certain altitude, then the missile enters the midcourse guidance section when the seeker intercepts the target. During the middle flight, the target information is detected by the missile seeker on the missile and the missile dives in the end. It is necessary to control the angle of sight near the Brewster Angle in the middle flight after turning.

According to the above requirements, the ballistic design scheme is shown in Figure 4.

According to the characteristics of the flight trajectory, it can be divided into the initial turn section, the middle guidance section and the end attack section [10].

3.1. The initial turn section

For the flight trajectory design for intercepting the ultra-low altitude target, the initial turning section is controlled by the program and has nothing to do with the target. After the missile is launched at a predetermined elevation angle, it reaches a certain altitude and turns toward the target. In this process, we can provide better initial conditions for pilot flight in the middle section by adopting the given design rules of the trajectory inclination angle through the reasonable parameter setting. The initial turn section is specifically divided into the following stages [11]:

(1) The booster

After launching the missile, the missile is accelerated according to the given trajectory inclination (the trajectory inclination angle is a constant value) under the influence of the booster. The working time of the booster is t_1, then the change law of the trajectory inclination angle and acceleration is:

$$\begin{cases} \theta_*(t) = \theta_0 & 0 \le t < t_1 \\ a_*(t) = a_m & 0 \le t < t_1 \end{cases} \tag{3}$$

(2)The gliding section

After the work of the booster is over, the missile keeps its speed unchanged and climbs upwards, then begins to turn toward the target after it rises to a certain height. Remember the starting turn time is t_2, then the change law of the trajectory inclination angle and acceleration is:

$$\begin{cases} \theta_*(t) = \theta_0 & t_1 \le t < t_2 \\ a_*(t) = 0 & t_1 \le t < t_2 \end{cases} \tag{4}$$

(3) The turning section

After the gliding segment was over, the missile begins to make a turn according to the predetermined procedure and eventually turned flat. The change law of the trajectory inclination angle of the turning section can be given in terms of flight time. Remember the ending time of the turning section is t_3. In order to meet the continuous change of the ballistic inclination angle and the ballistic inclination rate during the flight, the change law of the ballistic inclination of the turning section must meet the following conditions:

①At the beginning of the turning section, the ballistic inclination angle meets $\theta_*(t_1) = \theta_0$ and the change rate of the ballistic inclination angle meets $\dot{\theta}_*(t_1) = 0$;

② The change rate of the ballistic inclination meet $\dot{\theta}_*(t_2) = 0$ at the end of the turn.

3.2. The midcourse guidance section

Most of the active missiles adopt the classical proportional guidance law or the modified proportional guidance law. When the missile is guided by the proportional guidance law, the line of sight angle basically depends on the initial missile launch information, and there is no Brewster Angle constraint requirement. So it is unable to meet the Brewster Angle constraint. Therefore, based on the existing proportional guidance law, the proportional guidance law is modified by introducing the compensation amount so that the angle of sight meets the Brewster Angle constraint during the missile attack. The revised guideline can be written as[12-13]:

$$\frac{d\theta}{dt} = K\frac{dq}{dt} + x \tag{5}$$

In the formula (5), x is the compensation amount.

Assuming that at the initial time of the guidance section, the angle of sight is q_0, the change rate of the angle of sight is $\dot{q}_0$, and the angle of sight required by the Brewster Angle constraint is q_b. According to the Brewster Angle constraint requirement, the law of angular velocity change of the sight angle as shown in Figure 5 which needs to be designed. After the time t_f from the initial moment of the middle stage, the angle of sight changes from q_0 to q_b and the angle of sight angular velocity becomes smoothly 0 (namely $\dot{q}_e(t_f) = 0$, $\ddot{q}_e(t_f) = 0$); When $t > t_f$, keeping the line of sight unchanged ($\dot{q}_e(t) = 0$).
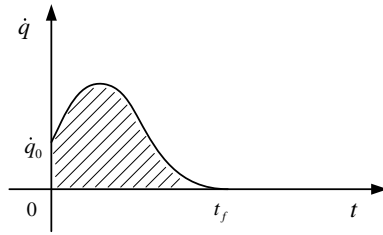

Figure 5. The line of sight angular velocity curve

From Figure 5, we can see that from time 0 to time t_f, the change law of the line-of-sight angular velocity satisfies the cubic spline curve, and its polynomial expression can be written as:

$$\dot{q}_e(t) = at^3 + bt^2 + ct + d \quad (0 < t \leq t_f) \tag{6}$$

Among them, a, b, c, d are the parameters to be determined. Therefore, the law of line-of-sight angle which satisfies Brewster Angle constraints can be expressed as:

$$\begin{cases} \dot{q}_e(t) = at^3 + bt^2 + ct + d & 0 < t < t_f \\ \dot{q}_e(t) = 0 & t \geq t_f \end{cases} \tag{7}$$

The derivation of the midcourse guidance law is as follows:

According to the missile-target relative motion equation, the amount of the change in the line-of-sight angular velocity caused by the compensation amount during Δt time can be expressed[14] as:

$$\begin{aligned} \dot{q}_e - \dot{q} &= \left[V\sin(q-\theta-x\Delta t) - V_T\sin(q-\theta_T)\right]/r - \\ &\quad \left[V\sin(q-\theta) - V_T\sin(q-\theta_T)\right]/r \\ &= \left[V\sin(q-\theta)\cos(x\Delta t) - V\cos(q-\theta)\sin(x\Delta t)/r\right]/r - \\ &\quad V\sin(q-\theta)/r \end{aligned} \tag{8}$$

During the missile attacks the target, because the radar seeker and the control system have higher operating frequencies, it can be assumed that the missile trajectory inc lination angle caused by the compensation amount changes a small amount in a relatively short period of time (or $x\Delta t$ is a small amount). Then:

$$sin(x\Delta t) \approx x\Delta t, \quad cos(x\Delta t) \approx 1 \tag{9}$$

The formula (8) can be written as:

$$\begin{aligned} \dot{q}_e - \dot{q} &= [V\sin(q-\theta) - V\cos(q-\theta)x\Delta t - V\sin(q-\theta)]/r \\ &= -V\cos(q-\theta)x\Delta t/r \end{aligned} \tag{10}$$

Among them, $\dot{q}_e$ is the line-of-sight angular velocity obtained by the modified guidance law. $\dot{q}$ is the line-of-sight angular velocity obtained according to the classical proportional guidance law. The amount of compensation that can be solved from the above formula is:

$$x = -\frac{r(\dot{q}_e - \dot{q})}{V\Delta t cos(q - \theta)} \tag{11}$$

3.3. The end attack segment

After entering the end, the seeker can effectively distinguish the target, so the Brewster Angle constraint can be dismissed and the missile's own guidance law can be used to design and achieve an effective attack on the target [15]. In general, the proportional guide law is:

$$\frac{d\theta}{dt} = K \frac{dq}{dt} \tag{12}$$

4. The ballistic simulation

In accordance with the trajectory scheme of the initial corner section, and the middle section guidance section and the end attack section designed in the previous section, the missile flight process was simulated and analyzed [16-17].

    Assuming that the ultra-low altitude target makes a uniform straight flight along the ground. The initial height $Y_t = 100\text{m}$, the initial position $X_t = 80km$, speed $V_t = 270m/s$, the trajectory inclination angle $\theta_t = 0°$; the missile launches vertically from the ground, the boost acceleration $a_{m1} = 30g$, the engine working time $t_1 = 6s$, the acceleration of turning section $a_z = g$, the starting time of turning section $t_2 = 8s$, the time at the end of the turning section $t_3 = 16s$, the trajectory inclination in the end $\theta_m = 0°$, the mid-adjustment time $t_f = 5s$, and the distance to the end $X_1 = 6km$;the Brewster's angle is 15° around [13-15].
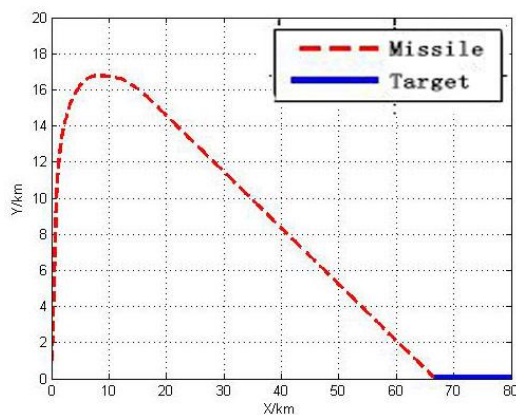

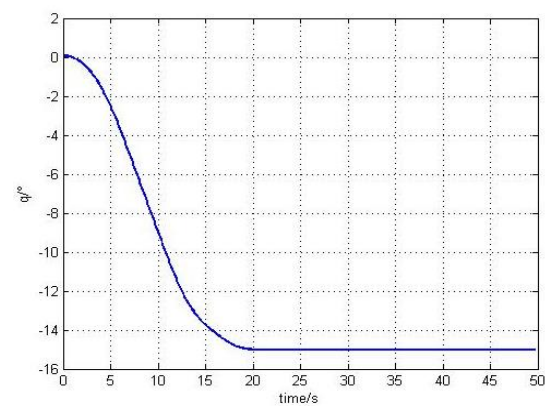**Figure 6.** The trajectory of missile and target

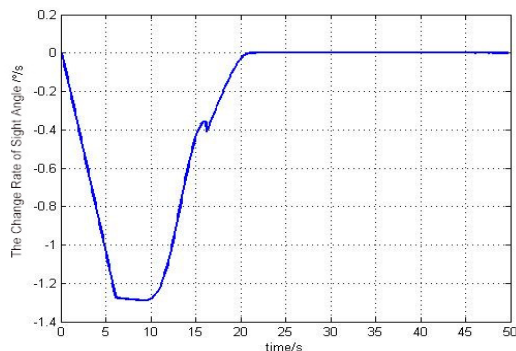**Figure 7.** The variation od the view angle

Figure 8. The variation of the sight angular rate
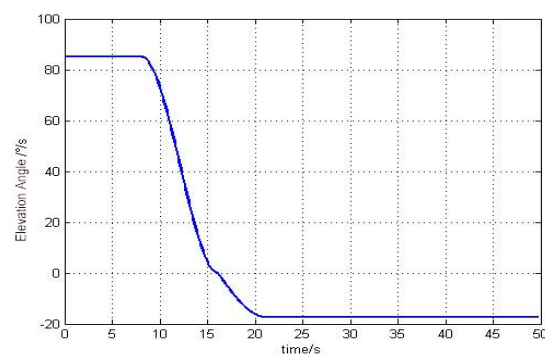

Figure 9. The variation of the angle of the trajectory

    It can be seen from Figure 6 that the missile is launched vertically from the ground, then goes through a turn of the initial stage program, the midcourse guidance and the end of the attack, finally intercepts the target successfully, and the designed trajectory is relatively flat. It can greatly enhance the maneuverability of the missile, reduce the overload and improve the guidance accuracy. From

Figure 7 and 8, it can be seen that after the turn is completed, the angle of sight can be quickly approached to the Brewster Angle and remain unchanged after arrival. The change of the line-of-sight angular rate also tends to 0. So it meets the guidance requirements. It can be seen from Figure 9 that the trajectory inclination of the missile is 0° after the turning of the program. It satisfies the set value, and the velocity inclination of the ballistic trajectory remains basically unchanged.

5. Conclusion

The article discusses the problem that the missile intercepts the ultra-low altitude target under vertical launch conditions. For the multi-path problem during the interception process, the position of the Brewster Angle is determined, and the interception trajectory model is established, and the improved proportional guidance law which constrains the angle of sight near the Brewster Angle is studied to make the density of ground and sea clutter environment minimum. Then the ballistic trajectory is divided into the initial turn section, the middle guidance section and the end attack section. The full range trajectory of the missile from the launch to the final hit is designed by simulation. The results show that the designed trajectory is relatively flat and meets the requirements, and the guidance accuracy is so high that it has a strong reference value.

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