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A FEM Based on the Parametric Variational Principle for the Cable-network Antenna's Form-finding Design

Tao Liu[1,2], Hao Li[2], Guoan Tang[1], Xin Zhou[2] and Liang Zhang[3]

[1]Department of Aeronautics and Astronautics, Fudan University, Shanghai 200433, China
[2] Aerospace System Engineering Shanghai, Shanghai 201109, China;
[3]College of Aerospace Engineering, Chongqing University, Chongqing 400044, China

*Corresponding author's Email: liutaobj@163.com

Abstract: According to the form-finding of the flexible cable-network antenna, a new form-finding analysis approach based on FEM is presented. A parametric variable and Co-rotational formulation are used to model antenna's constitutive nonlinearity and geometrical nonlinearity, and then the FEM governing equation is established. In order to improve the convergence ability, the Lemke algorithm combined with the improved Newton method is employed to solve the governing equation. The method is applied for nonlinear form-finding analysis of cable-network antenna and an ideal configuration is obtained. The research shows that the method can also be applied for studying other similar antennas.

1. Introduction

Cable-network antenna can be used as UHF, microwave and millimeter wave communications satellite antenna. It has many advantages such as high gain, light structural mass, high stowing ability and large aperture application rang. According to the nonlinear characteristics of flexible cable-network antenna, the form-finding of cable-network antenna is a great challenge to the design of deployable cable-network antenna [1]. Many researchers have done a lot of work on the form-finding of cable-network antenna. Their research methods mainly can be divided into two categories: linear form finding method and nonlinear form- finding method.

The force density method firstly given by Linkwitz [2] is the most widely used linear form-finding method. The basic principle of this method is to establish a force balance equation on each node of the cable-network and introduce the force density to transform the nonlinear equilibrium equation into a linear system of equations. On this basis, Morterolle [3] developed the force density method with iterative algorithm for the isotension design. Yang [4] proposed an optimized iterative design method combined with form-finding and force-finding based on the force density method. Lin [5] proposed a form-finding method of combing the improved fish swarm algorithm with the force density method. However, when solving large or irregular tension structures, the force density method is not efficient.

The finite element method is one of the general application nonlinear form- finding methods. Xia [6] used the structural reset method to optimize the pretension of the cable structure and proposed an iterative method for the optimal design of the pre tension. However, the distribution of the pretension in the iterative method depends on the initial value of the pretension, and the distribution of the pretension is difficult to control during the iterative solution, which makes it difficult to guarantee the uniformity of the pretension of the cable. Li [7] proposed a method for the cable pretension optimization and surface adjustment of combing genetic algorithm and finite element method combined. However, for large-scale cable antenna, the method suffers from several problems such as too many variables and convergence difficulty. The geometrical nonlinearity of cable antenna and the constitutive nonlinearity of cable are the difficulties of finite element calculation. Traditionally, The NR iteration technique is employed to solve the nonlinear finite element equations of the structure. In each iteration step, the tensile or compressive modulus is re-selected according to the current stress state of the cable. In the case of more cables, it is easy to bring instability and convergence difficulties. In order to improve the convergence of the finite element method to calculate the cable structure, many scholars have also carried on the corresponding research. Zhang [8] developed a new stable algorithm based on the parametric variational principle (PVP) for the geometric nonlinear analysis of bi modular truss and tenegrity structures. The method can improve the stability of the numerical calculation. Tan [9] applied PVP to the finite element analysis of cable-network antenna. He also used Total-Lagrangian column to describe the geometric nonlinearity of the cable net and NR iteration method to solve the equation. But the TL method is proved to be of poor accuracy and slow convergence speed in large deflection calculation [10]. NR method has local convergence property, if the initial configuration of the cable antenna and the initial stress are not ideal; the convergence of the iterative process is difficult to guarantee [11].

In this paper, according to the constitutive nonlinearity and geometrical nonlinearity of the flexible cable-network antenna, a finite element method based on parametric variational principle and Co-rotational formulation is used for the internal force calculation. Then the method combined with iterative algorithm is used for the modular antenna and the parabolic cylinder antenna' form-finding. The research shows that the method has good form-finding accuracy, and can be widely applied to the design of the cable-network antenna.

2. Description of the parametric variational principle

The antenna's cables can only sustain tension. In the presence of the initial pretension, the constitutive relationship can be expressed as:

$$\sigma = \begin{cases} E^+(\varepsilon + \dfrac{\sigma_0}{E^+}) & \varepsilon \geq -\dfrac{\sigma_0}{E^+} \\ 0 & \varepsilon < -\dfrac{\sigma_0}{E^+} \end{cases} \tag{1}$$

Where E^+ is the tensile elastic modulus; ε and σ are the axial strain and axial stress.

For the Eq. (1) to introduce a parametric variable λ, The formula can be expressed as follows:

$$\sigma = E^+(\varepsilon + \frac{\sigma_0}{E} + \lambda) \tag{2}$$

And λ should satisfy the following condition:

$$\lambda = \begin{cases} 0 & \varepsilon \geq -\dfrac{\sigma_0}{E^+} \\ -(\varepsilon + \dfrac{\sigma_0}{E^+}) & \varepsilon < -\dfrac{\sigma_0}{E^+} \end{cases} \tag{3}$$

By introducing a non-negative slack variable, the cable's tension and compression state control equation can be obtained:

$$\begin{cases} E^+(\varepsilon + \dfrac{\sigma_0}{E^+} + \lambda) - \nu = 0 \\ \lambda \geq 0, \nu \geq 0, \lambda\nu = 0 \end{cases} \tag{4}$$

where $\lambda\nu = 0$ is a complementary condition, when $\lambda = 0, \nu = 0$,The cable is in a non-deformed state, when $\lambda > 0, \nu = 0$,The cable is in a compressed state, when $\lambda = 0, \nu > 0$,The cable is in a stretched state.

3. A uniform description of the parameter variable and CR formulation

In this paper, the CR formulation is used to describe the geometric nonlinearity of the cable element. The CR formulation has a more concise column and higher computational efficiency in large deflection, large angle and small strain analysis. The tangent stiffness matrix of the cable element combined with the parametric variable and CR formula can be expressed as:

$$\boldsymbol{K}_T = \boldsymbol{K}_L + \alpha \boldsymbol{K}_N \tag{5}$$

$$\alpha = 1 + \frac{\lambda}{\varepsilon + \varepsilon_0} \tag{6}$$

Where $\boldsymbol{K}_L$ and $\boldsymbol{K}_N$ are linear and geometric stiffness matrix, ε is the element strain of current configuration, ε_0 is the initial strain caused by initial pretension.

By assembling the unit stiffness matrix $\boldsymbol{K}_T$, parametric variable λ and slack variable ν. The governing equations of the nonlinear finite element of the cable-net structure can be obtained

$$[\Delta \boldsymbol{P}] = [\boldsymbol{K}_T]\{\Delta \boldsymbol{\delta}\} \tag{7}$$

$$\begin{cases} \boldsymbol{A}(\boldsymbol{\lambda} + \boldsymbol{\varepsilon} + \boldsymbol{\varepsilon}_0) - \boldsymbol{\nu} = \boldsymbol{0} \\ \boldsymbol{\nu}^T \boldsymbol{\lambda} = \boldsymbol{0}, \boldsymbol{\lambda} \geq \boldsymbol{0}, \boldsymbol{\nu} \geq \boldsymbol{0} \end{cases} \tag{8}$$

where $\boldsymbol{A}$ is the $diag(E_i^+)$, $\boldsymbol{\lambda}$ is the $diag(\lambda_i)$, $\boldsymbol{\varepsilon}$ is the $diag(\varepsilon_i)$, $\boldsymbol{\varepsilon}_0$ is the $diag(\varepsilon_{0i})$, $\boldsymbol{\nu}$ is the $diag(\nu_i)$, i is the corresponding ith cable element. Eq. (8) is a linear complementarity problem and can be solved by Lemke algorithm. The NR method is the most commonly used iterative method for solving Eq. (7). However, the NR method is locally convergent, and it would be difficult to converge when the given initial pretension is not near the equilibrium state of the cable. In order to relax the selection of the initial value, the improved NR method is used in conjunction with the linear search method [11].

4. Form-finding of cable-network antenna

According to the above theory, the nonlinear finite element program was compiled on the Matlab. Then the program is used to the form-finding of the modular cable-network antenna shown in Fig.1. The model has a total of 181 nodes and 408 elements. The tensile modulus of the cable is 20GPa and the compressive modulus is zero. The nodes on the main radial direction of the back cable net are constrained.

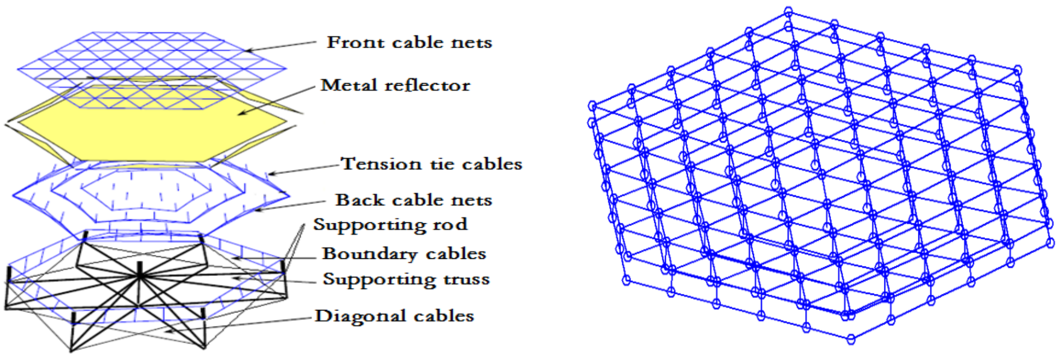


Figure.1 Structural configuration of the modular cable-network antenna

The form-finding process is as follows：

Step 1: Give the initial pretension of cable network T_0 .

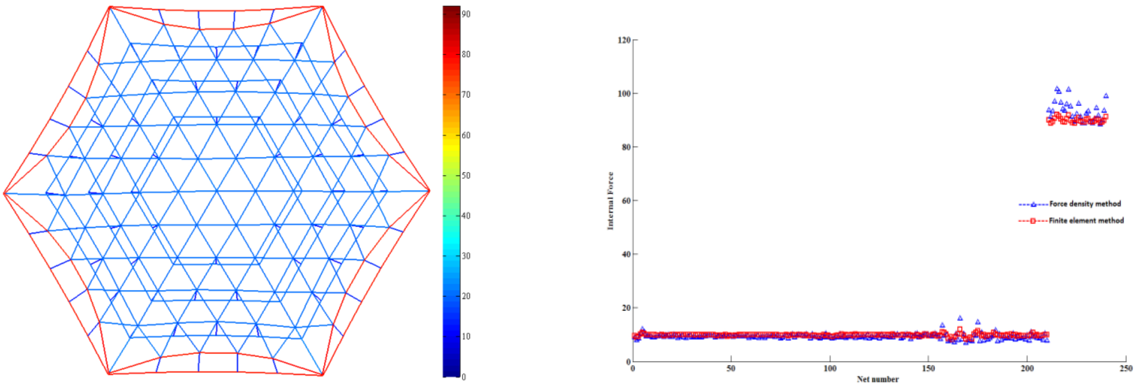
Step 2: Calculate cable network node coordinates (x,y,z) and cable tension T

Step 3: Calculate the error of the front cable network node coordinates between before and after $tol = \sum_{i=1}^{n} norm((x_i, y_i, z_i)_{i+1} - (x_i, y_i, z_i)_k)$, and calculate the rise-span ratio $\rho = \frac{\delta}{l}$.

Step 4: Determine whether T , ρ and tol sastify the constraint condition, if the condition is satisfied, turn to the follow. Otherwise, update the initial tension T_0 and coordinates of cables to and (x, y, z) , then turn to step 1.

Step 5: Output cable node coordinates and tension T, end.

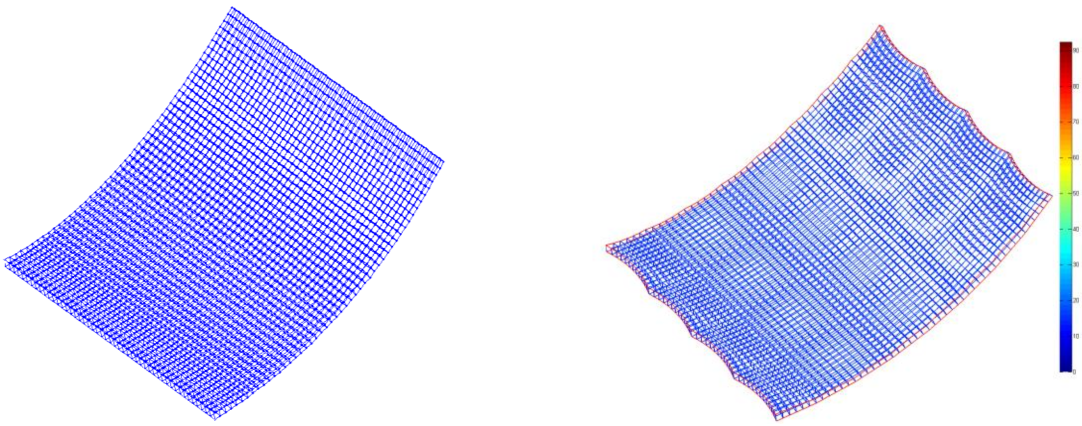
Under the above-mentioned index, the form-finding result shown in Fig. 2(a) is obtained. The results of the finite element method are compared with the traditional force density method. The internal force distribution of the front cables are shown in Fig. 2(b), it can be seen that the internal force of the two methods is consistent. It suggests that the calculation method adopted in this paper is feasible.



(a) Form-finding results of cable nets (b) Internal force of front cables

Figure.2 Internal force distribution of form-finding results of cable-network antenna

In order to prove the above method is also applicable to a more complex antenna. The form-finding design of the parabolic cylinder antenna has been carried out. Fig.3 (a) is the initial state of antenna with total of 4802 nodes and 11809 elements and Fig.3 (b) is the form-finding result. The result shows that the method can also be applied for studying other antennas.



(a)Initial state of cable nets (b)Form-finding result of cable nets

Figure.3 Form-finding design of the parabolic cylinder antenna

5. Conclusion

According to the constitutive nonlinearity and geometric nonlinearity characteristics of flexible cable-network antenna, the nonlinear FEM governing equation combine with the parametric variational principle and CR formulation is established. The Lemke algorithm combined with the improved Newton method is employed to solve the governing equation. The method is applied to the form-finding of the cable-network Antenna. Through analysis, The ideal configuration is obtained. In comparison with the force density method, it is proved that the method is feasible and has better uniformity of internal force distribution.

Further, the method in this paper can also be applied to form-finding under the flexible boundary and pretension optimization of cable-network antenna.

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