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Research on Cooperative Planning Technique of Gliding Trajectory for Multi-aircraft

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Abstract. In order to make multi-aircraft cooperating with each other and reaching the specified target area at the same time, the gliding trajectory cooperating planning is studied. The trajectory planning model is established. For a single aircraft, online trajectory generation based on the height vs velocity (H-V) profile is achieved, and a feedback linearization tracking guidance law is designed. For multi-aircraft, a multi-aircraft trajectory planning strategy based on time coordination is designed. Multi-aircraft trajectory cooperating planning under two different operational backgrounds is simulated. Simulation results show that the algorithm can achieve multi-aircraft trajectory cooperating planning, and multi-aircraft can reach the target area at the same time.

1. Introduction

With the strengthening of the aerial defence system, traditional single-aircraft is facing severe challenges in implementing penetrating and striking target. To achieve effective penetration and improve the effectiveness of cooperative operations, the US military took the lead in putting forward the concept of cooperative operations in the mid-1970s and achieved a series of research results.

At present, the main contents of multi-aircraft cooperative combat research include attack time coordination [1-5], attack angle coordination [6], attack time and attack angle coordination [7-11] and multi-aircraft formation control. In general, there are three deficiencies. First, most of the research objects are low-velocity aircraft, and they are considered to fly under uniform or uniform acceleration. There is a lack of research on cooperative operations for hypersonic vehicles. Second, it does not consider the effect of complex battlefield environment such as heat flow, dynamic pressure and overload. Third, the cooperative operation distance is close and the guidance is mostly terminal cooperative guidance.

In this paper, the cooperative engagement of multi-hypersonic gliding aircrafts are used as the study object. In order to make multi-aircraft reaching the specified target assembly area at the same time, the gliding trajectory cooperative planning is studied. Firstly, the trajectory planning model is established. Secondly, the trajectory of a single aircraft is generated online based on the HV flight profile. A nonlinear tracking law using feedback linearization method is used to track the standard trajectory. Finally, a multi-aircraft trajectory cooperative planning strategy is proposed. The simulation results are carried out in two different operational backgrounds. The illustrative examples show that the method is effective to solve the trajectory cooperative planning problem.

2. Dynamic equation

In order to simplify the calculation, the influence of the flattening rate is ignored and the three-degree-of-freedom motion equation of the aircraft is established as follows [12].

$$\begin{cases} \dot{r}=V\sin\theta \\ \dot{\lambda}=\dfrac{V\cos\theta\sin\sigma}{r\cos\phi} \\ \dot{\phi}=\dfrac{V\cos\theta\cos\sigma}{r} \\ \dot{V}=\dfrac{-D}{m}-g\sin\theta+\omega^2 r\cos\phi\left(\sin\theta\cos\phi-\cos\theta\sin\phi\cos\sigma\right) \\ \dot{\theta}=\dfrac{1}{V}\left(\dfrac{L}{m}\cos\nu-g\cos\theta+\dfrac{V^2}{r}\cos\theta+2\omega V\cos\phi\sin\sigma+\omega^2 r\cos\phi(\cos\theta\cos\phi+\sin\theta\sin\phi\cos\sigma)\right) \\ \dot{\sigma}=\dfrac{1}{V}\left(\dfrac{L\sin\nu}{m\cos\theta}+\dfrac{V^2}{r}\cos\theta\sin\sigma\tan\phi-2\omega V\left(\tan\theta\cos\phi\cos\sigma-\sin\phi\right)+\dfrac{\omega^2 r}{\cos\theta}\sin\phi\cos\phi\sin\sigma\right) \end{cases} \tag{1}$$

Where m is the mass of the aircraft, g is the gravity acceleration, V is the velocity; θ is the flight path angle; σ is the heading angle; r is the radial distance from the earth center to the aircraft. And $r = R_e + h$, where R_e is earth's average radius, h is the height of the aircraft from the local level; λ is the geographical longitude; ϕ is the geographical latitude; ν is the bank angle; $\omega_e = 7.292\times10^{-5}$ rad/s is the earth rotation. L and D are the aerodynamic drag force and lift force respectively. Their specific expression are given by

$$\begin{cases} L = 0.5\rho V^2 C_L S \\ D = 0.5\rho V^2 C_D S \end{cases} \tag{2}$$

Where S_M is the reference area. The atmospheric density $\rho = \rho_0 \exp\left(-\beta\left(r - r_0\right)\right)$, where ρ_0 =1.225kg/m^3 and β =1/7100. The drag coefficient C_D and lift coefficient C_L are related to the shape and flight state of the aircraft.

3. H-V flight profile generation and tracking law using feedback linearization

3.1. H-V flight profile generation

In general, the aircraft reentry process can be divided into three phases: the initial descent phase, the gliding phase, and the terminal guidance phase. The gliding trajectory is designed to two segments which can be obtained by cubic polynomial planning, namely

$$H_0 = c_1 + c_2 V + c_3 V^2 + c_4 V^3 \tag{3}$$

Where c_1, c_2, c_3, c_4 are the undetermined coefficients. The gliding trajectory is divided into two phases which are connected by control point. By changing the height and velocity of the control point, the shape of the two curves can be changed so that the trajectory of the entire gliding trajectory can meet the process constraints and task requirements and reach the target area.

The initial descent phase of the aircraft adopts the constant angle of attack and zero-bank angle to control. At the end of the descent phase, the pull-up point parameters including height H_d and speed V_d can be obtained. According to the terminal guidance requirements, the push-down point parameters including height H_f and speed V_f can be pre-determined. The speed of control point can be taken as $V_c = (V_d + V_f)/2$, and the height H_c must meet the constraints of the reentry corridor. The specific value is determined by the range to be flying. According to the parameters of the pull-up point, the control point, and that the change rate of the height vs the speed is equal to zero, the undetermined coefficient of the curve can be determined. Similarly, the curve form between the control point and the push-down point can be solved.

In the case of a small lateral manoeuvring range of the aircraft, the range is mainly determined by the longitudinal range. The longitudinal range can be determined based on the H-V flight profile. Without considering the earth rotation, the equation of longitudinal motion is simplified as following.

$$\begin{cases} \dot{r} = V\sin\theta \\ \dot{V} = -\dfrac{D}{m} - g\sin\theta \\ \dot{\theta} = \dfrac{1}{V}\left[\dfrac{L\cos\nu}{m} + (\dfrac{V^2}{r} - g)\cos\theta\right] \\ \dot{R} = V\cos\theta \end{cases} \tag{4}$$

Where R represents the range.

$$\frac{dR}{dV} = \frac{V\cos\theta}{-D/m - g\sin\theta} \approx -\frac{mV}{D} = -\frac{2m}{C_D\rho VS} \tag{5}$$

When the H-V flight profile is determined, the range can be estimated by integrating the formula above. Generally, the specific height H_c of the control point can be determined by the Newton iteration algorithm according to the range mission. At the same time, since the speed V of the control point is determined and the height H_c needs to meet the reentry corridor constraints, the height of the control point can also be determined by traversing the search. That is, when the speed of the control point is known, the height interval is determined according to the reentry corridor constraints. The search step length is set. Then, the control point height H_c is searched and determined.

3.2. Tracking law using feedback linearization

In order to track the H-V curve, a feedback linearization is used to design the tracking controller. According to the dynamic equation in longitudinal plane, the state variables are taken as $[r,V,\theta]$, and the control variable is taken as longitudinal lift.

$$u = Y\cos\nu \tag{6}$$

The output of the control system is

$$y = H \tag{7}$$

According to the above transformation, the H-V profile tracking problem is converted into a single-input single-output problem with input as u and output as H . The system equation can be expressed as

$$\begin{cases} \dot{y} = \dot{H} = \dot{r} = V\sin\theta \\ \ddot{y} = \ddot{H} = \dot{V}\sin\theta + V\dot{\theta}\cos\theta \end{cases} \tag{8}$$

The simplified dynamic equation is brought into the above system equation. Then

$$\ddot{y} = a(x) + b(x)u \tag{9}$$

Where $a(x) = (-\dfrac{X}{m} - g\sin\theta)\sin\theta + (\dfrac{V^2}{r} - \dfrac{\mu}{r^2})\cos^2\theta$, $b(x) = \dfrac{\cos\theta}{m}$.

Let $u = \frac{w - a(x)}{b(x)}$, the input-output relationship will become a simple linear double-integral relationship.

$$\ddot{y} = w \tag{10}$$

In order to ensure the stability of tracking, the error must meet the following requirements at the same time.

$$\delta\ddot{H} + 2\zeta\omega\delta\dot{H} + \omega^2\delta H = 0 \tag{11}$$

Where $\delta H = H - H_0, \delta\dot{H} = \dot{H} - \dot{H}_0, \delta\ddot{H} = \ddot{H} - \ddot{H}_0$, H_0 is the reference height, H is the current flight height.

Therefore, the control system input is given by

$$u = \frac{\ddot{H}_0 - 2\zeta\omega\delta\dot{H} - \omega^2\delta H - a(x)}{b(x)} \tag{12}$$

As can be seen from the relationship between input and output of the control system, if the angle of attack or bank angle is determined, then the other can be obtained. Assuming that the angle of attack profile is determined, the bank angle is calculated by

$$\nu_c = \arccos(\frac{u}{Y}) \tag{13}$$

For the control of the lateral trajectory, the method of designing the heading angle error corridor is usually used.

The height - velocity curve obtained by tracking is shown in figure 1.

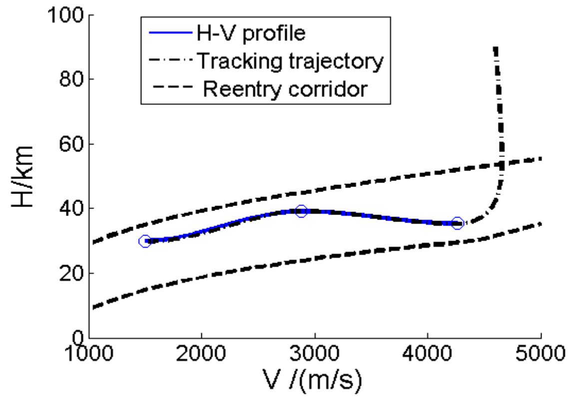

Figure 1. Velocity vs height curve.

4. Trajectory planning based on time coordination

In order to cope with the complex battlefield environment in the future and make multiple aircraft reach the attack target area at the same time, the following cooperative path planning process is designed on the basis of online trajectory generation method.

First, estimate the remaining flight time required for each aircraft to reach the target. The remaining flight time is available from the current time t and the terminal time t_f and given as $t_{go} = t_f - t$;

Second, determine whether cooperative is needed. Calculate the difference among the remaining flight time of each aircraft, and compare it with the pre-given time difference ε. If $\left|t_{go}^i - t_{to}^j\right| > \varepsilon$, the trajectory needs cooperative planning; otherwise, it does not need;

Third, do cooperative planning. Considering that the flight process is one no-power reentry and gliding phase, it is difficult to achieve acceleration for the aircraft. So the aircraft with the larger remaining flight time is used as a reference, and the remaining flight trajectories are regenerated and tracked online.

Without loss of generality, the trajectory online generation and cooperative planning process of two aircrafts can be shown in figure 2.

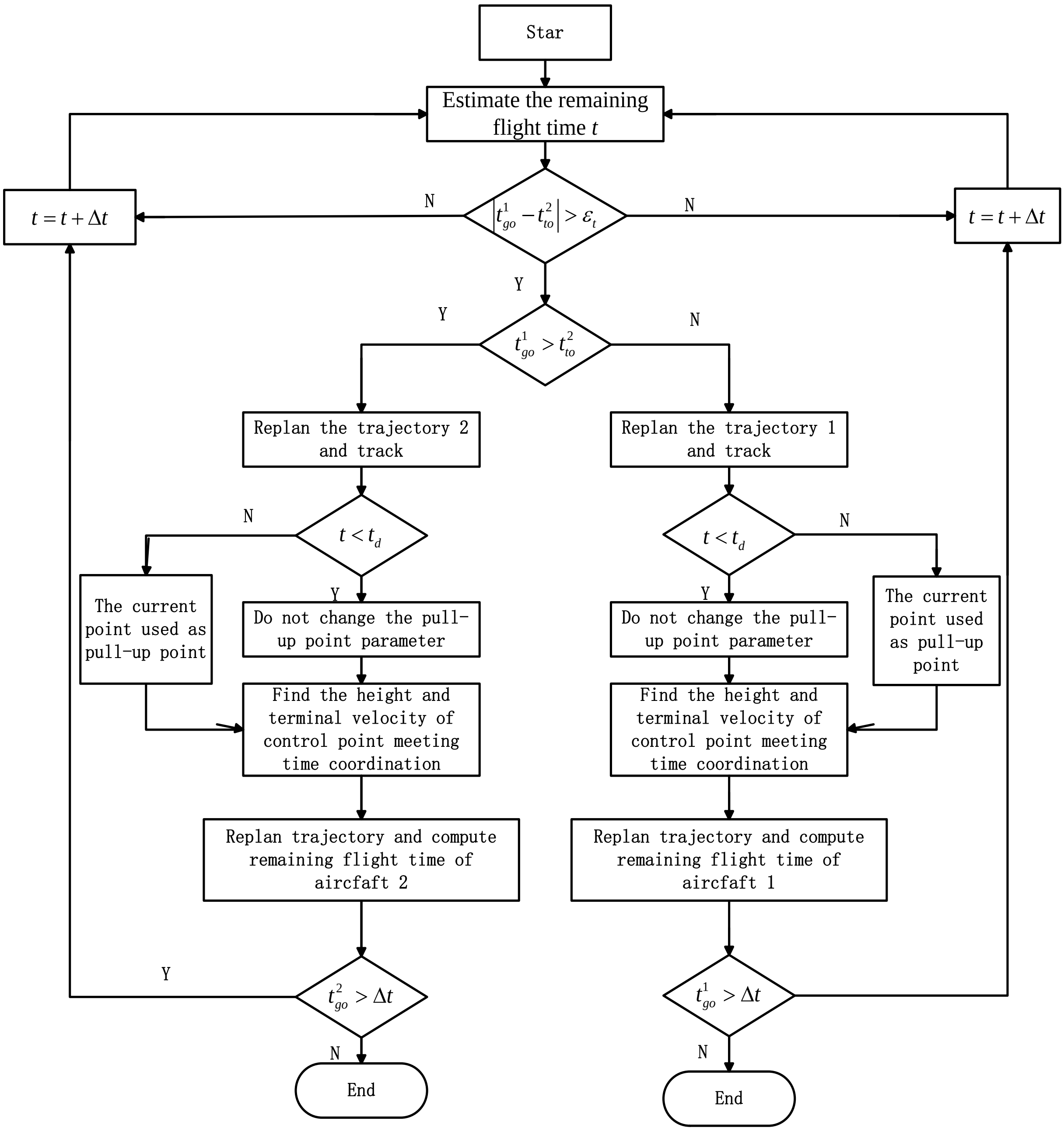

Figure 2. Flow chart of online generation and cooperative planning.

In the figure, it means that the cooperation is performed at this time Δt, which is the coordination time interval. t_d is time of the pull-up point.

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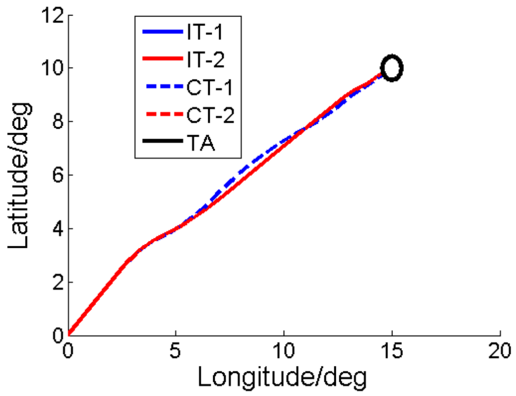
5. Experiments and results

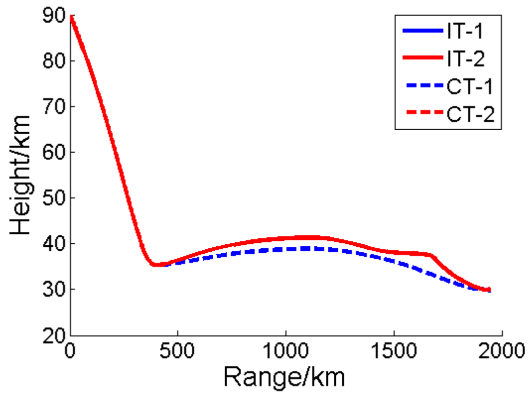
5.1. Launch at the same place but the different time

Assuming that the latitude and longitude of the launch point are set (0°E, 0°N). The initial velocity is 4600m/s, the height is 90km, the flight path angle is -6.5°, and the heading angle is 45°. Take the target latitude and longitude latitude as (15°E, 10°N). The push-down point satisfies the following constraints: the height is 30km, the velocity is between 1200m/s and 1800m/s, and the radius of the target area is 50km.

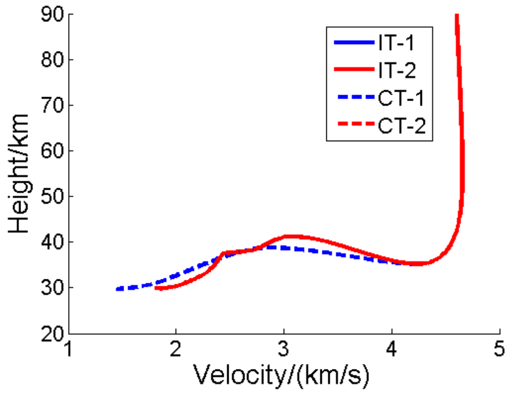
The time interval between the launch of the two aircrafts is 60s. The time-coordination interval of the two aircrafts is $\Delta t = 100s$. The allowable time difference is $\varepsilon_t = 1s$. The simulation results are shown in figure 3.



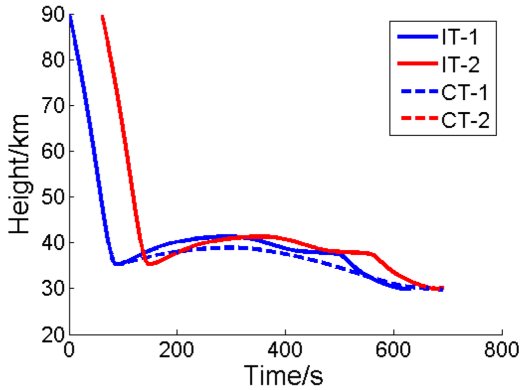
(a) Longitude-latitude curve.



(b) Range-height curve



(c) Velocity-height curve



(d) Time-height curve

Figure 3. Results for launching at the same place but the different time.

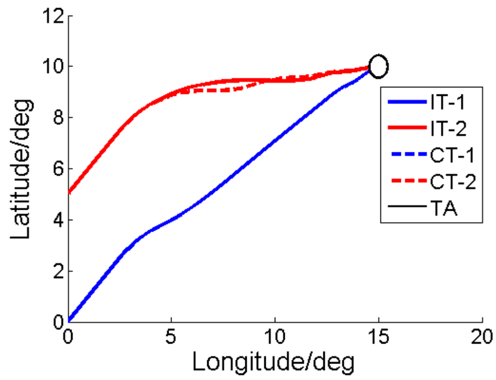
In figure 3, the denotations IT-1, IT-2, CT-1, CT-2 represent initial trajectory of aircraft 1, initial trajectory of aircraft 2, cooperative trajectory of aircraft 1 and cooperative trajectory of aircraft 2 respectively. The denotation TP represents the target area.

As can be seen from the figure 3, the initial state and the terminal state are the same before the two aircrafts cooperating. There is only a gap in time, so the trajectories obtained by online generation and tracking guidance are parallel to each other. On account of that the trajectory of aircraft 2 does not need to be replanned, the IT-2 and CT-2 are coincident. The number of trajectories seen from the figure is three.

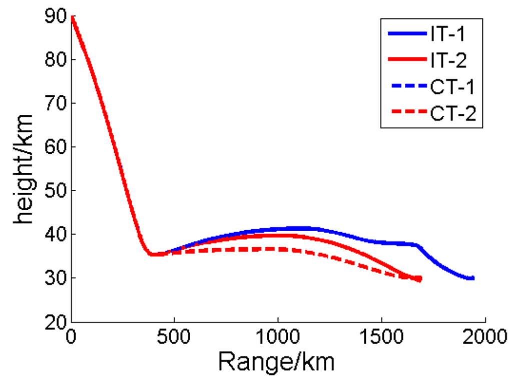
From figure 3a, it can be seen that after the cooperative trajectory planning, the two aircrafts can still reach the preset target area. But the flying height are different which can be seen from figure 3b.

Since the aircraft 1 is launched 60s earlier than 2, the height of the trajectory gliding phase of the aircraft 1 should be reduced as a whole during the cooperative planning in order to achieve the time coordination. At the same time, it can be seen from the terminal state that the two aircraft are consistent in range and have the same height, and satisfy the given terminal state constraints. From figure 3c, it can be seen that after the cooperative planning, the gliding height is reduced. The velocity decreases from 1800 m/s before the coordination to 1446 m/s. From figure 3d, it can be seen that after the cooperative planning, the two aircraft trajectories can reach the predetermined altitude at the same time, which meets the time coordination requirement.

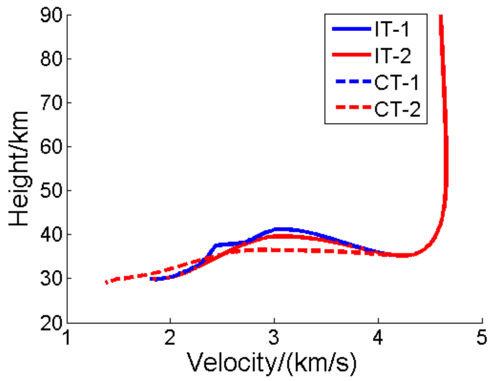
5.2. Launch at the same time but the different places
In order to strike the same target whose latitude and longitude is (15^{o}E, 10^{o}N), two aircrafts are launched from two positions whose latitudes and longitudes are (0^{o}E, 0^{o}N) and (0^{o}E, 5^{o}N) respectively. The starting height is 90km and the initial velocity is 4600m/s. The flight path angle is -6.5^o, and the heading angle is 45^o. The two aircrafts are required to reach the designated target area at the same time, and the push-down point should satisfy the constraints, including that the height is 30km, the velocity is between 1200m/s and 1800m/s, and the range to be flying is 50km. The coordination time interval is taken as $\Delta t = 100s$, and the allowable time difference is $\varepsilon_t = 1s$. The simulation results are shown in figure 4.



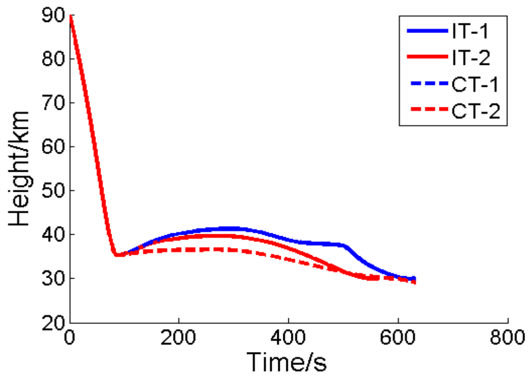
(a) Longitude-latitude curve.



(b) Range-height curve



(c) Velocity-height curve



(d) Time-height curve

Figure 4. Results for launching at the same time but the different place.

From the simulation results, it can be seen that before the cooperative is performed, the flight time of initial trajectory 1 time is 631.7 s, and the initial trajectory 2 is 564 s. The aircraft 2 reaches the target area earlier than aircraft 1. In order to make the two aircraft reach the target area at the same time, the trajectory of the aircraft 2 needs to be replanned online. Therefore, the deceleration of aircraft 2 is controlled until reaching the target area simultaneously with the aircraft 1. On account of that the trajectory of aircraft 1 does not need to be replanned, the IT-1 and CT-1 are coincident. The number of trajectories seen from the figure is three. From the flight time information of the two aircrafts, it can be seen that the time coordination control strategy successfully makes the two aircraft reach the target area at the same time. Finally, the two aircrafts complete the coordinated combat mission.

From the simulation results obtained in the above two different situations, we can see that the cooperative trajectory planning method can enable multiple aircrafts to reach the target area at the same time, which layes the foundation for the terminal guidance.

6. Conclusions

With the continuous development of hypersonic technology and the strengthening of the aerial defence system, multi-aircraft cooperative combat will become a new combat style. In view of the deficiencies in the current research, this paper does research on cooperative planning technology of gliding phase for multi-aircraft. Based on the online trajectory generation and a feedback linearization tracking guidance law for a single aircraft, a multi-aircraft trajectory planning strategy based on time coordination is designed under time coordination condition. Two cases are simulated and verified under different operational backgrounds. The simulation results show that the designed trajectory cooperative planning strategy can effectively achieve the multi-aircraft reaching the target area at the same time. The flight process meets the constraints of the reentry corridor. The trajectory satisfies the terminal constraint, which can lay the foundation for the subsequent terminal guidance.

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References

[1] Dhananjay N and Dhananjay N 2014 Accurate time-to-go estimation for proportional navigation guidance *Journal of Guidance Control & Dynamics.* **37** 1378-1383.

[2] Jeon I S, Lee J I and Tahk M J 2010 Homing guidance law for cooperative attack of multiple missiles *Journal of Guidance Control & Dynamics* **33** 275-280

[3] Sun X J, Zhou R and Hou D L 2014 Consensus of leader-followers system of multi-missile with time-delays and switching topologies *Vac Optik - International Journal for Light and Electron Optics.* **125** 1202-1208

[4] Zhang L M, Sun M W and Chen Z Q 2014 Receding horizon trajectory optimization with terminal impact specifications *Mathematical Problems in Engineering* 693-697

[5] Zhang Y A, Wang X L and Wu H L 2014 Impact time control guidance law with field of view constraint *Aerospace Science & Technology* **39** 361-369

[6] Song J H, Song S M and Xu S L 2016 Cooperative guidance law for multiple missiles with impact angle constraints *Journal of Chinese Inertial Technology* **24** 554-560

[7] Zhang C Y, Song J M and Hou B. 2016 Cooperative guidance law with impact angle and impact time constraints for networked missiles *Acta Armamentarii* **37** 431-438

[8] Fang Y, Ma K and Chen Y 2014 Cooperative guidance laws with constraints on impact time and terminal angle *Journal of System Simutation* **26** 2434-2441

[9] Jung B and Kim Y 2006 Guidance laws for anti-shipping missiles using impact angle and impact time *AIAA Guidance, Navigation, and Control Conferenc. Sci. Technol, Keystone, Colorado*

[10] Kumar S R and Ghose D 2015 Impact Time and Angle Control Guidance *AIAA Guidance, Navigation, and Control Conference*

[11] Fang Y, Ma Y and Chen Y 2014 Cooperative guidance laws with constraints on impact time and terminal angle *Journal of System Simulation* **26** 2434-2441

[12] Vinh N X 1981 *Optimal trajectories in atmosphere flight* (New York: Elsevier Scientific Publishing Company)