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A Novel Penetration-evade Guidance Law against Two Homing Missiles

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Abstract. A novel guidance for an evader which is exposed to a threat of two homing missiles is presented. Each missile is assumed to measure solely its own line-of-sight (LOS) angle and share it with other missile. Such information sharing enables the missiles to form a triangular measuring baseline relative to the evader and to improve their estimation accuracy. However, if the separation angle between the two LOS is small enough, the observability of such double-LOS measuring approach becomes weak. Motived by this observation, the idea of the proposed concept is to bring the missile on the same LOS with target in the first stage. As for the second stage, the evader perform an appropriately timed maximum evasive maneuver. The simulation results demonstrate that the proposed guidance law can reduce the estimate and guidance accuracy of interceptors and achieve the goal of penetration.

1. Introduction

Modern high-precision interceptors can easily track and destroy our high-value ballistic missiles. Mobile escape is the best way to get rid of the interceptor. It is a direct and effective attack strategy [1].

For the one-to-one case where the enemy's defense system uses a missile to intercept, the literature [2-4] has deduced the optimal escape strategy for the target. The assumption is that the missile has all the information of the target. However, in the real environment where there is noise interference, it is difficult to obtain all the states of the target. Therefore, many estimation algorithms have been derived [5]. In order to increase the probability of interception, the multiple-to-one attack method for launching multiple missiles is very effective in increasing the probability of intercepting the success rate. It can also restrict the interception time and the interception angle [6-8]. Most tactical missiles are equipped with infrared sensors that measure the LOS angle to measure the LOS between the missile and the target. Literature [9-11] studied the methods of tracking the target and enhancing the performance of the estimation in the context of multi-missiles using purely azimuth measurements. Through cooperative detection, the tracking accuracy of the target can be enhanced. The multi-missiles share the LOS angles measured by them in real time, and the missile-target triangulation structure [12] can greatly enhance the estimated performance of the guidance system. Literature [13] proved that this information sharing estimation method has a very good improvement in interception performance. However, the estimated performance of this method largely depends on the trajectory of the missile. Regardless of the filter used, if the difference between each missile and the target LOS angle is small, the relative kinematics relationship becomes difficult to estimate, the accuracy of the estimate decreases, and the missile guided system's interception decreases. Literature [14-15], from the perspective of guided missile interception, enhances the estimation accuracy by adjusting to the appropriate LOS angle difference.

The above studies all emphasize how to enhance the estimated performance of the interceptor. For the defense side, there is less research on one-to-two or one-to-many penetration methods. This paper proposes a missile penetration prevention escape strategy from the perspective of the penetration defense missile. The guidance law is divided into two phases. In the first phase, the penetration missile is intended to reduce the estimated performance of the interceptor in the context of multi- LOS measurement. Changing the interceptor's trajectory will make them equal to the sight line angle of the penetration missile. In this case, the distance between the missiles becomes difficult to estimate, which greatly increases the survivability of the missiles. At the right moment, it switches to the second stage, during which the missiles make maximum maneuvers to escape.

2. Problem statement

The considered engagement scenario consists of two interceptors and an evader. This paper considers the issue from the perspective of the target. The two-dimensional geometric relationship shown in Figure 1. $X-O-Y$ is Cartesian inertial coordinate system. The speed, normal acceleration and flight-path angle are V, a and γ respectively; Subscript $i\in\{1,2\}$ and t means the i missile and the target respectively; (x_j, y_j), $j\in\{1,2,t\}$ denotes the position of the j th vehicle, and the distance between the target and the i th missile is denoted as r_i. The LOS of i th missile is λ_i. The difference between the two line of sight angles is defined as $\Delta\lambda \triangleq \lambda_1-\lambda_2$, which will be used in the derivation of the guidance law.

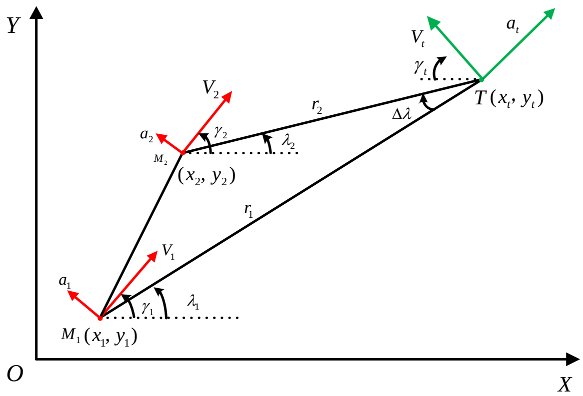

Figure 1. Planar geometric relationship

The kinematics relationship of the i th missile in the polar coordinate system (r_i, λ_i) is expressed as

$$\begin{cases} \dot{r}_i = V_{ri} \\ \dot{\lambda}_i = V_{\lambda i}/r_i \end{cases} \tag{1}$$

The speed component along and normal to the LOS are

$$\begin{cases} V_{ri} = -V_i\cos(\gamma_i-\lambda_i) - V_t\cos(\gamma_t+\lambda_i) \\ V_{\lambda i} = -V_i\sin(\gamma_i-\lambda_i) + V_t\sin(\gamma_t+\lambda_i) \end{cases} \tag{2}$$

Arbitrary-order linear dynamics of the j th vehicle is

$$\begin{cases} \dot{\boldsymbol{X}}_j = \boldsymbol{A}_j\boldsymbol{X}_j + \boldsymbol{B}_j u_j \\ a_j = \boldsymbol{C}_j\boldsymbol{X}_j + D_j u_j \ , \quad j\in\{1,2,t\} \\ \dot{\gamma}_j = a_j/V_j \end{cases} \tag{3}$$

The maximum maneuverability of the j th vehicle is

$$\left|u_j\right| \le u_{\mathrm{j\,max}}, \quad j \in \{1,2,t\} \tag{4}$$

The i th missile's interception time is t_{fi}. Under the assumption of linearization with a small angle, the interception time can be considered as fixed.

$$t_{fi} \triangleq -r_i / \dot{r}_i, \ \ \forall i \in \{1,2\} \tag{5}$$

The non-negative time-to-go of the i th missile is defined as

$$t_{goi} = \begin{cases} t_{fi} - t & t < t_{fi} \\ 0 & t > t_{fi} \end{cases} \quad \forall i \in \{1,2\} \tag{6}$$

When $t = t_{fi}$, $r_i(t_{fi})$ is the miss distance of the i th missile.

If at the end of the guidance stage, the deviation between the missile and the target and the respective collision triangle is very small, the linearization assumption can be established, and the linear plane geometry relationship between the missile and the target [16] is shown in Figure 2

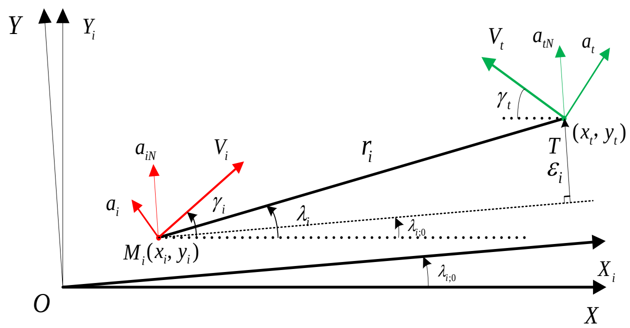

Figure 2. Linearized planar geometric relationship

The accelerations normal to initial LOS $LOS_{i;0}$ is a_{t_iN} and a_N, satisfying

$$a_{t_iN} = k_{ti} a_t, k_{ti} = \cos(\gamma_{t;0} + \lambda_{i;0}) \tag{7}$$

$$a_{iN} = k_i a_i, k_i = \cos(\gamma_{i;0} - \lambda_{i;0}) \tag{8}$$

The subscript 0 indicates the initial value at the start of linearization, and k_i and k_{ti} indicate the k th linearization parameter.

The LOS separation angle $\Delta\lambda$ is defined by the LOS angle in the nonlinear model (1). If $\left|\lambda_{2;0} - \lambda_{1;0}\right|$ is small enough, then $\Delta\lambda$ can be expressed approximately as

$$\Delta\lambda \approx \sin(\lambda_2 - \lambda_{2;0}) - \sin(\lambda_1 - \lambda_{1;0}) \triangleq x_\lambda \tag{9}$$

From Figure 2, we can see that x_λ can be represented by the linearization variables ε_1 and ε_2

$$x_\lambda = \varepsilon_2 / r_2 - \varepsilon_1 / r_1 \tag{10}$$

The distance r_i can be expressed approximately as

$$r_i \approx V_{ci} t_{goi} \tag{11}$$

Where V_{ci} is a constant approach speed

On substituting Eq. (12) in the derivative of Eq. (11), we get

$$\dot{x}_\lambda = \frac{\dot{\varepsilon}_2}{V_{c2} t_{go2}} + \frac{\varepsilon_2}{V_{c2}(t_{go2})^2} - \frac{\dot{\varepsilon}_1}{V_{c1} t_{go1}} - \frac{\varepsilon_1}{V_{c1}(t_{go1})^2} \tag{12}$$

Define the state vector as

$$\boldsymbol{x} \triangleq \left[\boldsymbol{x}_\varepsilon^T \ \dot{\boldsymbol{x}}_\varepsilon^T \ \boldsymbol{X}_m^T \ \boldsymbol{X}_t^T \ x_\lambda \right] \tag{13}$$

Where

$$\boldsymbol{x}_\varepsilon = [\varepsilon_1 \ \varepsilon_2]^T \triangleq [x_1 \ x_2]^T \tag{14}$$

$\dot{\boldsymbol{x}}_\varepsilon$ is the relative speed vector normal to the respective initial LOS. Then the equation of motion can be expressed as

$$\begin{cases} \dot{x}_1 = x_3 \\ \dot{x}_2 = x_4 \\ \dot{x}_3 = k_{t1}a_t - k_1 a_1 \\ \dot{x}_4 = k_{t2}a_t - k_2 a_2 \\ \dot{\boldsymbol{X}}_m = \boldsymbol{A}_m \boldsymbol{X}_m + \boldsymbol{B}_m \boldsymbol{u}_m \\ \dot{\boldsymbol{X}}_t = \boldsymbol{A}_t \boldsymbol{X}_t + \boldsymbol{B}_t \boldsymbol{u}_t \\ \dot{x}_\lambda = -\dfrac{x_1}{V_{c1}(t_{go1})^2} + \dfrac{x_2}{V_{c2}(t_{go2})^2} - \dfrac{x_3}{V_{c1}t_{go1}} + \dfrac{x_4}{V_{c2}t_{go2}} \end{cases} \tag{15}$$

Where $\boldsymbol{A}_m$ and $\boldsymbol{B}_m$ are diagonal matrices

$$\boldsymbol{A}_m \triangleq diag\left\{\boldsymbol{A}_1, \boldsymbol{A}_2\right\} \tag{16}$$

$$\boldsymbol{B}_m \triangleq diag\left\{\boldsymbol{B}_1, \boldsymbol{B}_2\right\} \tag{17}$$

Assume that the i th missile's guidance law can be expressed as the following linear form

$$u_i = K_{1i}\varepsilon_i + K_{2i}\dot{\varepsilon}_i + \boldsymbol{K}_{mi}\boldsymbol{y}_i + \boldsymbol{K}_{ti}\boldsymbol{y}_t + K_{ui}u_t \tag{18}$$

Equations (16) and (19) can be written in matrix form

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}(t)\boldsymbol{x}(t) + \boldsymbol{B}(t)u_t(t) \qquad \boldsymbol{x}(0) = \boldsymbol{x}_0 \tag{19}$$

Where

$$A(\mathrm{t}) = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & 0 \\ \boldsymbol{A_{21}} & \boldsymbol{A_{22}} & \boldsymbol{A_{23}} & \boldsymbol{A_{24}} & 0 \\ \boldsymbol{A_{31}} & \boldsymbol{A_{32}} & \boldsymbol{A_{33}} & \boldsymbol{A_{34}} & 0 \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A_t} & 0 \\ \boldsymbol{A_{51}} & \boldsymbol{A_{52}} & \boldsymbol{0} & \boldsymbol{0} & 0 \end{bmatrix} B(\mathrm{t}) = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{B_2} \\ \boldsymbol{B_3} \\ \boldsymbol{B_t} \\ 0 \end{bmatrix} \tag{20}$$

3. Penetration escape strategy design

Lemma 1 [12]: The distance $r_{i;k}^{+}$ between the target and the i missile can be calculated based on the measurement value $z_{i;k}, z_{j;k}$ of the i th missile and the j th missile, which contain noise, and its relative position (r_{ij}, λ_{ij}). And $r_{i;k}^{+}$ follows the distribution $r_{i;k}^{+} \sim \mathrm{N}(r_{i;k}, \sigma_{\rho i;k}^2)$, $r_{i;k}$ is the real distance at time t_k, $\sigma_{\rho i;k}^2$ is the non-stationary standard deviation of $r_{i;k}^{+}$.

$$r_{i;k}^{+} = r_{ij;k}\frac{\sin(\lambda_{ij;k} - z_{j;k})}{\sin(z_{i;k} - z_{j;k})}, i \neq j \tag{21}$$

$$\sigma_{\rho i} = r_{ij}\frac{\sqrt{\sigma_j^2 \sin^2(\lambda_{ij} - \lambda_i) + \sigma_i^2 \sin^2(\lambda_{ij} - \lambda_i)\cos^2(\lambda_j - \lambda_i)}}{\sin^2(\lambda_j - \lambda_i)} \tag{22}$$

Where $r_{ij;k} = \sqrt{x_{ij;k}^2 + y_{ij;k}^2}, \lambda_{ij;k} = \tan^{-1}(y_{ij;k} / x_{ij;k})$.

If we make $\left|\lambda_j - \lambda_i\right| \to 0$, we get

$$\lim_{\left|\lambda_j - \lambda_i\right| \to 0} \sigma_{\rho i} = \infty, \qquad i \neq j,\ r_{ij} \neq 0 \tag{23}$$

When the LOS separation angle $\Delta\lambda$ is close to zero, the two LOS angle λ_1 and λ_2 are equal. The variance $r_{i;k}^{+}$ increase, which in turn makes the distance of the i th missile inestimable and the measured values shared with each other will be equal. Deterioration of the missile's estimation accuracy, especially the estimation of the distance, the speed of the target, and the time-to-go, will lead to a decrease in the guidance accuracy of the missile. Motived by this observation, this section will design the guidance law by minimizing $\Delta\lambda$.

3.1 *First Stage-Optimal Guidance Law*

Defining the intercept time as:

$$t_f \triangleq \min_{i\in\{1,2\}} t_{fi} \tag{24}$$

Selecting the quadratic cost function J to minimize the LOS separation angle and the target's control effort as performance indicators

$$J = \frac{1}{2}\int_{t_0}^{t_s}[Q(\tau)x_\lambda^2(\tau) + Ru_t^2(\tau)]d\tau \tag{25}$$

Where $t_s \le t_f$ is the switching time from first stage to second stage and R is the weight of the target control effort. $Q(\tau)$ is the time-varying weight of the LOS separation angle.

$$Q(\tau) \triangleq \frac{\eta}{t_s - \tau} \tag{26}$$

For a linear time-varying system (20), in order to minimize the performance index (25), the optimal control is determined to be

$$u_t^1(t) = -R^{-1}\boldsymbol{B}^T(t)\boldsymbol{P}(t)\boldsymbol{x}(t), \quad t \in [0,\ t_s] \tag{27}$$

The nonnegative matrix P satisfies the differential matrix Ricatti equation

$$\dot{\boldsymbol{P}} = -\boldsymbol{A}^T\boldsymbol{P} - \boldsymbol{PA} + \boldsymbol{PB}R^{-1}\boldsymbol{B}^T\boldsymbol{P} - \boldsymbol{C}^TQ\boldsymbol{C} \tag{28}$$

Where $\boldsymbol{P}(t_s) = \boldsymbol{0}$, $\boldsymbol{C} = [\boldsymbol{0\ 0\ 0\ 0}\ 1]$.

3.2 *Second Stage-Escape Strategy*

At the end of the first stage, this paper will make the target do the maximum maneuver escape in the opposite direction of the acceleration command.

Defining the switching time as

$$t_s = t_f - \Delta t \quad 0 < \Delta t < t_f \tag{29}$$

Where the value of Δt determine the time of escape stage. And the acceleration command is

$$u_t^2(\mathrm{t}) = \begin{cases} +u_{t\max} & if\ \ u_t^1(\mathrm{t_s}) \le 0 \\ -u_{t\max} & else \end{cases} \tag{30}$$

Where $u_t^1(\mathrm{t_s})$ is the acceleration command of the target at t_s. The final Penetration escape strategy is summarized as:

$$u_t(\mathrm{t}) = \begin{cases} u_t^1 & if\ \ 0 \le t \le t_s \\ u_t^2 & else \end{cases} \tag{31}$$

4. Numerical simulation

This section demonstrates the effectiveness of the penetration escape strategy by numerical simulation. Facing the two homing missiles launched by the enemy's defense system, the penetration-escape strategy was employed to reduce the missile's estimated performance and guidance performance and achieve the purpose of penetration. The simulation scenario was set as follows: At the beginning of the simulation, two missiles were launched simultaneously. After the first missile passed the target, the simulation was continued until the end of the second missile. The initial conditions of the simulation and the relevant parameters in the guidance law are shown in Table 1 and Table 2, respectively.

Table 1. The values of initial conditions

initial conditions	values
$x_{1;0}, y_{1;0}; x_{2;0}, y_{2;0}$	(0,150); (0,50)
$x_{t;0}, y_{t;0}$	(15000,100)
$u_{1\max}, u_{2\max}, u_{t\max}$	45g,45g,10g
$V_1 = V_2 = V_t$	$2500\, m/s$
$\lambda_{1;0}, \lambda_{2;0}, \lambda_{t;0}$	0rad, 0rad, 0rad,

Table 2. The design parameters

parameters	values
R, η	$1{,}10^{10}$
τ_1, τ_2, τ_t	0.01,0.01,0.0 2
N_{PN}, ε'	4, 0

The target employs the guidance law proposed in this paper. The switching time Δt is set to 0.5s. Its trajectory are shown in Figure 3. It can be seen from the change of the LOS angle (Figure 4) that the guidance law is ideal for controlling the LOS separation angle.

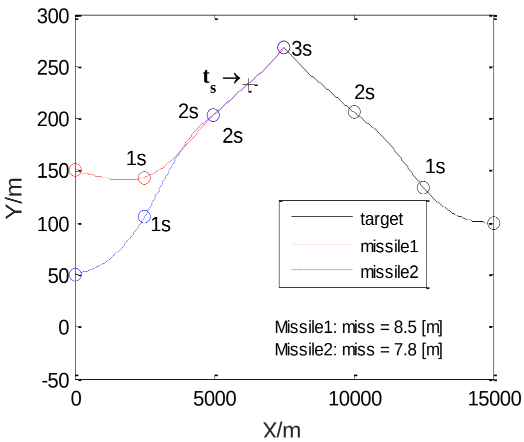

Figure 3. Planar trajectories. Δt=0.5

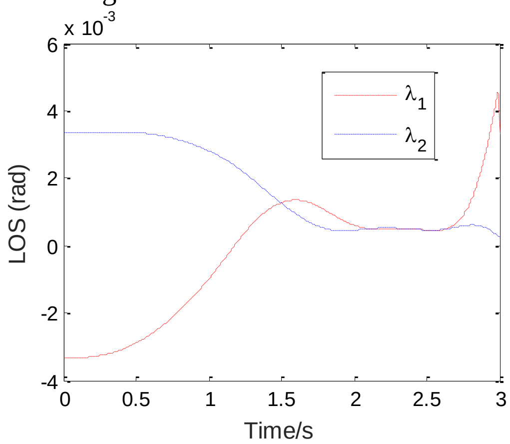

Figure 4. LOS of missiles

In order to analyse the two missiles' estimated performance against a target employing penetration-escape guidance law, the estimated performance is measured by calculating the standard deviation of the error between true and estimated values. Simulation results as shown in Figure 5. As Δt decreases, the duration of the first stage increases, and the accuracy of the missile's estimation of distance, speed, and time-to-go significantly decreases. At the end of the phase, the estimates of all variables deteriorate.

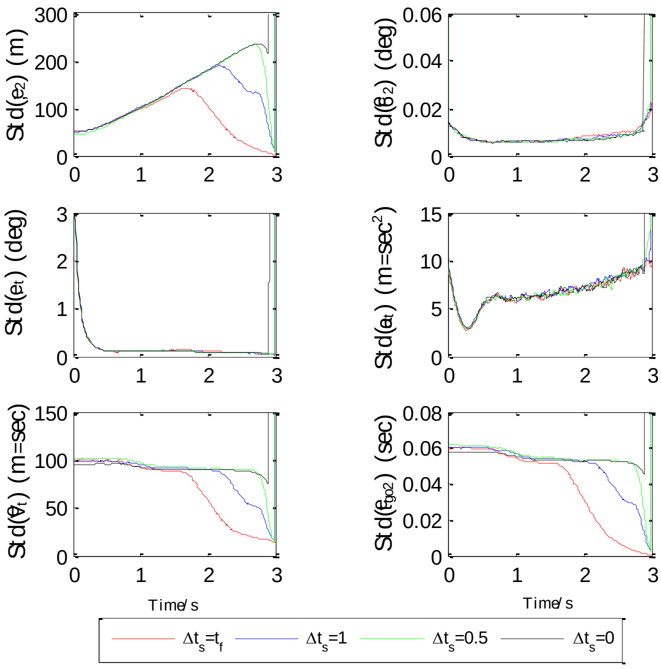

Figure 5. observability performance

5. Conclusion

This paper designs a penetration prevention strategy for the penetration defense missiles. In order to improve the accuracy of the estimation by intercepting the interceptors of the pure azimuth measurement to improve the estimation accuracy, a control strategy is proposed. The antimissile guided the law of equalization of the LOS angles of the two interceptors, thereby exacerbating the estimation accuracy of the interceptor and further reducing the guidance accuracy. Then choose the appropriate switching time to do the maximum escape maneuver to achieve the purpose of penetration. From the Monte Carlo simulation results, if the available overload of the penetration missile is determined, there is an optimal switching time to perform the maximum escape maneuver. Selecting the appropriate switching time can greatly enhance the chance of survival of the penetration missile.

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