

A dissipative constitutive model for the hysteretical behaviour of a woven composite fabric under large strain

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Abstract. Draping composite reinforcement on non-developable shapes necessarily leads to deformations in the plane generating large shears between warp and weft. Energy dissipation appears since the sliding between yarns and between fibers creates friction effect. The theory presented here is a constitutive model describing the dissipative behaviour of a 2D composite woven fabric. The model is based on two innovative points which are the Green – Naghdi additive decomposition for anisotropic material and the theory of nested surfaces according with Mroz studies. Both points lead to define the strong non-linearity of the problem and the hysteretical behaviour during the unloading phase. The dissipation process driven by fibers friction is exclusively associated with the in-plane shear deformation mode. The model is calibrated using standard methods such as the Picture Frame test. This model is also discretised to be integrated in a finite element calculation software.

1. Introduction

The composite fields offer a wide range of material configuration. Depending on the applications, these kinds of material can be from different natures, being from different elements and get specific geometric and mechanical characteristics. Depending on the type of composite used, shaping can vary from simple and mastered processes to much more delicate processes which are still subjects to many stages of research and discussion. In many cases, the composite material consists of a reinforcement that gives most of the mechanical characteristics to the material and a matrix that is injected onto the woven fabric to lock it. The object of this paper is then to study the dissipative behaviour of a woven fabric made by continuous fibre. One of the famous shaping process for composite reinforcement is the Resin Transfer Moulding [1], [2] which consist to shape the fabric and then inject the resin. The work in this paper will focus on the shaping phase where the fabric knows very large deformations and in some cases cyclical large deformations. Indeed, the material will undergo very high deformations to be superimposed on the shape of the matrix. Since the complexity of the piece's shape is in constant evolution, it is now, not unusual to get large variation of shear angle for example. However, to describe the shaping phase, only hyper-elastic or visco-elastic model already exist [3]–[8]. But these models do not consider dissipation. Then, for these reasons, the aim of this paper is to write a constitutive model which is considering the dissipation and the hysteretical behaviour of the material considering some assumptions. Indeed, during a cyclic loading, it is shown in this paper that the behaviour is following high non-linearities. Moreover,



since the fabric is composed of fibre, it will therefore deform according to different modes (elongations in the direction of yarns) and the in-plane shear mode [9]–[11]. As this type of material is usually made of fibres that have a very high stiffness (which prevents them from lying down) so most of the energy dissipation is due to the in-plane shear. Since the deformations are very high, the model presented here is written under large strains and considering the hypothesis that the fibres do not lengthen, the dissipation will be exclusively defined by a pure in-plane shear kinematics.

To write a model taking into this energy dissipation, the additive decomposition of Green-Naghdi applied for an anisotropic material under large strain will be used but not demonstrated here. The use of this decomposition gives a certain degree of freedom since the use of the intermediate configuration is possible and every fundamental tensor can be defined [12], [13]. On the other hand, this decomposition links all these tensors to the initial configuration, so that the formulation of this constitutive model is in total lagrangian formalism. Working on the initial configuration is easier since every quantity is known. Then, the first part of this paper is about the elaboration of an elasto-plastic model based on works already done [14]–[16]. It is shown that it is working well only if the load is monotone, however it does not describe the reality. Secondly, the hysterical effect of the material can be approached by several methods such as the fractional derivative. However, every work from fractional derivative is made under the condition of small perturbation and seems to be not very stable when the loading is negative [17]. In addition, it works when the unloading would be linear but for a woven fabric under large strain, the behaviour during an unloading phase is asymptotic. Then, the method presented here is based on the Mroz [18], [19] works who propose to improve a model by adding internal surfaces on it. By changing the evolution law of each surface, the model can describe high non-linearity and the asymptotic and hysterical behaviour. This is the strategy used here and it is shown that with only two nested surfaces, the result can be interesting.

Moreover, at the end of this article it is shown that all the constitutive model is only depending on only one variable. This is very interesting since a simple Newton-Raphson algorithm is enough to compute the model and find the solution. In addition, this model is only available for thick fabric. This strong hypothesis makes possible to simplify the study because of the in-plane stresses assumption.

Finally, at the end of this paper, a comparison between the experimental and the use of this model is made to show the capacity of this constitutive description of the material.

2. Model assumptions

The model presented in this paper can be only used under certain conditions:

- Traction-compression deformation modes (considered elastic) and in-plane shear (considered dissipative) are decoupled.
- Stress and strain tensors are assumed to be written with generalised quantities that have already been integrated into the thickness of the fabric.
- Since the carbon or glass fibres do not lengthen, the behaviour of the elongation/compression modes is purely elastic and does not dissipate energy. Therefore, after the first assumption, the dissipation process follows a kinematic of pure shear.
- The study presented here is made at the macroscopic scale. The fabric is then considered as an homogeneous material.

3. Kinematics of the deformations

In this section, the set of tensors necessary to write the model is defined. The coupling between the Green-Naghdi and the Kröner-Lee decomposition allows decoupling the total transformation and deformation tensors into an elastic part and a plastic part.

3.1. Decomposition and decoupling under large strain

The decomposition of Green-Naghdi is a relatively common formalism in the hypothesis of small perturbations and in the framework of crystalline materials. After a thermodynamical analysis inspired

by works already done, the compatibility of this decomposition with the field of finite strain and anisotropic structures allows to write the formulation defined by equation 1.

$$\bar{\bar{E}} = \bar{\bar{E}}_e + \bar{\bar{E}}_p \quad (1)$$

Coupled with the multiplicative decomposition of Kröner-Lee formulated by equation 2, this couple permits de define tensors in the initial configuration (Lagrangian configuration).

$$\bar{\bar{F}} = \bar{\bar{F}}_e \cdot \bar{\bar{F}}_p \quad (2)$$

The use of such decomposition makes possible to use the method of the intermediate configuration involving a configuration where residual stresses and plastic strains remain figure 1.

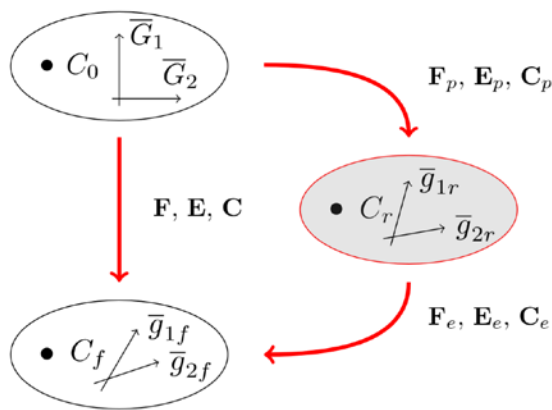


Figure 1. Intermediate configuration method and decoupling.

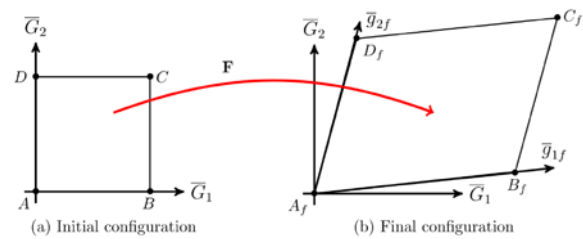


Figure 2. Kinematics of the total transformation.

The quantities defined on the figure 1 and figure 2 are:

- C_0 : Initial material configuration
- C_r : Relaxed configuration of the material. It is the so called intermediate configuration.
- C_f : Final material configuration.
- $\bar{\bar{F}}, \bar{\bar{F}}_e, \bar{\bar{F}}_p$: Respectively, the gradient of the total transformation, the gradient of the elastic transformation and the gradient of the plastic transformation.
- $\bar{\bar{E}}, \bar{\bar{E}}_e, \bar{\bar{E}}_p$: Respectively, the total Green-Lagrange deformation tensor, its elastic component and its plastic component.
- $\bar{\bar{C}}, \bar{\bar{C}}_e, \bar{\bar{C}}_p$: Respectively, the total right Cauchy-Green tensor, its elastic component and its plastic component.

3.2. Description of the kinematics

Total transformation is imposed by a test or a load as it can be seen on the figure 2. A user imposes this tensor and it can be described as the equation 3.

$$\bar{\bar{F}} = F_{ij}(\vec{G}_i \otimes \vec{G}_j) \quad (3)$$

\vec{G}_i and \vec{G}_j are the initial warp and weft direction of the material.

Plastic transformation is imposed by the assumption written above. Considering the last condition, it follows a pure shear kinematics. Figure 3 shows two possibilities of pure shear kinematics. For a question of simplicity, the kinematics shown on figure 3b is chosen since it is symmetric. However, if the kinematics of figure 3a would have been chosen, the result would be the same since the difference is only a pure rotation and since the model must satisfy the principles of thermodynamics and must follow the principle of objectivity.

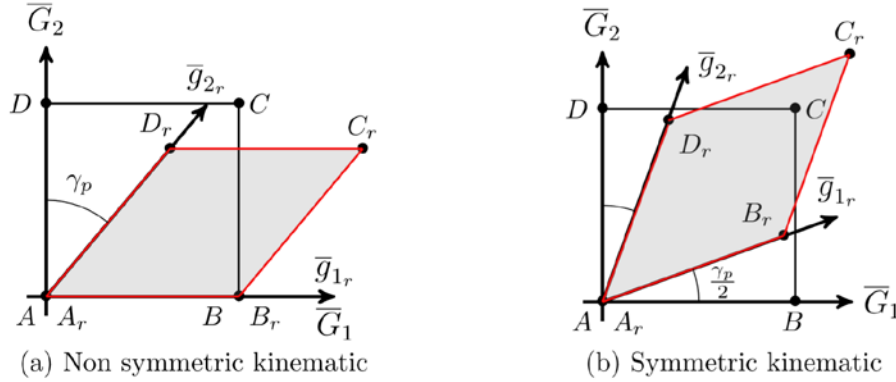


Figure 3. Two kinematics of the plastic transformation (pure shear).

The definition of this quantity is given by equation 4. The chosen kinematics gives a symmetrical tensor.

$$\bar{\bar{F}}_p = F_{pij}(\vec{G}_i \otimes \vec{G}_j) = \cos\left(\frac{\gamma_p}{2}\right)(\vec{G}_1 \otimes \vec{G}_1 + \vec{G}_2 \otimes \vec{G}_2) + \sin\left(\frac{\gamma_p}{2}\right)(\vec{G}_1 \otimes \vec{G}_2 + \vec{G}_2 \otimes \vec{G}_1) \quad (4)$$

Considering this tensor, it is now possible to calculate $\bar{\bar{C}}_p$ and $\bar{\bar{E}}_p$ given by equation 5 and equation 6.

$$\bar{\bar{C}}_p = \bar{\bar{F}}_p^t \cdot \bar{\bar{F}}_p = \bar{\bar{I}} + 2\cos\left(\frac{\gamma_p}{2}\right)\sin\left(\frac{\gamma_p}{2}\right)(\vec{G}_1 \otimes \vec{G}_2 + \vec{G}_2 \otimes \vec{G}_1) \quad (5)$$

$$\bar{\bar{E}}_p = \frac{1}{2}(\bar{\bar{C}}_p - \bar{\bar{I}}) = \frac{1}{2}\sin\left(\frac{\gamma_p}{2}\right)(\vec{G}_1 \otimes \vec{G}_2 + \vec{G}_2 \otimes \vec{G}_1) \quad (6)$$

Elastic deformation tensor can be deduced using equation 1 and equation 2. Using the Green-Naghdi additive decomposition for anisotropic field, it follows:

$$\bar{\bar{E}}_e = \frac{1}{2}\bar{\bar{F}}_p^t \cdot (\bar{\bar{F}}_e^t \cdot \bar{\bar{F}}_e - \bar{\bar{I}}) \cdot \bar{\bar{F}}_p \quad (7)$$

From an energetical point of view, the dual of the quantities presented above are used. Since the model is in total lagrangian, the tress tensor used for the elasto-plastic law described just after this section is Piola-Kirchhoff (PKII) written with the letter S in this paper. Its energetical strain dual is the tensor of Green-Lagrange E. The next section is about the elasto-plastic behaviour.

4. Elasto-plastic law

To write an elasto-plastic constitutive model, three quantities are necessary:

- A potential of free energy Ψ , which leads to define the stress tensor.
- A plasticity criterion f_s , directly connected to as dissipative potential D.

- A plastic flow law λ , defined from the derivation of the dissipative potential D by its stress space.

Moreover, some assumptions are added to write the inequality of Clausius-Duhem under large strain:

- Temperature is constant during the elastic transformation $T : \dot{T} = 0$.
- Temperature is homogeneous during the elastic transformation.
- A single internal variable is used here, the dissipative part of the Green-Lagrange tensor.

To determine the potential of free energy, it is necessary to start with the energy of the transformation Φ_0 which is the sum of an elastic potential Ψ and a dissipative potential D as following:

$$\Phi_0 = \Psi + D \quad (8)$$

This free energy potential leads to define the stress tensor considering the inequality of Clausius-Duhem under the assumption taken before:

$$\bar{\bar{S}} : \dot{\bar{\bar{E}}} - \mu \frac{d\Psi}{dt} \geq 0 \quad (9)$$

In this inequality it appears:

- The rate of total deformation $\dot{\bar{\bar{E}}}$.
- The surface density μ .
- The power per surface unit $\frac{d\Psi}{dt}$.

Using the principle of the local state and according to the additive decomposition of Green-Naghdi, it comes:

$$\left(\bar{\bar{S}} - \mu \frac{d\Psi}{d\bar{\bar{E}}_e} \right) : \dot{\bar{\bar{E}}}_e + \bar{\bar{S}} : \dot{\bar{\bar{E}}}_p \geq 0 \quad (10)$$

Given the formalism describes in the second section, the simplification of the inequality before leads to:

$$\bar{\bar{S}} = \mu \frac{d\Psi}{d\bar{\bar{E}}_e} \quad (11)$$

4.1. Calculation of the stress tensor: Piola-Kirchhoff II

To determine the stress tensor, it is necessary to define the potential of free energy firstly. As it was said in the introduction, there are two main in-plan deformation mode associated to this macroscopic model. One in elongation and one in shear. Since the assumption of decoupled mode is made, the behavior can be described using an elastic potential using these following invariants.

$$I_{11} = \bar{\bar{E}}_e : (\vec{G}_1 \otimes \vec{G}_1) \quad (12)$$

$$I_{22} = \bar{\bar{E}}_e : (\vec{G}_2 \otimes \vec{G}_2) \quad (13)$$

$$I_{12} = \bar{\bar{E}}_e : (\vec{G}_1 \otimes \vec{G}_2) + \bar{\bar{E}}_e : (\vec{G}_2 \otimes \vec{G}_1) \quad (14)$$

Then, it now possible to define the potential of free energy, and therefore the Piola-Kirchhoff tensor.

$$\Psi(I_{11}, I_{22}, I_{12}) = \frac{1}{2} \cdot (K_1 I_{11}^2 + K_2 I_{22}^2 + K_{sh} I_{12}^2) \quad (15)$$

With K_1 , K_2 , K_{sh} , are material parameters concerning the rigidity of each transformation modes. By derivating equation 15 by the elastic part of the Green-Lagrange tensor, it comes equation 16.

$$\bar{\bar{S}} = \mu \left[\begin{aligned} &K_1 \left(\bar{\bar{E}}_e : (\vec{G}_1 \otimes \vec{G}_1) \right) \cdot (\vec{G}_1 \otimes \vec{G}_1) + K_2 \left(\bar{\bar{E}}_e : (\vec{G}_2 \otimes \vec{G}_2) \right) \cdot (\vec{G}_2 \otimes \vec{G}_2) + \\ &K_{sh} \left(\bar{\bar{E}}_e : (\vec{G}_1 \otimes \vec{G}_2 + \vec{G}_2 \otimes \vec{G}_1) \right) \cdot (\vec{G}_1 \otimes \vec{G}_2 + \vec{G}_2 \otimes \vec{G}_1) \end{aligned} \right] \quad (16)$$

It is important to notice that $K_{sh'} = 0.5 \cdot K_{sh}$. Indeed, because of the shear invariant definition and since the elastic part of Green-Lagrange tensor is symmetric, the component E_{e12} appears twice in each direction.

4.2. Plasticity criterion

As it was said before, the plasticity criterion can be written using the dissipative part D from equation 8. Given the macroscopic approach and being in total lagrangian (reasoning on the initial configuration), the definition of the dissipative potential is following:

$$D = \bar{\bar{S}} : \dot{\bar{\bar{E}}}_p \quad (17)$$

Using equation 6, the equation above can be rewritten:

$$D = \bar{\bar{S}} : \left[\frac{\dot{\gamma}_p}{2} \cos\left(\frac{\gamma_p}{2}\right) (\vec{G}_1 \otimes \vec{G}_2 + \vec{G}_2 \otimes \vec{G}_1) \right] \quad (18)$$

As the variation of the dissipative part of Green-Lagrange is only composed by the components 12 and 21, then by duality, only the component 12 and 21 of the stress tensor will contribute to the energy. The plasticity criterion without hardening evolution is following:

$$f_s^0(\bar{\bar{S}}) = \left| \bar{\bar{S}} : (\vec{G}_1 \otimes \vec{G}_2 + \vec{G}_2 \otimes \vec{G}_1) \right| - S_y \quad (19)$$

Since, f_s is depending on a yield S_y , there are different domains in which the value of the criterion can be located:

- The elastic domain such as $f_s < 0$
- The limit of the elastic domain such as $f_s = 0$
- The plastic domain such as $f_s > 0$

4.3. Hardening function

There is two hardening function for this kind of behaviour. The first one is the kinematic hardening which represents the displacement of the centre of the yield surface. This is commonly written trough a second-order tensor noted X and this tensor must be chosen to be able to define the reduce stress, commonly noted $\bar{\bar{S}} - \bar{\bar{X}}$ inside the criterion:

$$f_s^X(\bar{\bar{S}}, \bar{\bar{X}}) = \left| (\bar{\bar{S}} - \bar{\bar{X}}) : (\vec{G}_1 \otimes \vec{G}_2 + \vec{G}_2 \otimes \vec{G}_1) \right| - S_y \quad (20)$$

Such as:

$$\bar{\bar{X}} = \frac{1}{2} q(\gamma_p) (\vec{G}_1 \otimes \vec{G}_2 + \vec{G}_2 \otimes \vec{G}_1) \quad (21)$$

Where $q(\gamma_p)$ is a function determinate experimentally which represent the evolution of the surface center. The second is the isotropic hardening which increase or decrease the plasticity yield S_y . The representative function is α and the formalism is following:

$$f_s^\alpha(\bar{\bar{S}}) = \left| \bar{\bar{S}} : (\vec{G}_1 \otimes \vec{G}_2 + \vec{G}_2 \otimes \vec{G}_1) \right| - (S_y + \alpha(\gamma_p)) \quad (22)$$

Given that the projection is equal in both direction, it comes:

$$\bar{\bar{X}} : (\vec{G}_1 \otimes \vec{G}_2 + \vec{G}_2 \otimes \vec{G}_1) = 2 \frac{1}{2} q(\gamma_p) = q(\gamma_p) \quad (23)$$

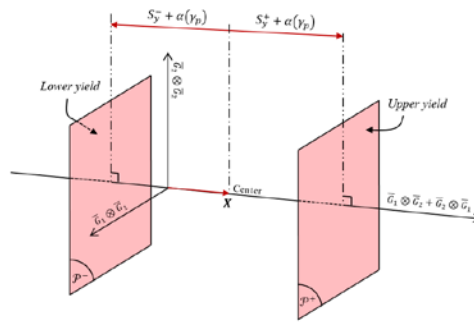


Figure 4. The yield criterion in the 3D stresses space (with γ_p supposed to be greater than zero).

Finally, the total yield criterion is only depending on a single variable which is the angle of plastic deformation γ_p .

$$f_s(\gamma_p) = \left| \bar{\bar{S}} : (\vec{G}_1 \otimes \vec{G}_2 + \vec{G}_2 \otimes \vec{G}_1) - q(\gamma_p) \right| - (S_y + \alpha(\gamma_p)) \quad (24)$$

5. Calibration by a Picture Frame experimentation

The elasto-plastic model described above needs to be compared to experimental results to be calibrated. To do that, a Picture Frame experimentation is made imposing this following tensor of total transformation F:

$$\bar{\bar{F}} = \cos\left(\frac{\gamma_t}{2}\right) (\vec{G}_1 \otimes \vec{G}_1 + \vec{G}_2 \otimes \vec{G}_2) + \sin\left(\frac{\gamma_t}{2}\right) (\vec{G}_1 \otimes \vec{G}_2 + \vec{G}_2 \otimes \vec{G}_1) \quad (25)$$

Equation 25 coupled with equation 24 leads to:

$$f_s(\gamma_p) = \left| 2\mu K_{sh} (\sin(\gamma_t) - \sin(\gamma_p)) - q(\gamma_p) \right| - (S_y + \alpha(\gamma_p)) \quad (26)$$

The experimental results are shown in figure 5:

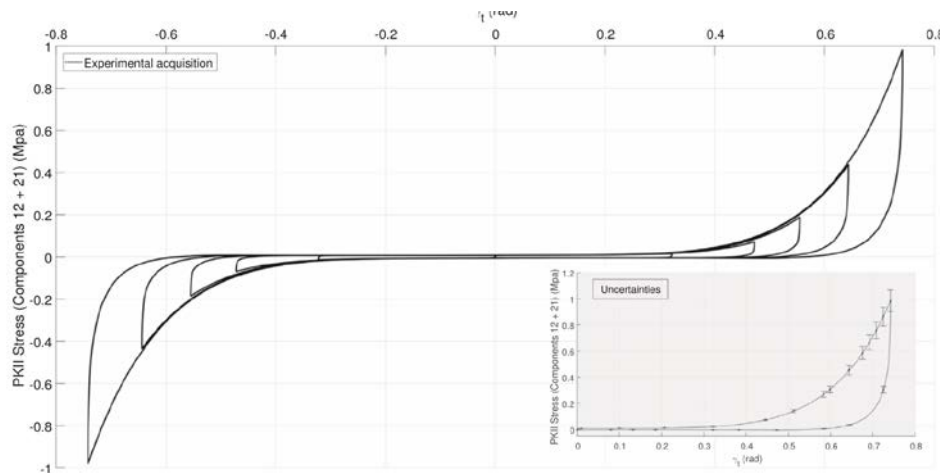


Figure 5. Experimental behaviour during cycling loading and its measurement uncertainties.

Through the behavior presented in figure 5, it is possible to deduce the equation describing the hardening functions (Kinematics and Isotropic).

$$q(\gamma_p) = \sum_{i=1}^8 Q_i \gamma_p^i \quad (27)$$

$$\alpha(\gamma_p) = \sum_{i=1}^8 A_i \gamma_p^i \quad (28)$$

Such as

Table 1. Kinematics and Isotropic hardening parameters.

i	Q_i	A_i	i	Q_i	A_i
1	0.01	0.0016	5	2.853	0.0013
2	-0.00008	-0.018	6	-0.004	3.962
3	0.0084	-0.0002	7	3.184	-0.00126
4	0.000982	0.926	8	1.810	1.907

Using the parameters presented in table 1 and $K_1 = K_2 = 10000$ and $K_{sh} = 142$ the simulation result is following:

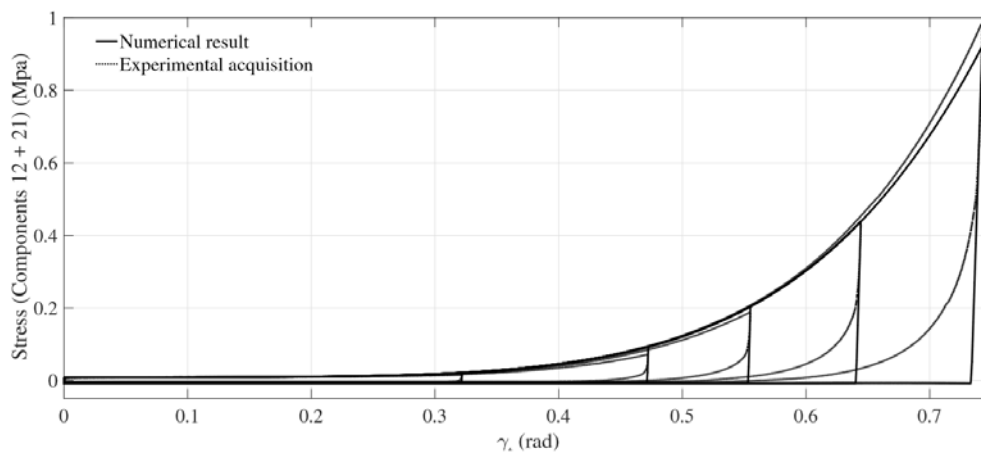


Figure 6. Experimental result and comparison with numerical result.

As it can be shown on the figure 6, the numerical result is satisfying during the loading phase but does not give efficient results during the unloading phase. To improve this model, the strategy adopted here is the use of nested surface as Mroz described it in 1967. Considering the figure 4, where only one surface was used and adding two other surfaces inside which are following specific evolution laws (linear) it would be possible to get the hysteretical behaviour of the material.

6. Improvement of the model using nested surfaces

As it was said before, the constitutive model is good enough for the loading phase but adapted for the unloading phase. In 1967 Mroz proposed a possibility to fit an elastoplastic behaviour with several surfaces. The idea in this section is to improve the general modelisation by adding two surfaces inside the surface presented before. The evolution laws of internal surfaces are supposed to be linear but are different following different case. Every laws and cases are explained below.

6.1. Formalism with three surfaces

The improved model is composed of three surfaces which are presented on figure 7.

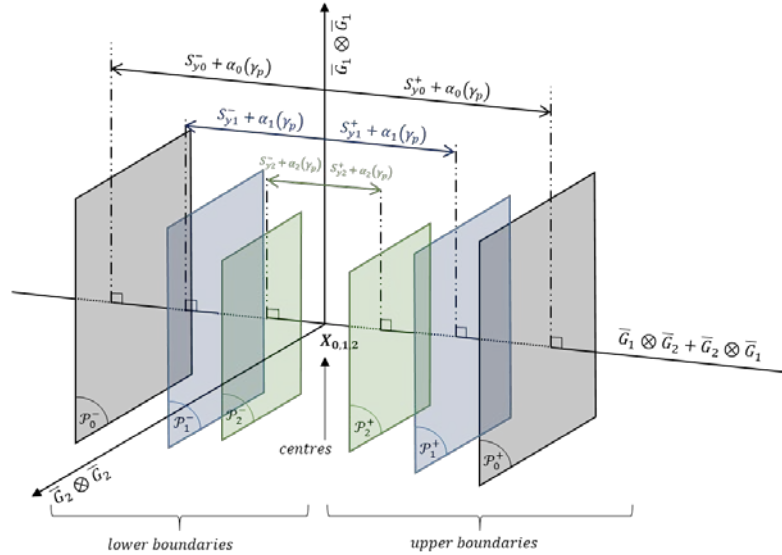


Figure 7. Improved constitutive model with two internal surfaces.

In figure 7, several surfaces are presented:

- The external yield surface (denoted 0) in black (or denoted 0) on the figure 7.
- An internal yield surface (denoted i with $i \in [1, n - 1]$ with n the total number of internal surfaces in blue (or denoted 1) on the figure 7.
- The smallest yield surface (denoted n) in green (or denoted 2) on the figure 7.

The first possibility is when the smallest surface is active. This surface is active at the beginning of the simulation or at the transition between the loading and the unloading phase.

6.2. The smallest surface is active

In this case, only one surface has to be updated, and its yield function is described by this following equation:

$$f_s(\gamma_p) = \left| \bar{\bar{S}} : (\vec{G}_1 \otimes \vec{G}_2 + \vec{G}_2 \otimes \vec{G}_1) - q_n(\gamma_p) \right| - (S_{yn} + \alpha_n(\gamma_p)) \quad (29)$$

Thus, it is sufficient to update the active surface n by its proper linear evolution laws:

$$\begin{cases} \alpha_n(\gamma_p) = a_n \alpha_0(\gamma_p) \\ q_n(\gamma_p) = h_n(\Delta\gamma_p) \end{cases} \quad (30)$$

Then, the second possibility is when a surface i between the smallest and the external surfaces is active.

6.3. A middle surface is active

In this case, more surfaces have to be updated. Indeed, in figure 7 if the surface 1 is active, then the surfaces 1 and 2 must be updated. Concerning the active surface i , the specific laws are following:

$$f_s(\gamma_p) = \left| \bar{S} : (\vec{G}_1 \otimes \vec{G}_2 + \vec{G}_2 \otimes \vec{G}_1) - q_i(\gamma_p) \right| - (S_{yi} + \alpha_i(\gamma_p)) \quad (31)$$

The coefficients are updated by these following equations:

$$\begin{cases} \alpha_i(\gamma_p) = a_i \alpha_0(\gamma_p) \\ q_i(\gamma_p) = h_i(\Delta\gamma_p) + q_i(\gamma_p - \Delta\gamma_p) \end{cases} \quad (32)$$

Concerning the surfaces j smaller than the active surface such as $j \in [i + 1, n]$ their coefficients of the evolution laws are updated as following:

$$\begin{cases} \alpha_j(\gamma_p) = a_j \alpha_0(\gamma_p) & a_i > a_j > a_n \quad \forall j, a_j \in [0, 1] \\ q_i(\gamma_p) = \bar{S} : (\vec{G}_1 \otimes \vec{G}_2 + \vec{G}_2 \otimes \vec{G}_1) - \text{sgn}(\Delta\gamma_p) (S_{yj} + \alpha_j(\gamma_p)) \end{cases} \quad (33)$$

As it can be seen, the evolution laws are made to keep the contact between the surface in order to be fully representative. The latest case is when the active surface is the external one.

6.4. The external surface is active

In this case, the external evolution laws are following the laws presented on equations 26, 27 and 28. However, it is still necessary to update the internal surfaces j such as $j = [1, n]$ by the same evolution laws presented in equations 33.

6.5. Final result of the final model

The result is presented on figure 8. As it can be shown, the model is compatible for positive and even negative loading and unloading. All parameters used on the constitutive equations and on the evolution laws are presented in table 2.

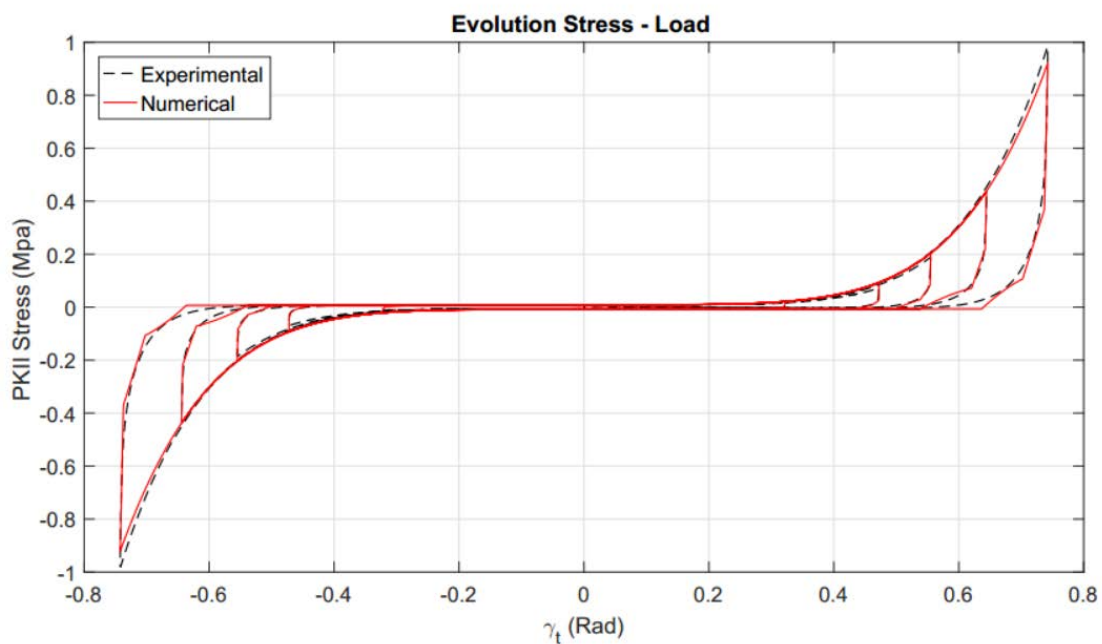
Table 2. Evolution laws coefficients.

$ \gamma_t $ (Deg	a_0	a_1	a_2	h_0	h_1	h_2
[00, 19[1	0.9	0.85	1	0.2	2
[19, 28[1	0.9	0.75	1	1	2
[28, 32[1	0.9	0.6	1	1.5	4
[32, 37[1	0.9	0.5	1	2	8
[37, 40+[1	0.99	0.45	1	4	10

The material coefficients are presented in table 3.

Table 3. Material coefficients.

	<i>Value</i>
K_1	10000
K_2	10000
K_{sh}	142
Sy_0	0.0081
Sy_1	0.006
Sy_2	0.0025

**Figure 8.** Final result of the improved constitutive model.

The model, using three nested surfaces makes possible to approach the experimental acquisition correctly. The result would be even better by adding other surfaces. However, it is necessary to find a compromise between the number of surfaces and the simulation time. Indeed, in this case, having very small and very close plasticity yields impose small increments at the beginning to avoid instabilities between the surfaces.

7. Conclusion

The principal objective of this paper is to propose an elasto-plastic constitutive model considering the hysterical effect of the material during an unloading phase. To do that, two major innovations were written. Firstly, the additive decomposition of Green-Naghdi has been adapted for anisotropical material knowing high geometrical non-linearities (under large strains). On the other hand, the use of the Mroz theory about nested surfaces gives the possibility to approach the hysterical behaviour of the material. As it is possible to see on the figure 8, this model is good enough, but it needs a large quantity of parameters which can be difficult to identify. However, it is very easy to compute this model since it depends by only one variable. Then, a simple Newton-Raphson algorithm permits to get a solution very quickly. Finally, this model is a first step to describe dissipative effect and high non-linearity for woven

fabric during its shaping phase. It completes the models already written who are, for a majority of them, hyper-elastic or written in the context of small perturbation.

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