

Meso-scale model for the forming process of biaxial reinforced weft-knitted fabrics

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Abstract. Numerical modelling of textile materials is important for developing new forming processes of textile reinforcements for composite parts, namely to decide on forming tool geometry and forming process parameters such as blank holder forces. A numerical model reduces the application of trial-and-error, which significantly reduces costs and material resources. The magnitude and distribution of blank holder forces play a decisive role in the quality of the formed textile. In general, no or even low blank holder forces will lead to wrinkles in the useful part of the preform. This reduces the mechanical quality of composite parts reinforced by textiles. In contrast, excessive blank holder force will cause damage to the textile. To observe these effects in complex forming situations, a material model has to be found which takes all forming modes of textile materials into account. A meso-scale model for biaxial reinforced weft-knitted fabrics is introduced, in which present yarns are simplified and discretized by beam elements. A single yarn is considered as a chain of multiple beam elements. Methods of modelling the complex geometry of the reinforced knitted structure are presented. The models are suitable for virtual mechanical tests and large scale forming simulations.

1. Introduction

The nature resources for economic development are limited. To achieve a sustainable development, the energy consumption, the emission of harmful substances and the use of material should be reduced. Fibre-reinforced polymer composites (FRP), which are increasingly used, are one of the innovative development to meet the mentioned needs. Continuous fiber-reinforced composites consist of a tensile load-absorbing textile reinforcement structure and a shaping, compressive-load-absorbing matrix material. FRPs have excellent properties, such as high specific strength and stiffness, good damping properties, chemical resistance and low thermal expansion. In comparison to conventional materials, particularly based on metals, they are distinguished by outstanding corrosion resistance and weight reduction. In order to fully exploit the potential of the fiber reinforcement in the composite, the fibers in the preform have to be arranged along the main load direction and suitably embedded into the matrix [1]. Vallons et al. [2] concluded that the strength of a biaxial carbon/epoxy non crimp fabric composite decreased approximately 20% under tensile stress due to a 5° misalignment of the fibre.

The forming of complex 3D components from 2D textile material with a load path optimized textile reinforcement structure requires innovative and reliable manufacturing processes at a reasonable price. To boost the usability of this material a reliable process design is necessary, which can only be achieved with the help of numerical methods. Many production parameters of the forming process have to be determined by “trial and error” experiments, for example the blank holder forces required for forming.



Excessive blank holder forces will pull the textile apart at a short travel distance of the stamp. Low blank holder forces lead to wrinkles on semi-finished product, which would reduce the mechanical properties of the composite part [3]. With the help of numerical simulation tools, the time and cost intensive “trial and error” experiments can be avoided.

There are some computing models available for analyzing the forming behavior of textile structures, such as woven fabrics [4–9], biaxial and multiaxial non-crimp fabrics [6,10], weft-knitted fabrics [11,12], and multiaxial warp-knitted fabrics [13]. The textile structure, in general, can be modelled at three different levels of objectivity: micro-scale (Figure 1), meso-scale (Figure 2) and macro-scale (Figure 3).

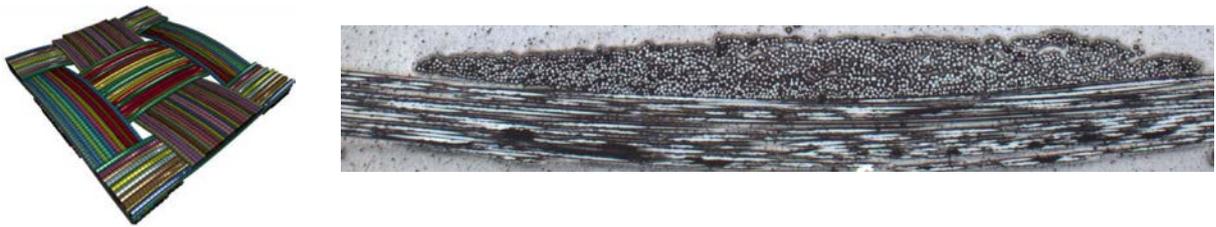


Figure 1. Micro-scale model of a plain woven fabric unit cell with 30 beam element chains for 1 yarn (left) and the cross-section of the real fabric (right) [14]

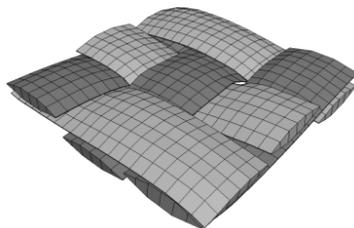


Figure 2. Meso-scale model of a twill woven fabric unit cell with shell elements [15]

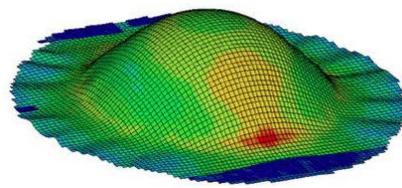


Figure 3. Macro-scale model of textile fabrics during forming process [16]

At the micro-scale, a single filament of the technical multifilament yarns can be modelled by beam or solid elements. But for the sake of simplification the quantity of representative beam or solid elements in the model is far less than the reality as shown in Figure 1. The models based on this method are closest to reality. The computational costs are highest compared to the other levels. For mechanical characterization of the filament, additional properties of the material such as the contact friction should be determined.

The models on the meso-scale level are simpler than the previous one. The smallest unit in these models is the yarn, not the filament itself. Depending on the diameter and the estimation of the behavior of yarn, beam elements [17], shell elements [15] or solid elements [18,19] are used. The computational cost of these models is lower than on the micro-scale but still relatively high compared to a continuum macro model. The contact between the single yarns, as well as the contact between the yarns and other objects like forming tools still has to be computed. For every new binding type, a new model for the textile is required. Figure 2 shows a meso-scale model of twill woven fabric unit cell with shell element [15]. Despite its simplicity, the meso-scale model can be used for mechanical [20,21], permeability [22,23] and structural analysis [24].

The macro-scale models have the least computational cost, which is significantly lower in comparison to the meso-scale and the micro-scale models. The textile structure is summarized in a continuum as shown in Figure 3. For a flat textile structure, shell elements can be applied for modelling. The textile structure plays no role while building the macro-scale models. The required parameters for the material model are the force-strain tensile behavior in warp and weft direction, the shearing behavior as well as

the bending stiffness. Those properties have to be determined by physically or virtually testing the fabrics [25]. Additionally, according to Boisse et al. [26] the tensile behavior of the fabric, where the yarn systems are interwoven, is biaxial, i. e. the tension-deformation states in the warp or weft directions depend on the other direction. Therefore, this biaxial tensile behavior also needs to be tested on a biaxial tensile testing device. Despite its simplicity the macro-scale model can describe the shearing of the yarn system and typical failure modes during forming of textile such as wrinkle formation.

The development process of complex composite structures requires a higher level of objectivity, which the meso-scale model can offer. In this paper a simplified meso-model for biaxial reinforced weft-knitted fabrics is introduced, in which even yarn slippage and sliding are taken into account. Yarns are simplified and discretized by beam elements. A single yarn is considered as a chain of multiple beam elements. Methods of modelling, validation and the application of this model will be presented.

2. Material

The biaxial reinforced weft-knitted fabrics in this paper are made of commingled hybrid yarns of carbon fibre and polyamide 6.6. Folded yarn of glass fibre and polyamide 6.6 was used as the knitting yarn as shown in Figure 4. The fabrics were fabricated on a modified flat-bed knitting machine [27].

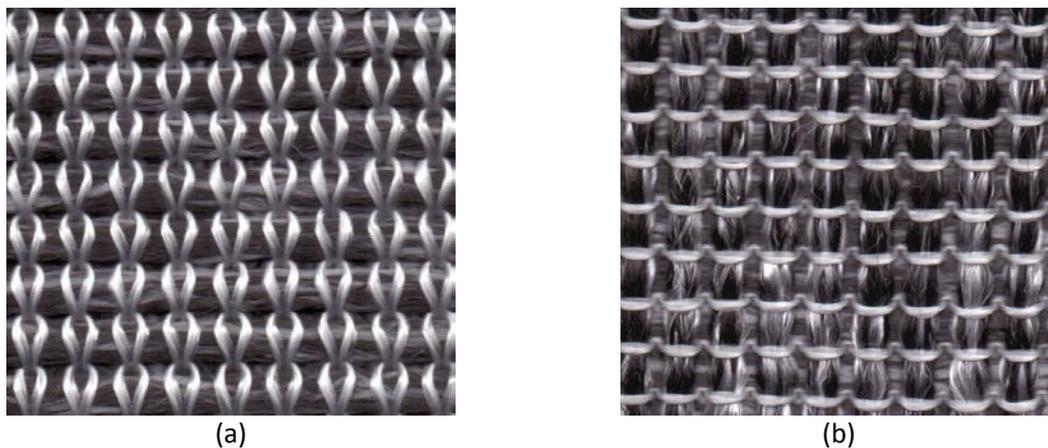


Figure 4. Biaxial reinforced weft-knitted fabric: (a) Right side (b) Left side

3. Methods

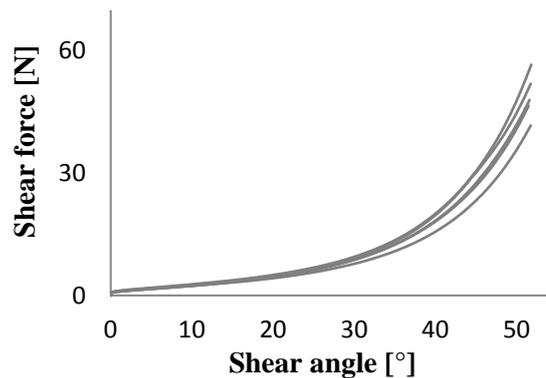
3.1. Experimental testing of the textile material

To characterize the textile material, typical tensile experiments for yarns (ISO 3341 [28]) and textile stripes (DIN EN ISO13934-1 [29]) were carried out. Additionally, shear tests with a picture frame of size 200 mm \times 200 mm and cantilever bending tests (DIN 53362 [30]) help to determine the shear resistance and bending rigidity of the fabrics accordingly. A summary of the textile characteristic is presented in Table 1.

The CF/PA hybrid yarn has a fineness of 1200 tex and is actual a folded yarn from four smaller commingled hybrid yarns of 300 tex fineness. It has a non-linear mechanical characteristic as shown in Figure 16. On the contrary the knitting yarn, which is a folded yarn of glass fibre of 68 \times 2 tex and polyamide 6.6 of 31.2 tex, possesses a linear mechanical characteristic as shown in Figure 17. The result of tensile textile stripes test and shear test also shows non-linear characteristic as shown in Figure 19 and Figure 5 accordingly.

Table 1. Characteristics of the biaxial reinforced weft-knitted fabrics

| Parameter | Value |
|--|----------------------------------|
| Reinforcing yarn (commingled hybrid yarn and folded) | CF/ PA 6.6 (4 × 300 tex) |
| Distance between reinforcing yarns | 3.6 mm |
| E-Modulus of reinforcing yarn | 69.6 GPa |
| Knitting yarn (folded yarn) | GF (68×2 tex)/ PA 6.6 (31.2 tex) |
| E-Modulus of knitting yarn | 68.4 GPa |
| Flat density of the fabrics | 824.2 g/m ² |
| Thickness of fabric according to ISO 5084 [31] | 1.9 mm |
| Max. tensile force of fabric according to DIN EN ISO 13934-1 [29] | 7530 N |
| Fracture strain of fabric according to DIN EN ISO 13934-1 [29] | 1% |
| Overhang length of fabric in Cantilever test according to DIN 53362 [30] | 165 mm |
| Loop length | 14.4 mm |

**Figure 5.** Shear force-shear angle curve for picture frame test of fabrics

Although the biaxial reinforced weft-knitted fabric is classified as a non-crimp fabric, the reinforcing yarns in the structure are actually not perfectly non-crimp. Due to the interaction between knitting yarn and reinforcing yarn during knitting process, the reinforcing yarns will be slightly crimped. To determine the crimp ratio of reinforcing yarn, the standard DIN 53852 [32] is applied. The crimp ratio is calculated as follows

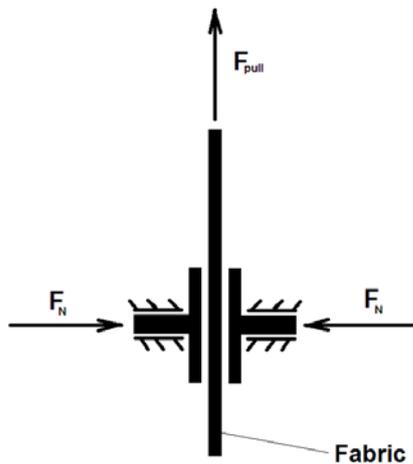
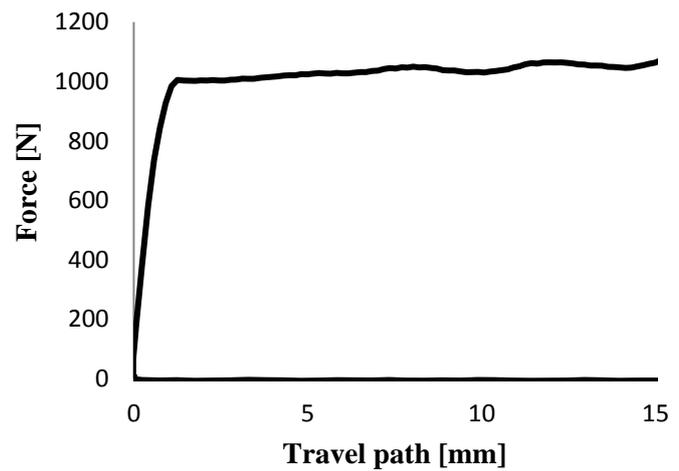
$$E = \frac{l_f - l_w}{l_f} = 1 - \frac{l_w}{l_f} \quad (1)$$

where E [-] is the crimp ratio, l_f [mm] is the length between two markings on the yarn after taking from the textile structure and stretching under conditions as in German Standard DIN 53 852 [32], l_w [mm] is the length between two markings on the yarn before stretching. The crimp ratio of warp and weft yarns are shown in Table 2.

The friction between the textile and metal part was tested on a biaxial tensile test machine. The principle of the test is shown in Figure 6. The textile stripe sample is pulled with force F_{pull} along its length while being pressed with a predefined force F_N in the vertical direction. The experimental result of the friction test is shown in Figure 7.

Table 2. Crimp ratio of warp and weft reinforcing yarns

| | Crimp ratio E [-] |
|-----------|-------------------|
| Warp yarn | 0.003496 |
| Weft yarn | 0.002597 |

**Figure 6.** Testing principle of friction between textile and metal part on the biaxial tensile machine**Figure 7.** Experimental result of the friction test

Forming experiments of the textile fabric with L-profile were used to validate the usefulness of the model [33]. An overview of the stamp forming device with eight blank holder segments is shown in Figure 8. A formed sample with the L-Profile forming device for the biaxial reinforced weft-knitted fabric is shown in Figure 9. This sample was scanned with a laser device to determine the yarn orientation in the formed structure, which will be used as a comparison to the forming simulation using the textile meso-scale model.

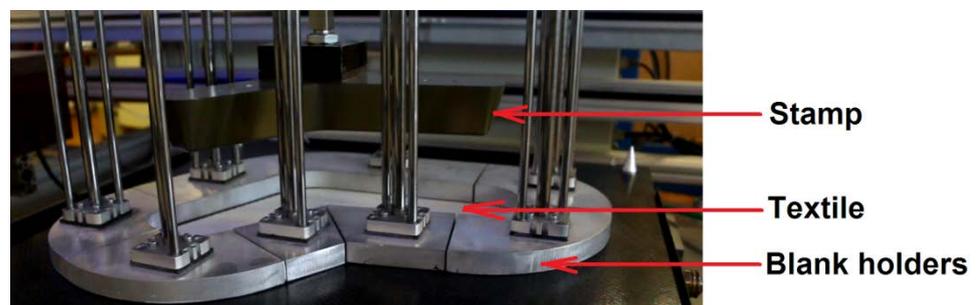
**Figure 8.** Forming device with L-shape [33]



Figure 9. Formed sample with the L-Profile forming device for the biaxial reinforced weft-knitted fabric

3.2. Modelling

In general the yarns are simplified and discretized by beam elements. A single yarn is considered as a chain of multiple beam elements. At the recent state of development, linear elastic material model is used. By the application of Belytschko-Schwer resultant beam formulation [34], the bending rigidity of the yarn can be described correctly. The diameter of the beam element can be chosen freely and does not affect beam bending rigidity. The methods of modelling the geometry of yarns will be presented in the next parts.

3.2.1. Modelling geometry of the knitting yarn system

The knitting yarn system in the biaxial reinforced weft-knitted fabric is a single jersey structure, which can be described mathematically by the equations of Choi and Lo [35]. The construction of one knitting loop is governed by the following equations:

$$x(t) = at^3 - 1.5at^2 + 0.5(a + w)t \quad (2)$$

$$y(t) = 0.5(c + 2e)(1 - \cos(\pi t)) \quad (3)$$

$$z(t) = 0.5(t_h - d)(1 - \cos(2\pi t)) \quad (4)$$

where x, y, z [mm] are coordinates of every single point on the knitting loop, w [mm] is the loop width, $(c+2e)$ [mm] is the loop height, e [mm] is the adjacent loop overlapping distance, t_h [mm] is the fabric thickness, d [mm] is the yarn diameter, a [-] is a free parameter, t [-] ($t \sim [0,1]$) is a parameter of the space curve. An example of a simple single jersey structure modelled by these equation is shown in Figure 10.

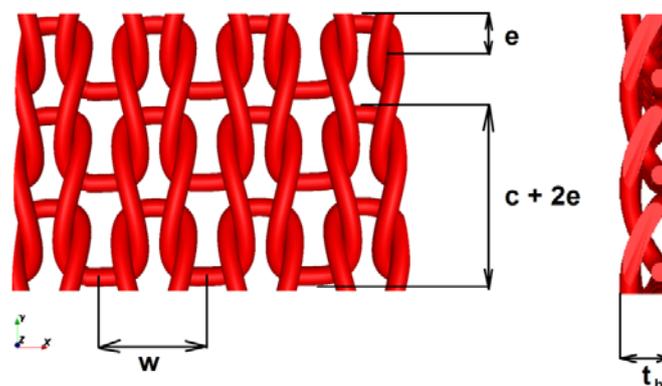


Figure 10. Single jersey weft-knitting structure according to Choi and Lo [36]

3.2.2. Modelling geometry of the reinforcing yarns

To implement the crimp of the yarns into the model, it is assumed that these yarns have only vertical waviness along its main axis in a harmonic manner as shown in Figure 11.



Figure 11. Reinforcing yarn within the fabric with waviness

The relationship between the coordinators of the yarn is described as follows:

$$z(y) = A \cos(y) \quad (5)$$

where z [mm] is the z coordinate of a point on the reinforcing yarn, y [mm] is the y coordinate of a point on the reinforcing yarn, and A is a waviness parameter. The length of a cosine wave arc is from 0 to 2π and is determined with:

$$\int_0^{2\pi} \sqrt{1 + A^2 \sin^2 y} dy \quad (6)$$

$$= 4 \int_0^{\frac{\pi}{2}} \sqrt{1 + A^2 \sin^2 y} dy \quad (7)$$

Equation (7) is a Legendre complete elliptic integral of the second kind [36] and can be evaluated with functions available in the mathematical library of SymPy for Python. From Equation (1) we can calculate the crimp ratio of the beam chain in the model with the following formula:

$$E = 1 - \frac{4 \int_0^{\frac{\pi}{2}} \sqrt{1 + A^2 \sin^2 y} dy}{2\pi} \quad (8)$$

By solving Equation (8) for each value of crimp ratio E of warp yarn and weft yarn, the waviness parameter A for each reinforcing yarn system can be determined.

4. Result

The diameter of yarns was chosen as 0.75 mm for reinforcing yarns and 0.25 mm for knitting yarns. An overview of the meso-scale model is shown in Figure 12 and Figure 13.

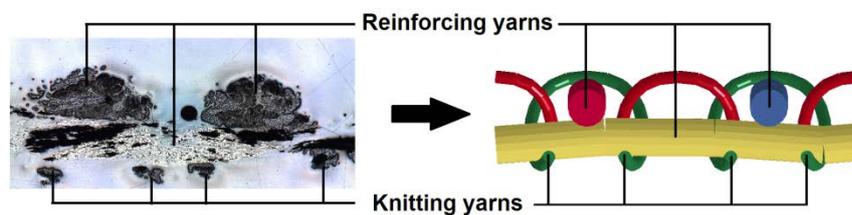


Figure 12. Cross-section of yarns in the fabric and its representative beams in the model

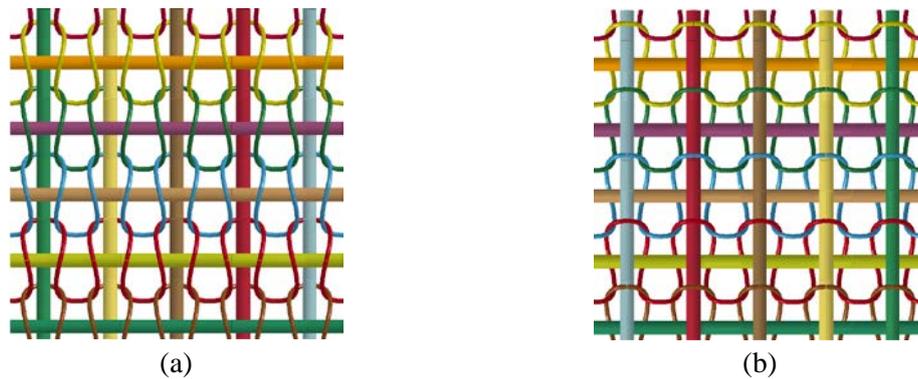


Figure 13. Overview of the meso-scale model: (a) Right side (b) Left side

A simple simulation of the cantilever test for a single yarn helps to verify the bending rigidity of the beam element in the model as shown in Figure 14. For knitting yarn an analogous simulation was also made for the same purpose.

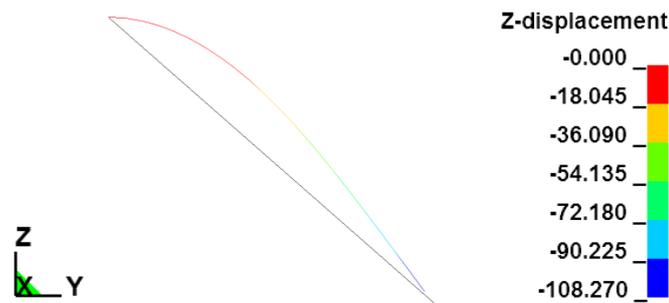


Figure 14. Simulation of cantilever test to verify the bending rigidity of beam element representative for reinforcing yarn in model

Another simulation of cantilever test for the textile fabric was used to verify the bending rigidity of the whole textile meso-scale structure in model as shown in Figure 15. Both bending rigidities – yarn and fabric – were verified with those simulations.

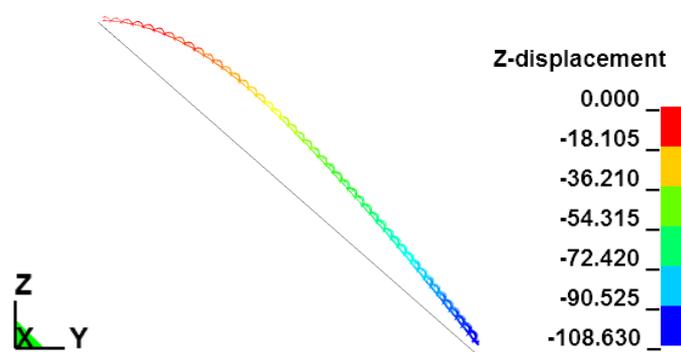


Figure 15. Simulation of cantilever test to verify the bending rigidity of the biaxial reinforced weft-knitted fabric in model

The simulation of tensile test helps to validate the force-strain behavior of the beam element with linear elastic material model. While this material model satisfied the requirement of yarn material with

linear characteristic such as knitting yarn (Figure 17), it causes a noticeable error by the yarn material with non-linear characteristic such as the commingled hybrid reinforcing yarn (Figure 16).

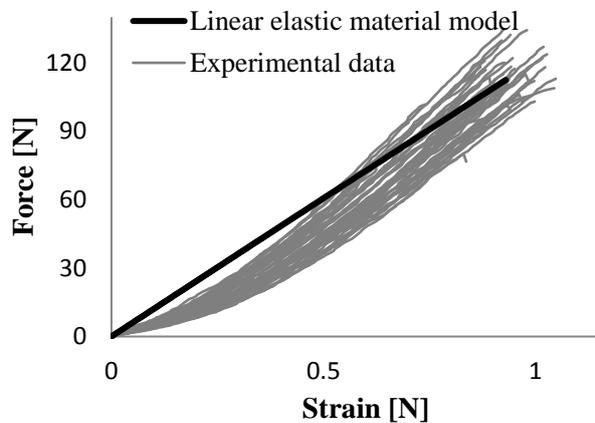


Figure 16. Comparison of force-strain curves of linear elastic material model and experimental data of reinforcing yarn

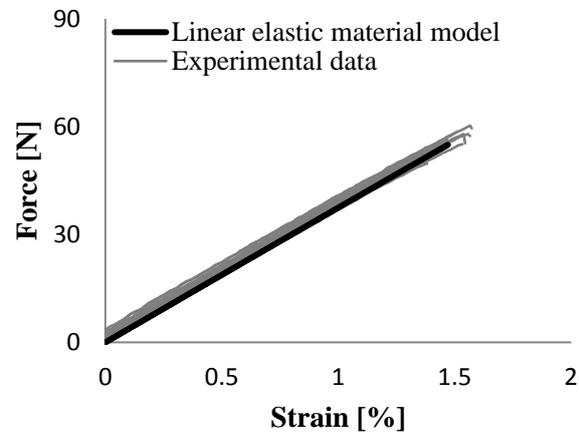


Figure 17. Comparison force-strain curves of linear elastic material model and experimental data of knitting yarn

A simulation of the fabric tensile test helps to check the correct mechanical behavior of the textile fabric model. Here, the fabric stripes with the dimension of 200 mm × 50 mm, consist of 14 yarns in the tensile direction. Due to the simplification of the yarn at the meso-scale, the main reinforcing yarn system in the model along the tensile direction takes most of the tensile stress. Figure 18 shows the axial force of the beam elements in the structure just before reaching fracture strain. Figure 19 shows the comparison of force-strain curves between simulation and experimental tensile tests.

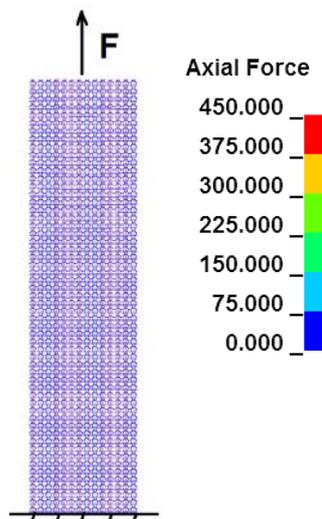


Figure 18. Axial force of the beam elements in structure just before reaching fracture strain

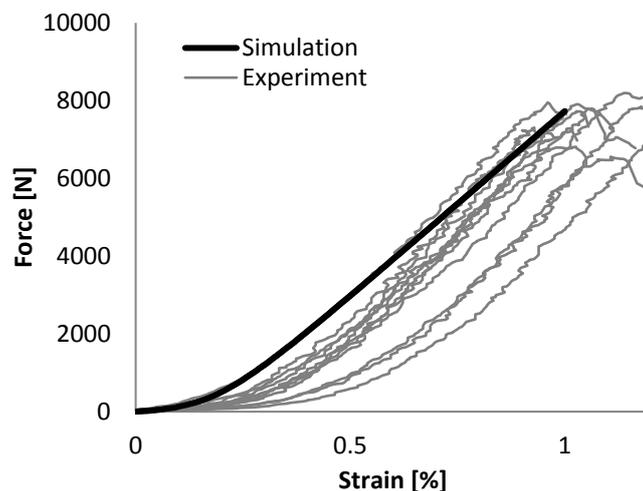


Figure 19. Comparison of force-strain curves between simulation of tensile test for textile stripes and experiment

A forming simulation using the L-profile was performed to check the usefulness of the textile meso-scale model as shown in Figure 20. The simulation was done with the same process parameters, which were used for the experiment (Figure 9), namely force of 44N was applied homogeneously for each

blank holder segment. The result of simulation shows the yarn run of the warp reinforcing system as in Figure 21. For model validation with experimental test, the experimentally formed sample as shown in Figure 9 was scanned with laser triangulation to record the yarn run (Figure 22). An optical comparison is shown in Figure 23.

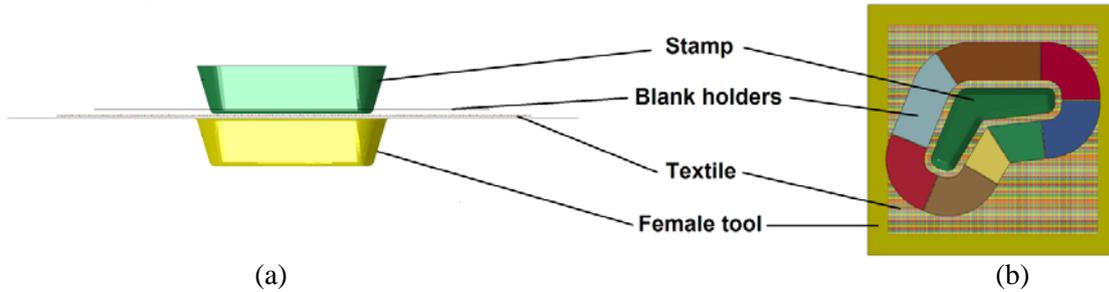


Figure 20. Forming simulation for L-Profile: (a) Front view (b) Top view

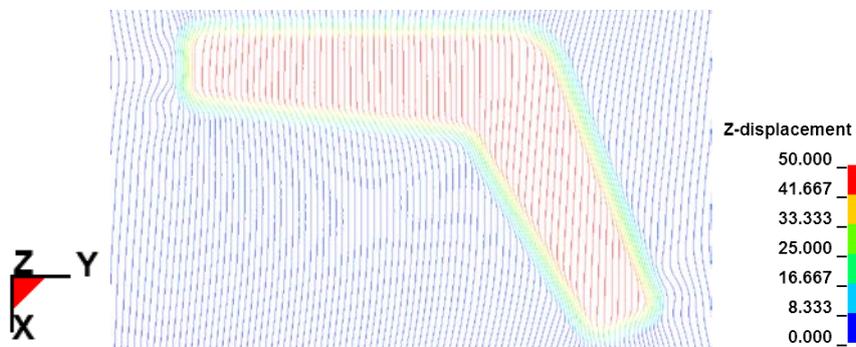


Figure 21. Yarn run of warp reinforcing system according to simulation result

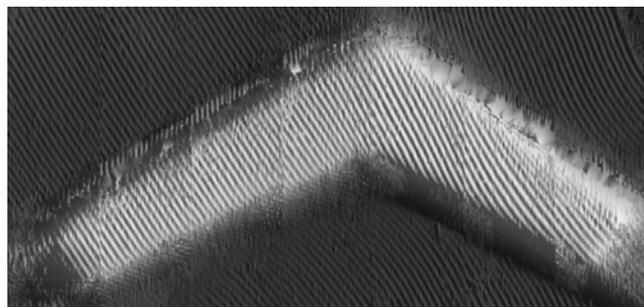


Figure 22. Scanned result of the formed sample with yarn run

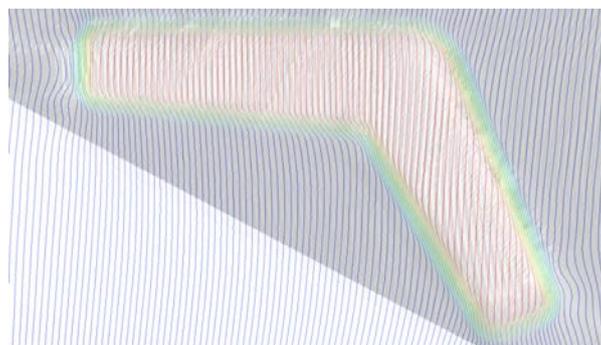


Figure 23. Optical comparison of yarn run between simulation result and scanned data

To quantitatively compare the yarn direction between model and experiment, the angle of each yarn run on top area of the formed part with the horizontal direction was measured as shown in Figure 24. The quantitative direction comparison of 70 warp yarns on the top side of the L-profile part is shown in Figure 25. At the left end of the L-profile, strong sliding of warp yarn system can be observed in both experiment and model as shown in Figure 26. Comparison between experiment and forming simulation using the meso-scale approach show very good agreement.

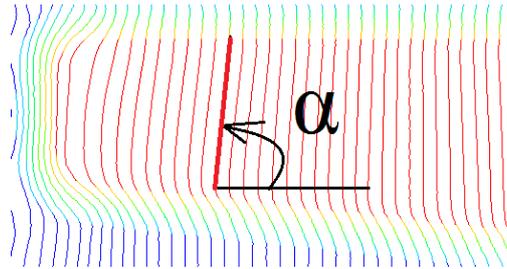


Figure 24. Measuring method of the direction of the warp yarn

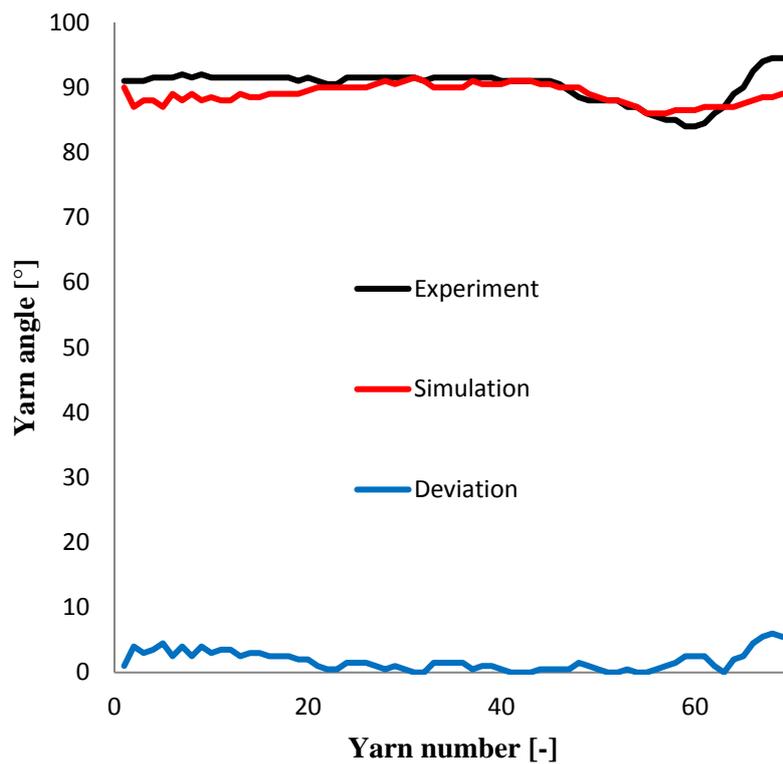


Figure 25. Quantitative direction comparison of 70 warp yarns

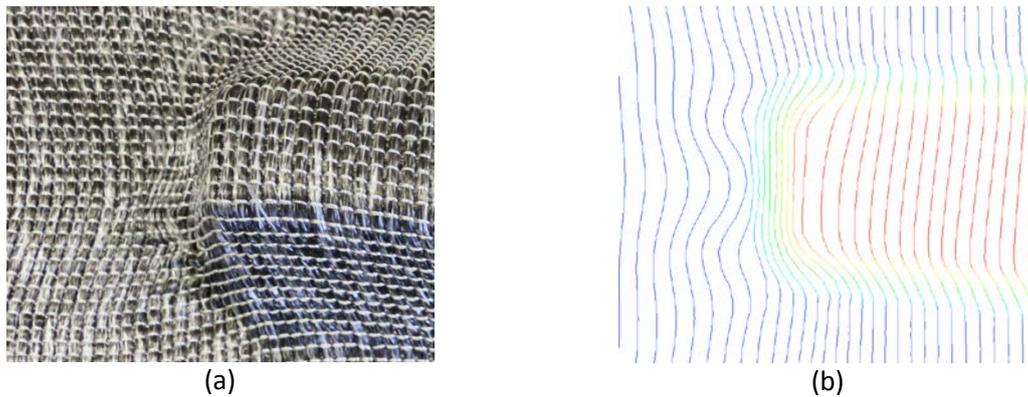


Figure 26. Strong sliding of warp yarns at left end of L-profile: (a) Experiment (b) Simulation

5. Conclusions

A meso-scale model of biaxial reinforced knitted fabric was introduced. The modelling and validating methods were presented. The model is simple and capable of taking into account most of the mechanical behaviours of the fabric. The usefulness of the model is proved by the application to determine the yarn misalignment during preforming process for an L-profile. The large-scale simulation predicted the direction of yarn with a slight deviation, this direction plays a significant role in the end properties of the composite part. The sliding of yarn is observable in the simulation results. The shear angle of reinforcing yarn system can also be validated, once the yarn run of the weft yarn system can be determined, which would be part of future studies.

Acknowledgements

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