

Buckling Analysis of FGM Plates by thin plate spline RBF based Meshfree Approach

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Abstract- Present article attempts to analyze the buckling response of FGM plates. Hamilton Principle is employed to derive the governing differential equations of theories. Discretization of governing differential equation is done using a thin plate spline RBF based meshless method. For analysis of buckling load, numerical results are obtained. Further, for purpose of validation, obtained results are compared with analytical results wherein effect of grading index and span to thickness ratio is observed.

Keywords: Meshfree, Buckling, RBF, FGM Plate

1. Introduction

FGM's have played a very intensive role in the recent development of structure and materials. They are heterogeneous materials in which the material properties vary continuously from one phase to other phase. High resistance against heat and capacity to withstand high temperature gradient has led to its intensive engineering application in the field of aircraft industry, power generation plants, computer parts, robot, nuclear power plants, automobile industry, biomedical industry etc. Owing to their unique properties many researchers have inclined their efforts on Buckling analysis of FGM materials. Yang et al. [1] carried out buckling analysis of FG plates using FSDT. Bodaghi and Saidi [2] analyzed buckling behavior of FGM plates by implementation of higher order shear deformation theory. Hosseini-Hashemi et al. [3], by implementation of Mindlin plate theory, studied buckling of in-plane loaded isotropic rectangular plates using various boundary conditions. Ghannadpour et al. [4], using finite strip method analyzed buckling behaviour of rectangular FG plates with thermal load. Thai and Choi [5], by implementation of a simple refined theory, analyzed the buckling behaviour of functionally graded plates using classical plate theory. Implementing small strain elasticity theory, Uymaz and Aydogdu [6] analyzed buckling response of rectangular FG plates using various axial loadings and boundary conditions. Chu et al. [7] used RBF collocation method and Hermite collocation method, initially developed for buckling analysis of functionally graded thin plates with inplane material inhomogeneity. Do and Thai [8] used radial point interpolation function to combine Kirchhoff plate theory and meshfree method and hence determined various results for isotropic and sandwich plates.

2. Mathematical Formulation

The coordinate system is considered consists of a rectangular plate with edge lengths 'a' and 'b' along x and y-axes, and thickness 'h' along z-axis respectively. Here the mid plane coincides with x-y plane. Figure 1 shows the geometry of FGM plate.



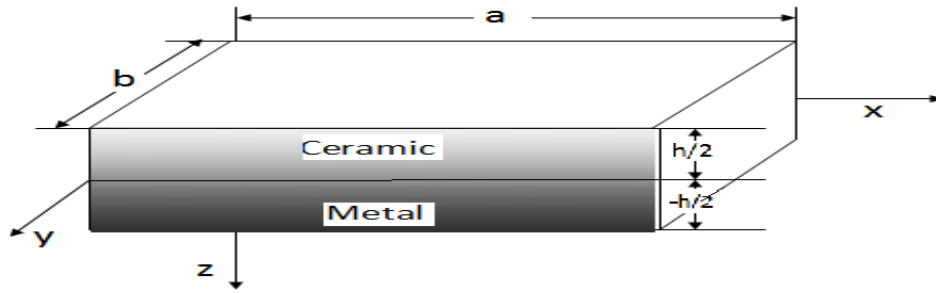


Figure 1 Geometry of rectangular FGM plate in rectangular coordinate system

The displacement field at any point in plate of uniform thickness is expressed as Reddy[9]

$$\begin{Bmatrix} U \\ V \\ W \end{Bmatrix} = \begin{Bmatrix} u_0(x, y) - z \frac{\partial w(x, y)}{\partial x} + z \left[1 - \frac{4}{3h^2} z^2 \right] \psi_x(x, y) \\ v_0(x, y) - z \frac{\partial w(x, y)}{\partial y} + z \left[1 - \frac{4}{3h^2} z^2 \right] \psi_y(x, y) \\ w_0(x, y) \end{Bmatrix} \quad (1)$$

The governing differential equations of plate using Hamilton's principle is obtained by collecting the coefficients of displacement field variables and expressed as:

$$\delta u_0 : \frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \quad (2)$$

$$\delta v_0 : \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} = 0 \quad (3)$$

$$\delta w_0 : \frac{\partial^2 M_{xx}}{\partial x^2} + \frac{\partial^2 M_{yy}}{\partial y^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} = N_{xx}^b \frac{\partial^2 w}{\partial x^2} + N_{yy}^b \frac{\partial^2 w}{\partial y^2} + 2 N_{xy}^b \frac{\partial^2 w}{\partial x \partial y} \quad (4)$$

$$\delta \psi_x : \frac{\partial M_{xx}^f}{\partial x} + \frac{\partial M_{xy}^f}{\partial y} - Q_x^f = 0 \quad (5)$$

$$\delta \psi_y : \frac{\partial M_{xy}^f}{\partial x} + \frac{\partial M_{yy}^f}{\partial y} - Q_y^f = 0 \quad (6)$$

Where, N_{xx}^b , N_{yy}^b and N_{xy}^b are the applied in-plane compressive loading in x and y direction and shear loading respectively.

$$N_{ij}, M_{ij}, M_{ij}^f = \int_{-h/2}^{+h/2} (\sigma_{ij}, z \sigma_{ij}, f(z) \sigma_{ij}) dz \quad (7)$$

$$Q_x^f, Q_y^f = \int_{-h/2}^{+h/2} (\sigma_{xz}, \sigma_{yz}) \left(\frac{\partial f(z)}{\partial z} \right) dz \quad (8)$$

The material property gradation considering power law is expressed as:

$$E(z) = [E_c - E_m] \left(\frac{2z+h}{2h} \right)^n + E_m \quad (9)$$

Here 'E' denotes the modulus of elasticity, 'h' is the thickness of the plate, E_m and E_c are the corresponding Young's modulus of elasticity of metal and ceramic, respectively, and 'z' is the thickness coordinate ($-h/2 \leq z \leq h/2$), 'n' is volume fraction exponent or grading index

The stiffness coefficients are:

$$A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij} = \int_{-h/2}^{h/2} \left\{ \left([Q_{ij}^c - Q_{ij}^m] \left(\frac{2z+h}{2h} \right)^n + Q_{ij}^m \right) (1, z, z^2, f(z), z f(z), f^2(z)) \right\} dz \quad (10)$$

for i, j = 1, 2, 6

$$A_{ij} = \int_{-h/2}^{h/2} \left\{ \left([Q_{ij}^c - Q_{ij}^m] \left(\frac{2z+h}{2h} \right)^n + Q_{ij}^m \right) \left(\frac{\partial f(z)}{\partial z} \right)^2 \right\} dz \quad (11)$$

for i, j = 4, 5

Here superscript c stands for ceramic and m stands for metal.

The boundary conditions for simply supported edges are:

$$\begin{aligned} x=0, a: v=0; \psi_y=0; w=0; M_{xx}=0; N_{xx}=0 \\ y=0, b: u=0; \psi_x=0; w=0; M_{yy}=0; N_{yy}=0 \end{aligned} \quad (12)$$

3. Solution methodology

Governing differential equations Eqn. and boundary conditions Eqn. (12) are discretized using polynomial radial basis function. A two dimensional rectangular domain with any arbitrary boundary having NB boundary nodes and NI domain interior nodes is shown in **Figure 2**.

For present analysis thin plate spline radial basis function

$$g \|X - X_j, m\| = \log(r) r^{2m} \quad (13)$$

$$\text{Where, } r = \|X - X_j\| = \sqrt{(x - x_j)^2 + (y - y_j)^2}$$

and 'm' is shape parameter. In present analysis value of m=3 is taken after validation and convergence study. The field variables (displacements) in terms of radial basis function are expressed as:

$$u = \sum_{j=1}^N \alpha_j^u g(\|X - X_j\|), v = \sum_{j=1}^N \alpha_j^v g(\|X - X_j\|), w = \sum_{j=1}^N \alpha_j^w g(\|X - X_j\|), \quad (14)$$

$$\psi_x = \sum_{j=1}^N \alpha_j^{\psi_x} g(\|X - X_j\|), \psi_y = \sum_{j=1}^N \alpha_j^{\psi_y} g(\|X - X_j\|)$$

$\delta = \alpha_j^u, \alpha_j^v, \alpha_j^w, \alpha_j^{\psi_x}, \alpha_j^{\psi_y}$ are unknown coefficient to be determined. In the eigenvalue

problems, the objective is to obtain eigenvalues (λ) and corresponding eigenvectors. The eigenvalue problem can be expressed as:

$$\begin{bmatrix} L \\ B \end{bmatrix}_{5N \times 5N} \{\delta\}_{5N \times 1} = \lambda \begin{bmatrix} A \\ 0 \end{bmatrix}_{5N \times 5N} \{\delta\}_{5N \times 1} \quad (15)$$

$$\text{Here } \{\delta\} = [\alpha^{u_0} \alpha^{v_0} \alpha^{w_0} \alpha^{\psi_x} \alpha^{\psi_y}]^T$$

Equation (15) is solved by standard eigen solvers of computational software in order to obtain eigenvalues and eigenvectors.

4. Results and discussion

A simply supported isotropic plate has been taken for convergence and validation purpose. The buckling load parameter is non-dimensionalised as $\lambda = N_x a^2 / \pi^2 (Eh^3 / 12(1 - \nu^2))$ where

$E=3 \times 10^6$ psi and $\nu=0.316$. The results are obtained using AHSST. From table 1 which is depicted in Figure 2, it can be seen that for all the four modes, a good convergence is achieved which is within 1 percent for most of the cases beyond 13×13 nodes.

Table 1 Non dimensional buckling load parameters $\lambda = N_x a^2 / \pi^2 (Eh^3 / 12(1 - \nu^2))$ of simply supported isotropic square plate under uniaxial load.

Mode	Number of Nodes						Timoshenko[10]
	7x7	9x9	11x11	13x13	15x15	17x17	
1	44.094	40.629	39.959	39.723	39.616	39.559	39.4761
2	59.491	60.879	61.294	61.448	61.515	61.548	61.9814
3	90.064	103.186	106.801	108.101	108.658	108.931	109.656
4	164.761	161.655	159.523	158.620	158.191	157.965	157.904

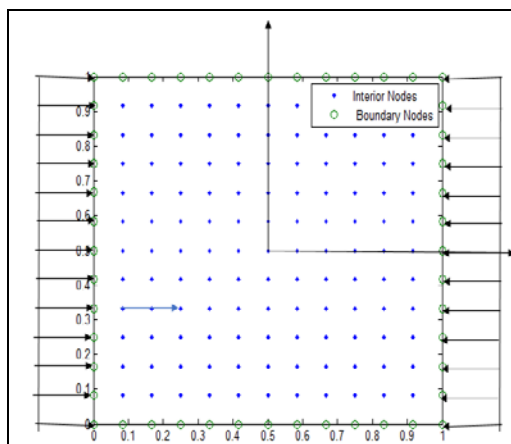


Figure-2: Uniaxial compressive load on FGM plate.

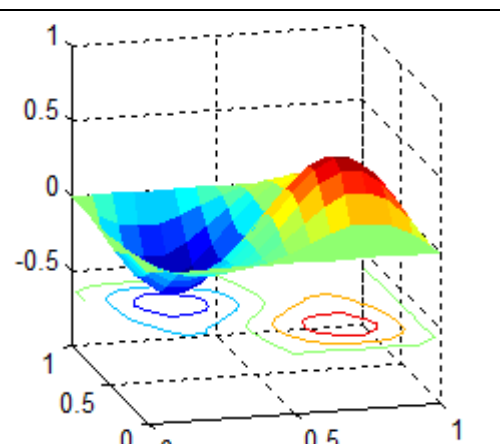


Figure-3: Deformed shape of mode-4 for FGM plate.

Figure 2 Non dimensional buckling load parameter(λ) of simply supported isotropic square plate under uniaxial load.

For further analysis functionally graded materials are taken with following properties:
FGM: Ceramic- $E_c = 200 \text{ GPa}$, $\nu_c = 0.3$, Aluminum (Al) - $E_m = 70 \text{ GPa}$, $\nu_m = 0.3$

The plate is of square shape with unit dimensions.

The buckling load parameter is non-dimensionalised as $\lambda = N_x a^2 / (E_m h^3)$

It can be seen from **Figure 4** that the critical buckling parameter increase from thick to thin FGM plate and after $a/h=40$, the effect of critical buckling parameter almost negligible. Buckling load parameter decreases as grading index increases and effect decreases as the value of 'n' increases.

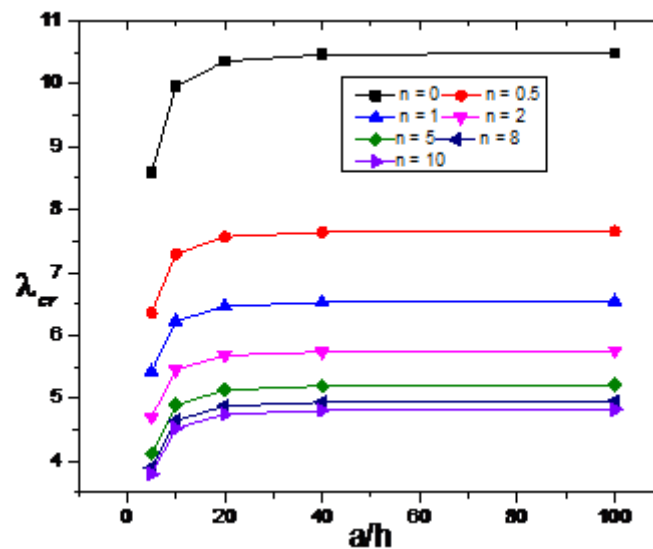


Figure 4 The critical buckling load parameter λ_{cr} of square FGM simply supported plate under uniaxial load.

5. Conclusion

Buckling loads of thick to thin FGM plates are obtained. The results herein agree well with the analytical results indicating the efficacy of present solution methodology. Effect of span to thickness ratio becomes less dominant after $a/h=40$. Buckling load parameter decreases as grading index increases. It is more dominant initially and effect decreases as index increases.

6. References

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