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Improving Vibration analysis of laminated composite plate by using WU-C2 RBF based Meshfree Method

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Abstract. The paper presents free vibration analysis of laminated composite plates using WU-C2 RBF based meshfree method. Analysis of vibration response of laminated plate using three transverse shear functions is carried out using simply supported boundary conditions. The governing equations are based on HSDT and Hamilton principle. The effect of side to thickness ratio can be accurately analysed using the present meshless method. Performance of the present method is illustrated by investigating characteristic properties of laminated composite through numerical examples.

Keywords: Meshfree, RBF, laminated composite, vibration

1. Introduction

Composite materials have been playing an essential role in various research areas owing to development of material science and technology. To achieve desired properties, layers of orthotropic materials are assembled systematically in different possible orientation and thus laminated composites are manufactured. Aeronautical industry, Structural industry, Power generation plants, Automobile industry etc. including various other domains involve intensive utilization of various laminated materials. Dai et.al[1] used HSDT for analysis of static deflection and vibration of laminated composite plates . Bui et.al [2] carried out vibration analysis of a laminated plate by application of Kriging interpolation method. Fallah N. et.al[3] studied the natural frequency of laminated plates using a meshless finite volume formulation. Ferriera et.al[4] analyzed the vibration of thick and symmetric laminated plates using FSDT based on MQ radial basis function. Wang Xinwei[5] used discrete singular convolution (DSC) method for investigation of free vibration analysis of three-layer angle-ply symmetric laminated plates. Bouazza et.al[6] analyzed natural frequency of laminated plates by employing nth order HSDT. Kulikov GM et.al[7] implemented the strong formulation for the 3D vibration analysis of laminated plates.

2. Mathematical Formulation

A rectangular laminated composite plate of length ‘a’, breadth ‘b’ along x, y-axes respectively and thickness ‘h’ in z direction. The midplane coincides with x-y plane of the coordinate system. It is assumed that there is perfect bonding between the layers of laminated plates. **Figure 1** shows the geometry of laminated plate in the rectangular coordinate system.

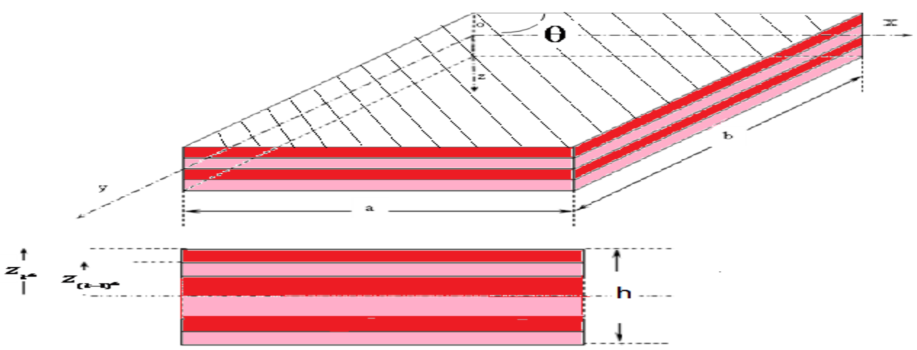

Figure 1 Geometry of laminated plate in rectangular coordinate system

Displacement field at any point on the plate, made up of uniform thickness, is expressed as

$$\begin{Bmatrix} U_x \\ U_y \\ U_z \end{Bmatrix} = \begin{Bmatrix} u_x(x,y) - z\dfrac{\partial u_z(x,y)}{\partial x} + f(z)\,\psi_x(x,y) \\ u_y(x,y) - z\dfrac{\partial u_z(x,y)}{\partial y} + f(z)\psi_y(x,y) \\ u_z(x,y) \end{Bmatrix}$$

here $f(z)$ is the transverse shear function.

Table 1. Transverse shear function $f(z)$.

S.No.	**f (z)**	**Abbreviation**	
1	$z.\left[1-\frac{4}{3h^2}z^2\right]$	HSDT 1	Levinsion [8], Reddy[9]
2	$z.m^{\left(-2\left(z/h\right)\right)^2}$, $m = 3$	HSDT 2	Mantari et.al [10]
3	$z.e^{\left(-2\left(z/h\right)\right)^2}$	HSDT 3	Karama et.al [11]

U_x, U_y and U_z are the in-plane and transverse displacements in the plate at any point (x, y, z) in x, y and z-direction respectively. And u_x, u_y and u_z are the displacements at midplane of the plate at any point (x, y) in x, y, respectively. The functions ψ_x and ψ_y are the higher rotations of the normal to the midplane due to shear deformation about x and y-axes, respectively.

Governing differential equations of the plate are obtained by collecting coefficients of δu_x, δu_y, δu_z, $\delta\psi_x$ and $\delta\psi_y$ which can be expressed as:

$$\delta u_x: \quad \frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = \left(I_0 \frac{\partial^2 u_x}{\partial \tau^2} - I_1 \frac{\partial^3 u_z}{\partial x \partial \tau^2} + I_3 \frac{\partial^2 \psi_x}{\partial \tau^2} \right)$$

$$\delta u_y: \quad \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} = \left(I_0 \frac{\partial^2 u_y}{\partial \tau^2} - I_1 \frac{\partial^3 u_z}{\partial y \partial \tau^2} + I_3 \frac{\partial^2 \psi_y}{\partial \tau^2} \right)$$

$$\delta u_z: \frac{\partial^2 M_{xx}}{\partial x^2} + \frac{\partial^2 M_{yy}}{\partial y^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} - N_x{}^b \frac{\partial^2 w_0}{\partial x^2} - N_y{}^b \frac{\partial^2 w_0}{\partial y^2} - 2N_{xy}{}^b \frac{\partial^2 w_0}{\partial x \partial y}$$

$$= I_0 \frac{\partial^2 w_0}{\partial t^2} + I_1 \left(\frac{\partial^3 u_x}{\partial x \partial \tau^2} + \frac{\partial^3 u_y}{\partial y \partial \tau^2} \right) - I_2 \left(\frac{\partial^4 u_z}{\partial x^2 \partial \tau^2} + \frac{\partial^4 u_z}{\partial y^2 \partial \tau^2} \right) + I_4 \left(\frac{\partial^4 \psi_x}{\partial x^2 \partial \tau^2} + \frac{\partial^4 \psi_y}{\partial y^2 \partial \tau^2} \right)$$

$$\delta \psi_x: \quad \frac{\partial M_{xx}^f}{\partial x} + \frac{\partial M_{xy}^f}{\partial y} - Q_x^f = \left(I_3 \frac{\partial^2 u_x}{\partial \tau^2} - I_4 \frac{\partial^3 u_z}{\partial x \partial \tau^2} + I_5 \frac{\partial^2 \psi_x}{\partial \tau^2} \right)$$

$$\delta \psi_y: \quad \frac{\partial M_{xy}^f}{\partial x} + \frac{\partial M_{yy}^f}{\partial y} - Q_y^f = \left(I_3 \frac{\partial^2 u_y}{\partial \tau^2} - I_4 \frac{\partial^3 u_z}{\partial y \partial \tau^2} + I_5 \frac{\partial^2 \psi_y}{\partial \tau^2} \right)$$

Force and moment resultants in the plate are expressed as:

$$N_{ij},\, M_{ij},\, M_{ij}^f = \int_{-h/2}^{+h/2} (\sigma_{ij},\, z\sigma_{ij},\, f(z)\sigma_{ij})\, dz \quad \text{And}$$

$$Q_x^f,\, Q_y^f = \int_{-h/2}^{+h/2} \left(\sigma_{xz},\, \sigma_{yz}\right) \left(\frac{\partial f(z)}{\partial z} \right) dz$$

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Laminated plate stiffness coefficients are expressed as:

$$A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij} = \sum_{k=1}^{nn} \int_{z_k}^{z_{k+1}} \bar{Q}_{ij} \left(1, z, z^2, f(z), z\, f(z), f^2(z)\right) dz \qquad \text{i, j = 1, 2, 6}$$

$$A_{ij} = \sum_{k=1}^{nn} \int_{z_k}^{z_{k+1}} \bar{Q}_{ij} \left(\frac{\partial f(z)}{\partial z}\right)^2 dz \qquad \text{i, j = 4, 5}$$

2.1 Boundary Conditions

Consistent boundary conditions for an arbitrary edge with simply supported edge conditions are obtained and expressed as:

$$at \quad x = 0, a: \ N_{xx} = 0, \ u_y = 0, \ u_z = 0, \ M_{xx} = 0, \ \psi_y = 0$$

$$at \quad y = 0, b: \ u_x = 0, \ N_{yy} = 0, \ u_z = 0, \ \psi_x = 0, \ M_{yy} = 0$$

2.2 Solution Methodology

WU-C2 Radial Basis Function (RBF) $g = (1-cr)^5 \left(8 + 40cr + 48(\mathrm{cr})^2 + 25(\mathrm{cr})^3 + 5(\mathrm{cr})^4\right)$ is used to discretize the governing differential equations in the space domain. Where, $r = \|X - X_j\| = \sqrt{(x-x_j)^2 + (y-y_j)^2}$ is the radial distance between nodes and 'm' and 'c' are shape parameters. In this work, in order to obtain the shape parameter 'c', an expression is as followed: $c = \dfrac{1.01N^{(1/25)N}}{\sqrt{m^{\left(\frac{N}{m}\right)}}}$

Value of 'm' is taken as 7.8 for WU-C2 RBF after validation and convergence study. The unknown field variables u_x, u_y, u_z, ψ_x and ψ_y appearing in governing differential equations is assumed in terms of radial basis function as:

$$u_x(x,y) = \sum_{j=1}^{N} \alpha_j^{u_x} g\left(\|X - X_j\|\right); \qquad u_y(x,y) = \sum_{j=1}^{N} \alpha_j^{u_y} g\left(\|X - X_j\|\right) \qquad u_z(x,y) = \sum_{j=1}^{N} \alpha_j^{u_z} g\left(\|X - X_j\|\right)$$

$$\psi_x(x,y) = \sum_{j=1}^{N} \alpha_j^{\psi_x} g\left(\|X - X_j\|\right); \qquad \psi_y(x,y) = \sum_{j=1}^{N} \alpha_j^{\psi_y} g\left(\|X - X_j\|\right)$$

The governing differential equations are discretized and finally expressed in compact matrix form as:

$$\left[[\mathrm{K}]\right]\{\delta\} = [\mathrm{M}]\{\delta\}$$

Here,

$\{\delta\} = [\alpha^{u_x}\ \alpha^{u_y}\ \alpha^{u_z}\ \alpha^{\psi_x}\ \alpha^{\psi_y}]^T$, $[\mathrm{K}]$ is Stiffness Matrix and $[\mathrm{M}]$ is mass matrix

The discretized governing equations for linear free vibration analysis can be written as:

$$\left[[K]_{5N\times 5N} + \omega^2 [M]_{5N\times 5N}\right]\{\delta\}_{5N\times 1} = 0$$

Using standard eigenvalue, the frequency is calculated as:

[V, D] = eig ([K], [M]);

Frequency (ω) = $\sqrt{D}$

3. RESULT AND DISCUSSION

To demonstrate the accuracy of present formulation, an RBF based meshless code in MATLAB is developed following the analysis procedure as discussed above. Several examples have been analyzed and compared with the published results. Based on convergence study, a 15×15 node is used throughout the study.

Table 2 Convergence study for free vibrations of simply supported laminated plate. (E1/E2=40, HSDT 2)

a/h	**13x13**	**15x15**	**17x17**	**19x19**	**Matsunaga**[12]
5	10.45818	10.47012	10.49048	10.50049	10.6876
10	15.01458	15.04751	15.05954	15.06507	15.0721
20	17.60653	17.63393	17.64396	17.64824	17.6369
25	18.02616	18.05104	18.06009	18.06392	18.0557
50	18.64462	18.66276	18.66904	18.67157	18.6702
100	18.81421	18.82859	18.83314	18.83473	18.8352

For verification of solutions obtained for vibration from the proposed WU-C2 RBF based meshfree method, a simply supported laminated plate is considered. A four layer [0/90/90/0] laminated composite plate used, has the material parameters: E1 = 40E2, G12 = G13 = 0.6E2, G23 = 0.5E2, υ12 = 0.25, ρ = 1. The convergences of non-dimensional free vibration parameters $\varpi = \left(\frac{\omega a^2}{h}\right)\sqrt{\frac{\rho}{E2}}$ for different theories are shown in Table 2. It can be seen that convergence is achieved within 2 % at 15×15 nodes.

Table 3 Comparison study of non-dimensional frequency parameters $\varpi = \left(\frac{\omega a^2}{h}\right)\sqrt{\frac{\rho}{E2}}$ of simply supported square plate.

Method	**a/h**					
	5	**10**	**20**	**25**	**50**	**100**
HSDT 1	10.449	15.022	17.622	18.042	18.660	18.828
HSDT2	10.470	15.048	17.634	18.051	18.663	18.829
HSDT3	10.459	15.036	17.629	18.048	18.662	18.828
Matsunaga, (M1) [12]	10.688	15.072	17.637	18.056	18.670	18.835
Wu et.al,(W1) [13]	10.682	15.069	17.636	18.055	18.670	18.835
Zhen W et.al, (Z1) [14]	10.729	15.166	17.804	18.240	18.902	19.157
Cho KN et.Al, (C1) [15]	10.673	15.066	17.535	18.054	18.670	18.835

A comparison study of non-dimensional frequency parameter with various span to thickness ratios (a/h) is presented in Table 3. It can be seen that the result of the non-dimensional natural frequency with various span to thickness ratio (a/h) are compared with Matsunaga (M1)[12], Wu et.al (W1)[13] , Zhen W et.al (Z1)[14],Cho et. al(C1)[15]. The present results of different theories HSDT 1, HSDT 2, HSDT 3 have good agreement with other published results.

Table 4 A non-dimensional frequency parameter $\varpi = \left(\frac{\omega a^2}{h}\right)\sqrt{\frac{\rho}{E2}}$ of a [0/90/90/0] SSSS laminated square plate (a/h = 10).

Theory	**E1/ E2**			
	10	**20**	**30**	**40**
HSDT 1	9.813367	12.16927	13.79573	15.02168
HSDT2	9.818596	12.18196	13.81549	15.04751
HSDT3	9.816299	12.17657	13.8071	15.03642

In Table 4, the first normalized frequency obtained from the present different theories HSDT1, HSDT2 and HSDT3 is listed with respect to a/h = 10 and various modulus ratios $E1/E2$ = 10, 20, 30, 40.

4. CONCLUSION

Using different HSDT, free vibration response of laminated plate is determined. The present WU-C2 RBF based meshfree method is in good agreement with the published results. For different theories, in case of thin plates the effect on frequency parameter is negligible, while there is considerable effect in case of thick plates. The effect of the span to thickness ratio decreases for a/h ≥ 40.

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