

Heat and Mass Transfer Effects on MHD Oscillatory flow of a Couple Stress fluid in an Asymmetric Tapered channel

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Abstract. In this paper it is proposed to discuss the effects of heat and mass transfer effects on oscillatory flow of an incompressible electrically conducting viscous fluid under the influence of couple stress force in an asymmetric channel filled with porous medium. The fluid is assumed to be electrically conducting in the presence of externally applied uniform magnetic field. The governing equations of flow consisting of momentum, energy equation and concentration have been formulated under Boussinesq approximation and non-dimensionalized with suitable boundary conditions. The closed form solutions for velocity temperature and concentration are obtained. The effect of various parameters on the fluid velocity temperature and concentration are analysed using graphs and tables.

Keywords: Oscillatory flow, couple-stress, Asymmetric Channel, Heat, Mass Transfer.

1. Introduction

Heat transfer in magnetohydrodynamic (MHD) flows emerge in various innovative applications. The MHD flow in the planar channels prompts a start-up procedure suggesting in this manner a viscous layer at the limit is all of a sudden set into movement and ends up imperative in the utilization of different branches of geophysics, astronomy and liquid building. A field in which MHD will assume a basic part is atomic combination, where it is engaged with no less than two distinct issues: the control and elements of plasma, and the conduct of the fluid metal amalgams utilized in a portion of the right now considered outlines of tritium reproducing covers. A hypothetical investigation of the oscillatory flow of a couple stress impacts, in a rotating, channel, has been directed by [1]. [2] discussed the consistent hydromagnetic flow of a couple stress liquid affected by a uniform magnetic field. A hypothetical investigation of the heat



and mass transfer impacts on an unsteady flow of a couple-stress liquid in a horizontal wavy permeable space with travelling thermal waves employed by [3]. [4] studied effect of chemical reaction of a couple stress fluid on an inclined asymmetric channel. The oscillatory flow in the optically thin thermal radiation limit has been studied by [5]. The peristaltic motion of a couple stress fluid in a porous medium is studied by [6]. [7] studied the effects of chemical reaction in oscillatory flow and mass transfer of in an asymmetric wavy channel. Sunspots are caused by the solar magnetic fields, the sun power is also governed by MHD. The effect of MHD oscillatory flow through porous medium with heat source has been investigated by [8]. [9] studied the effects heat and mass transfer in the oscillatory flow of blood. The oscillatory flow have extraordinary pertinence with applications in oil-penetrating, fabricating, preparing of nourishments, oil investigation, and polymer enterprises. The oscillatory channel flow in viscous fluid is extended to a non-Newtonian Jeffrey fluid model discussed by [10]. [11] analysed the oscillatory flow in an asymmetric channel under the effects of chemical reaction and heat transfer. Couple pressure fluid theory, created by [12], is one among the polar fluid theory which considers couple stresses notwithstanding the traditional Cauchy stress. The cardiovascular framework is delicate to changes in the earth, and flow qualities of blood are adjusted to fulfill changing requests of the life form. For numerous reasons, uses of MHD in physiological stream issues are of developing interest. Oscillatory flow of a fluid and heat transfer with porous under the magnetic field is discussed by [13]. The problem of an oscillatory MHD convective flow in a vertical porous channel is discussed by [14]. Tapered channel may fill in as a model for the intrauterine fluid movement in a sagittal cross-segment of the uterus under tumour treatment. [15] discussed the effects of magnetic field in the peristaltic flow in the tapered-asymmetric channel. [16-18] discussed effects of chemical reaction and thermal radiation on MHD oscillatory flow in a porous medium.

2. Formulation of the problem

Considering the viscous incompressible flow, of electrically conducting couple stress fluid in an asymmetric tapered wavy channel .

The geometry of the wall is given as

$$\begin{aligned} H_1 &= -d - m'X - a_1 \sin\left[\frac{2\pi}{\lambda}(X) + \phi\right] \\ H_2 &= d + m' + a_2 \sin\left[\frac{2\pi}{\lambda}(X)\right] \end{aligned} \quad (1)$$

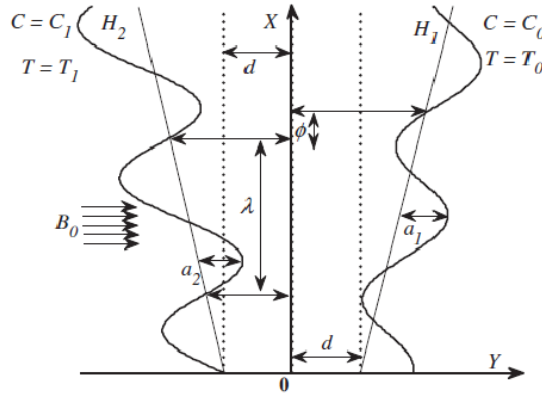


Figure 1. Geometry of the asymmetric tapered channel

in which a_1 and a_2 are the amplitudes of the left and right of the walls and $m' (\ll 1)$ is the non-uniform parameter of the channel, ϕ varies in the range $0 \leq \phi \leq \pi$, when $\phi = 0$ corresponds to the symmetric channel.

a_1, a_2, d and ϕ must satisfy the condition of the channel

$$a_1^2 + a_2^2 + 2a_1a_2\cos(\phi) \leq (2d)^2$$

The governing equations are given by

Equation of momentum

$$\frac{\partial u}{\partial t} = \frac{-1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\eta^*}{\rho} \frac{\partial^4 u}{\partial y^4} - \frac{\sigma B_0^2 u}{\rho} + g\beta_T(T - T_0) + g\beta_C(C - C_0) \quad (2)$$

Energy equation

$$\frac{\partial T}{\partial t} = \left(\frac{k}{\rho c_p} \right) \frac{\partial^2 T}{\partial y^2} + \frac{1}{\rho c_p} 4\alpha^2 (T - T_0) \quad (3)$$

Concentration equation

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial y^2} \quad (4)$$

The boundary conditions are

$$y = H_1 : u = 0, T = T_1, C = C_1 \quad (5)$$

$$y = H_2 : u = 0, T = T_2, C = C_2 \quad (6)$$

The radiative heat flux is given as

$$\frac{\partial q}{\partial y} = 4\alpha^2 (T_2 - T) \quad (7)$$

where $\alpha^2 = \int_0^\infty K_{\lambda w} \frac{\partial e_{b\lambda}}{\partial T} d\lambda$, $K_{\lambda w}$ is the absorption coefficient, $e_{b\lambda}$ is a plank's function.

We introduce the following non-dimensional quantities:

$$x' = \frac{X}{\lambda}, y' = \frac{Y}{d}, t' = \frac{\nu t}{d^2}, u' = \frac{ud}{\nu}, h_1 = \frac{H_1}{d}, h_2 = \frac{H_2}{d}, M^2 = \left[\frac{\sigma}{\rho\nu} \right],$$

$$Gr = \frac{g\beta_T d^3 (T - T_0)}{\nu^2}, Gc = \frac{g\beta_C d^3 (C - C_0)}{\nu^2}, Sc = \frac{D}{\nu}, \theta = \frac{(T - T_0)}{(T_1 - T_0)}$$

$$\phi = \frac{(C - C_0)}{(C_1 - C_0)} \quad (8)$$

The channel wall equations in non-dimensional form becomes

$$h_1 = -1 - mx' - b \sin(2\pi x' + \phi)$$

$$h_2 = 1 + mx' + a \sin(2\pi x')$$

The governing equations in dimensionless form together with appropriate boundary conditions as follows:

$$Re \frac{\partial u'}{\partial t'} = \frac{\partial^2 u'}{\partial y'^2} - \eta' \frac{\partial^4 u'}{\partial y'^4} - \frac{\partial p'}{\partial x'} - M^2 u' + Gr\theta + Gc\phi \quad (9)$$

$$Pe \frac{\partial \theta'}{\partial t'} = \frac{\partial^2 \theta'}{\partial y'^2} + N^2 \theta \quad (10)$$

$$\frac{\partial \phi'}{\partial t'} = Sc \frac{\partial \phi'^2}{\partial y'^2} \quad (11)$$

The corresponding non-dimensional quantities are:

$$y' = h_1 : u' = 0, \theta = 1, \phi = 1 \quad (12)$$

$$y' = h_2 : u' = 0, \theta = 0, \phi = 0 \quad (13)$$

Stress-free conditions

$$\frac{\partial^2 u'}{\partial y'^2} = 0 \text{ for } y = h_1, h_2$$

3. Solution of the problem

The system of equations (9)-(11) can be reduced to dimensionless form by assuming the following:

$$\frac{\partial p}{\partial x} = \lambda e^{i\omega t} \quad (14)$$

$$u(y, t) = u_0(y) e^{i\omega t}$$

$$\theta(y, t) = \theta_0(y) e^{i\omega t}$$

$$\phi(y, t) = \phi_0(y) e^{i\omega t} \quad (15)$$

Now substituting equations (15) and (16) in equations (9)-(11) we obtain the following equations:

$$\eta' \frac{d^4 u_0}{dy'^4} + \frac{d^2 u_0}{dy'^2} + (Rei\omega + M^2)u_0 = Gr\theta_0 + Gc\phi_0 - \lambda \quad (16)$$

$$\frac{d^2 \theta_0}{dy'^4} + (N^2 - i\omega Pe)\theta_0 = 0 \quad (17)$$

$$\frac{d^2 \phi_0}{dy'^2} - \left(\frac{i\omega}{Sc}\right)\phi_0 = 0 \quad (18)$$

subject to the boundary conditions

$$\begin{aligned} y = h_1 : u_0 &= 0, \phi_0 = 1, \theta_0 = 1 \\ y = h_2 : u_0 &= 0, \phi_0 = 0, \theta_0 = 0 \end{aligned} \quad (19)$$

Stress free conditions

$$y = h_1, h_2 : u_0'' = 0 \quad (20)$$

substituting the above boundary conditions(20)-(21) in the equations (17)-(19) we obtain the following:

Velocity distribution

$$\begin{aligned} u(y, t) = & \left[A_1 e^{\alpha y} + B_1 e^{-\alpha y} + C_1 e^{\beta y} + D_1 e^{-\beta y} + \left(\frac{Gr\theta_0}{\eta' P^4 - P^2 + Q^2} \right) \right. \\ & \left. + \left(\frac{Gc\phi_0}{\eta' L^4 - L^2 + Q^2} \right) - \frac{\lambda}{Q^2} \right] e^{i\omega t} \end{aligned} \quad (21)$$

Temperature distribution

$$\theta = \left[\frac{\sin(P(y - h_2))}{\sin(P(h_1 - h_2))} \right] e^{i\omega t} \quad (22)$$

Concentration distribution

$$\phi = \left[\frac{\sinh(L(y - h_2))}{\sinh(L(h_1 - h_2))} \right] e^{i\omega t} \quad (23)$$

$$(24)$$

The rate of heat transfer

$$Nu = \left(\frac{\partial \theta}{\partial y} \right)_{y=h_1, h_2} = - \left[\frac{\cos P(y - h_2)}{\sin P(h_1 - h_2)} \right] e^{i\omega t}$$

$$Nu = - \left[\frac{\cos P(h_1 - h_2)}{\sin P(h_1 - h_2)} \right] e^{i\omega t} \quad (25)$$

$$Nu = - \left[\frac{1}{\sin P(h_1 - h_2)} \right] e^{i\omega t} \quad (26)$$

The rate of mass transfer

$$Sh = - \left[\frac{\partial \phi}{\partial y} \right]_{y=h_1, h_2} = - \left[\frac{L \cosh[L(y - h_2)]}{\sinh[L(h_1 - h_2)]} \right] e^{i\omega t}$$

$$Sh = - \left[\frac{\cosh(L(h_1 - h_2))}{\sinh(L(h_1 - h_2))} \right] e^{i\omega t} \quad (27)$$

$$Sh = - \left[\frac{1}{\sinh(L(h_1 - h_2))} \right] e^{i\omega t} \quad (28)$$

4. Results and discussions

The graphical distribution for the magnetic parameter M , Grashoff number Gr , radiation parameter N , frequency of oscillation ω , Peclet number Pe , Reynolds number Re , Schmidt number Sc on velocity u , temperature θ , and concentration ϕ .

Fig.2 depicts the response of velocity to the increasing value of Gr . It shows that there is a backflow instigated in the left wall and significant flow in the right wall.

Fig.3 demonstrates that increase in Sc instigates a perceivable decrease in the velocity over the walls near the left wall velocity stays positive further from the left wall as we approach the wall focus and from that point the right wall, there is impressive hindrance on the stream prompting to significant backflow.

In fig.4 velocity profile for increase in Peclet number has been discussed. Increase in Peclet number accelerate the velocity in the left wall but decelerates the velocity in the right wall.

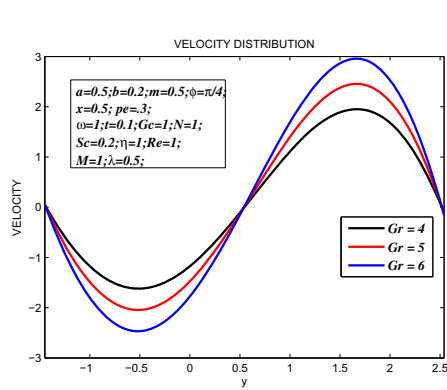
In fig.5 we can see that there is a backflow of velocity induced in the left wall and significant flow in the right wall.

Fig.6 shows the effect of radiation parameter N , on temperature profile. When there is an increase y , there is an increase of the effect of N on temperature.

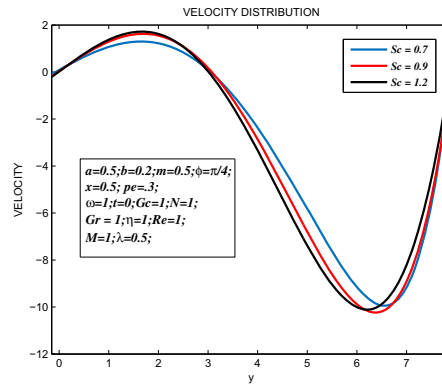
Fig.7 depicts the Peclet number effect on temperature profile. When the Peclet number increases the slope decreases. Fig.8 discusses the frequency of oscillation in temperature profile. It shows that increase in frequency of oscillation decreases the temperature. Similarly increase in Sc increases the temperature profile.

When there is an increase in frequency of oscillation there is a decrease in concentration distribution which is shown in fig.9 and in fig.10 concentration increases for increasing Sc .

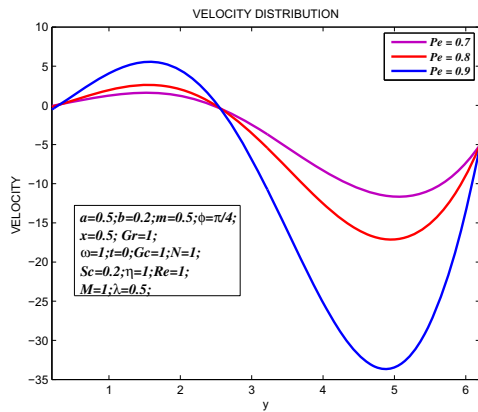
In fig.11 and fig.12 for various values of Pe , there is a noticeable decrease of heat transfer on the wall $y=h_1$ and $y=h_2$. Fig.13 and Fig.14 shows that increase in N increases the heat transfer at the walls $y=h_1$ and $y=h_2$. In fig.15 and fig.16 increase in Sc increases the mass transfer at the walls $y=h_1$ and $y=h_2$.



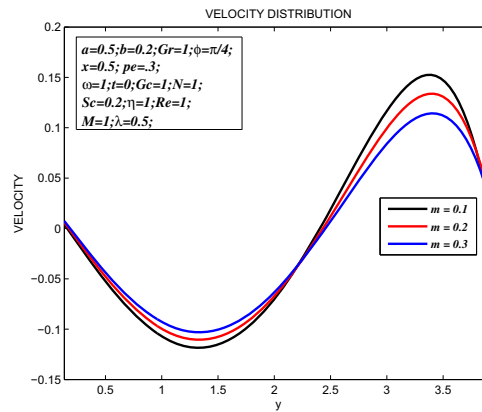
(a) FIGURE:2 Velocity profile for various values of Gr



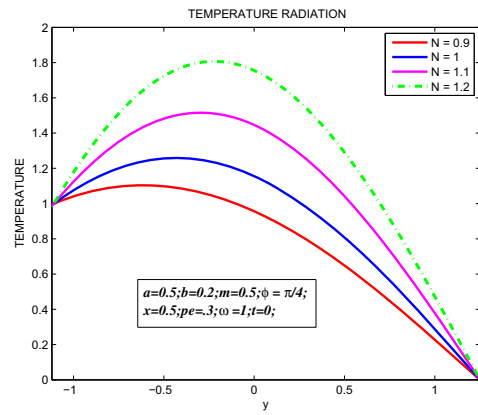
(b) FIGURE:3 Velocity for various values of Sc



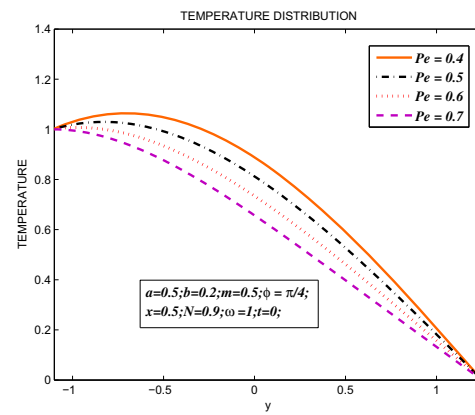
(c) FIGURE:4 Velocity profile for various values of Pe



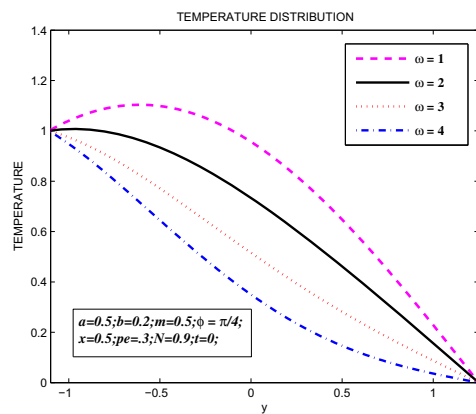
(d) FIGURE:5 Velocity profile for various values of m



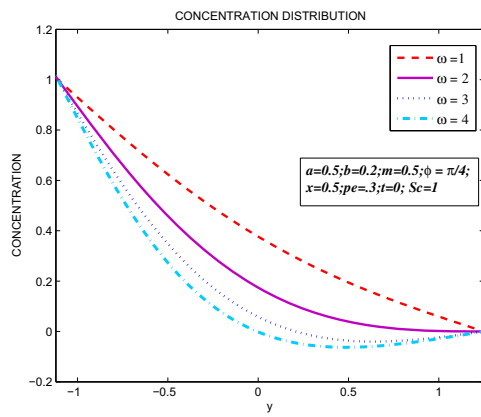
(e) FIGURE:6 Temperature profile for various values of N



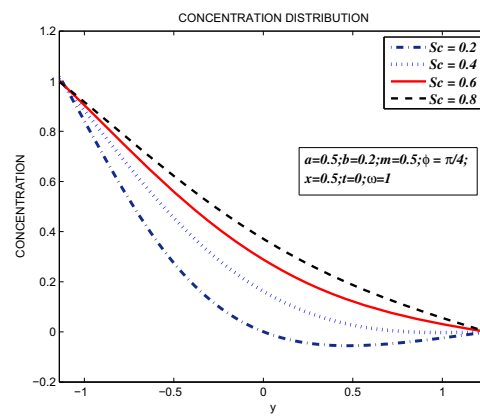
(f) FIGURE:7 Temperature profile for various values of Pe



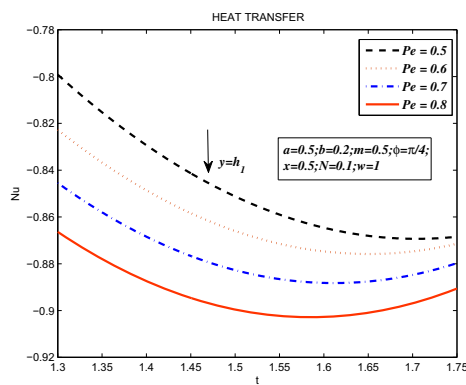
(g) FIGURE:8 Temperature profile for various values of ω



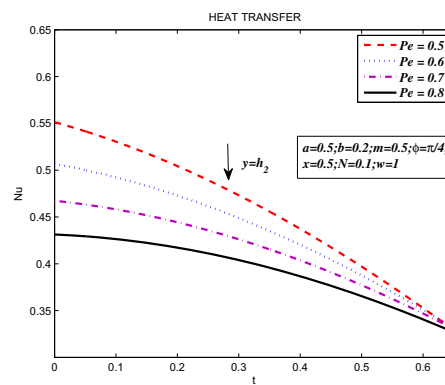
(h) FIGURE:9 Concentration profile for various values of ω



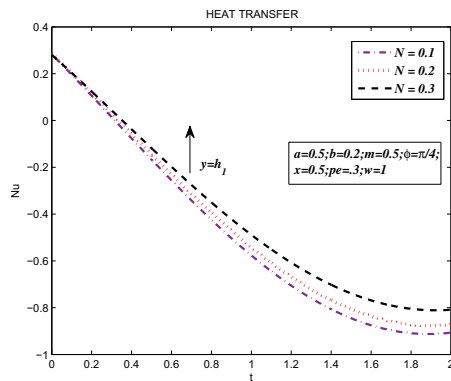
(i) FIGURE:10 Concentration profile for various values of Sc



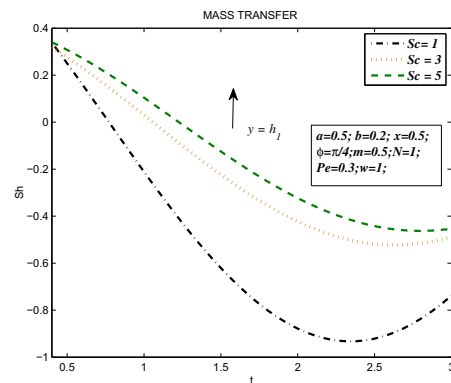
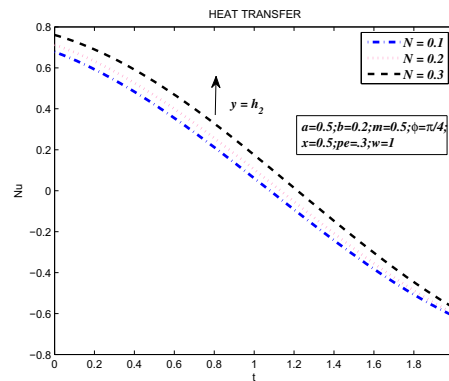
(j) FIGURE:11 Heat transfer at the wall $y=h_1$ for various values of Pe



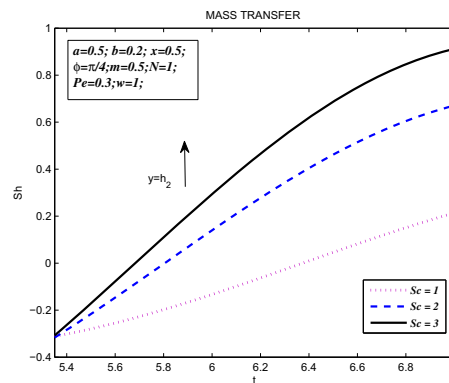
(k) FIGURE:12 Heat transfer at the wall $y=h_2$ for various values of Pe



(l) FIGURE:13 Heat transfer at the wall $y=h_1$ (m) FIGURE:14 Heat transfer at the wall $y=h_2$ for various values of N



(n) FIGURE:15 Mass transfer at the wall $y=h_1$ for various values of Sc (o) FIGURE:16 Mass transfer at the wall $y=h_2$ for various values of Sc



5. Conclusion

A theoretical analysis on effects of heat and mass transfer in MHD oscillatory flow of a couple stress fluid in an asymmetric tapered channel has been discussed in this paper.

- Velocity profile increases for increasing value of non-uniform parameter m , Peclet number Pe , Schmidt number Sc , and Grashoff number Gr on the upper wall $y = h_1$.
- Velocity decreases for increasing value of m , Pe , Sc , and Gr on the lower wall $y = h_2$.
- Temperature decreases for increasing value of Pe , and increases for radiation N .
- Concentration decreases for increasing value of ϕ and increases for increasing Sc .
- At both the walls $y = h_1$ and $y = h_2$ heat transfer increases while varying t and increasing value of N and decreases at both the walls for increasing Pe .

6. Nomenclature

a_1, a_2 - Amplitudes of right and left wall
 B_0 - Uniform magnetic field
 g - Acceleration due to gravity
 Gr - Grashof number
 Gc - Modified Grashof number
 M - Hartmann number
 m, m' - Non-uniform parameters
 q_r - The radiative heat flux
 Re - Reynolds number
 D - Mass diffusion coefficient
 N - Thermal radiation
 C_0, C_1 - Concentration of the walls
 T_0, T_1 - Temperature of the walls
 u - Dimensionless velocity component
 η - Couple stress parameter
 k - Porous permeability
 t', t - Dimensional and dimensionless time
 Pe - Peclet number

7. Greek symbol

θ - Temperature
 κ - Permeable parameter
 λ - Wavelength
 ϕ - Phase difference
 β_C - Coefficient of mass expansion
 ν - Kinematic viscosity
 ω - Frequency of oscillation

8. Appendix

$$\begin{aligned}
 \alpha^2 &= \frac{-1 + \sqrt{1 - 4\eta'Q^2}}{2\eta'}, \quad \beta^2 = \frac{-1 - \sqrt{1 - 4\eta'Q^2}}{2\eta'}, \quad Q^2 = (Rei\omega + M^2), \\
 P^2 &= N^2 - i\omega Pe, \quad L^2 = \frac{i\omega}{Sc}, \\
 I &= \frac{Gr}{\eta'P^4 - P^2 + Q^2}, \quad J = \frac{Gc}{\eta'L^4 - L^2 + Q^2}, \\
 A_1 &= \frac{-e^{\alpha h_2} \beta^2 J Q^2 - I e^{-\alpha h_2} \beta^2 J Q^2 + e^{-\alpha h_2} \beta^2 \lambda + e^{-\alpha h_2} L^2 J Q^2 - I e^{-\alpha h_2} P^2 Q^2 - \beta^2 \lambda e^{-\alpha h_2} P^2 Q^2}{2(\alpha^2 - \beta^2) \sinh[\beta(h_1 - h_2)]}
 \end{aligned}$$

$$B_1 = \frac{1}{2\sinh[\alpha(h_1 - h_2)]} \left[\frac{\lambda}{Q^2} (e^{\alpha h_2 - e^{\alpha} h_1}) + e^{\alpha h_2} (I + J) - C_1 e^{\alpha h_1 + \beta h_2} - e^{\alpha h_2 + \beta h_1} - D_1 e^{\alpha h_1 - \beta h_2} - e^{\alpha h_2 - \beta h_1} \right]$$

$$C_1 = \frac{-\lambda \alpha^2}{Q^2(\alpha^2 - \beta^2)} e^{-\beta h_2} - D_1 (e^{-2\beta h_2})$$

$$D_1 = \frac{\left((\alpha^2 + P^2)I + (\alpha^2 - L^2)J + \frac{\lambda \alpha^2}{Q^2} \right) e^{\beta h_2} - \frac{\lambda \alpha^2}{Q^2} e^{\beta h_1}}{2(\alpha^2 - \beta^2) \sinh[\beta(h_1 - h_2)]}$$

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