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Shear locking reduction in family of plane quadrilateral elements

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Abstract. There are certain cases where FEM gives erroneous results due to the problem of locking. Locking phenomenon has been classified into three basic categories. shear, volumetric and membrane locking. This project discusses the causes, effects and remedies for shear locking in conventional FEM using lower order elements for bending dominated problems. New quadrature rule is considered in this project which will overcome the hour glassing effect that arises in Gauss one-point integration while solving the bending dominated problem.

1. Introduction

In finite element analysis, shear locking gives erroneous results for the bending dominated problems or in some numerical analysis cases [1]. The most common approach to solve the shear locking problem is by using the higher order elements. However, utilizing these higher order elements will consume more computational time and resources. Although 8 nodded bilinear solid finite elements with 24 degrees of freedom are commonly used for analysis, higher order elements are time integration methods.

This time consumption can be reduced or optimized by making use of lower order elements. But usage of these lower order elements causes shear locking. This indicates that when fully integrated lower order elements are used, shear locking must be taken into account. Locking phenomenon can be reduced or minimized by opting for the one-point integration scheme, but this scheme or method suffers from hour glassing effect or zero energy modes. So, this locking is reduced or minimized by using the method called selective reduced integration which overcomes the hour glassing effect or zero energy modes.

However, locking is depends on the aspect ratio, i.e. if the aspect ratio is more the produced element will pretend to be stiffer [2]. To minimize this problem of locking three methods are discussed in this paper. 1. Refinement of mesh 2.Selective reduced integration 3.Higher order elements. Refinement of mesh is a trial and error method that cannot predict the exact meshing at which answers will be obtained. Opting for Higher order elements proves to be time-consuming. Hence, lower order elements with selective reduced integration are used to overcome the problem of shear locking in less computational time.

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1.1. Shear locking

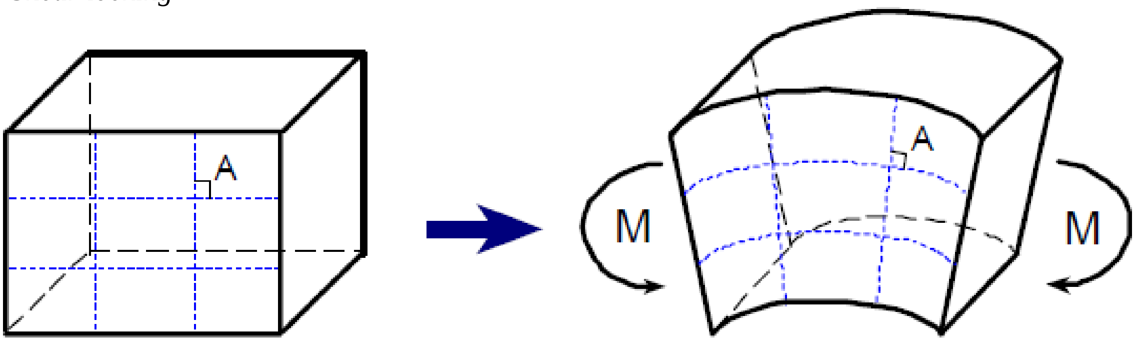

Figure 1. Behavior of an element under the moment in ideal condition.

In an ideal condition, (pure bending) applied moment causes the element to behave as shown in Figure1, i.e. when moment is applied to the element the top and bottom surfaces follow the curvature and right and left surfaces are constant, as shown in the above figure 1.[3]

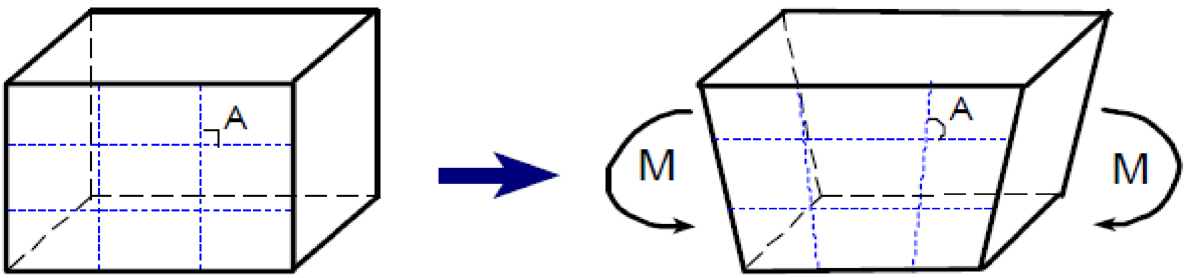

Figure 2. Behavior of the fully integrated first order element under moment.

For the case of pure bending, if fully integrated bi-linear four nodded quadrilateral element is used, for the applied moment the top and bottom surfaces behave linearly due to the bi-linearity nature of the quad 4 elements. This does not exist in normal condition. Because of this behavior the FEM codes are arrived with erroneous answers.
It is important to consider this problem while using FEM for analyzing bending dominated problems.

1.2. Shear Locking Methodology

In order to examine this locking problem fully integrated four node quadrilateral element is considered.

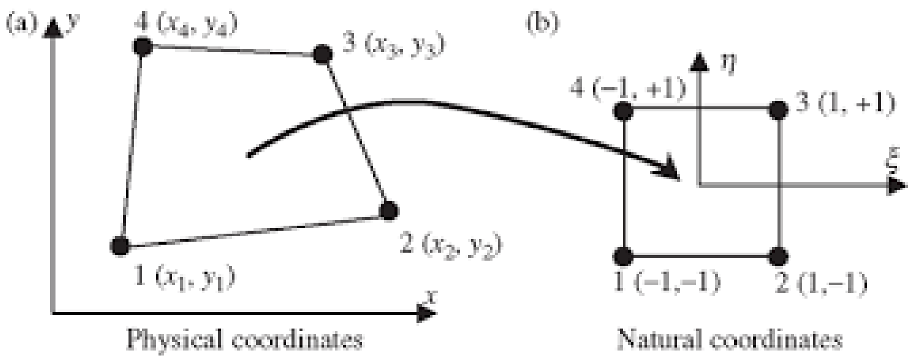

Figure 3. Four nodded quadrilateral element.

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Shape functions of Figure: 3are used as follows:

$$\begin{aligned} N_1 &= \frac{1}{4}(1-\xi)(1-\eta) \\ N_2 &= \frac{1}{4}(1+\xi)(1-\eta) \\ N_3 &= \frac{1}{4}(1+\xi)(1+\eta) \\ N_4 &= \frac{1}{4}(1-\xi)(1+\eta) \end{aligned} \tag{1}$$

These shape functions are calculated in natural coordinate system at different positions.

$$\begin{aligned} (\xi, \eta) &= (-1, -1); \\ (\xi, \eta) &= (1, -1); \\ (\xi, \eta) &= (1, 1); \\ (\xi, \eta) &= (-1, 1); \end{aligned} \tag{2}$$

Displacement field $$U^e(\xi) = \begin{pmatrix} u \\ v \end{pmatrix} \tag{3}$$

Where u, v are displacements along x and y directions in global coordinate system and displacements along ξ and η in local coordinate system respectively.

Strain displacement field

$$\varepsilon = B(\xi)U^e \tag{4}$$

B is strain displacement matrix

$$B = \begin{Bmatrix} \in_x \\ \in_y \\ \in_{xy} \end{Bmatrix} \tag{5}$$

This matrix consists of strains along x-axis, along y-axis and along xy-direction. For pure bending problems, the shear component in B matrix should be zero. Here strain along xy-direction is not zero which causes the element to behave stiffer than desired and leads to wrong answers with finite element method (FEM) codes.

B=B1*B2*B3

$$B_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \tag{6}$$

$$B_2 = \begin{bmatrix} J_{inv11} & J_{inv12} & 0 & 0 \\ 0 & 0 & J_{inv21} & J_{inv22} \\ J_{inv21} & J_{inv22} & 0 & 0 \\ 0 & 0 & J_{inv11} & J_{inv12} \end{bmatrix} \tag{7}$$

Where B1 is constant matrix and B2 is depend on Jacobian matrix.

Where J = Jacobian matrix.

Where D is material matrix in plain strain condition. This contains the material properties of elements such as Young's modulus E and Poisson's ratio v as shown in equation (8).

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$$D = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1+\nu & \nu & 0 \\ \nu & 1+\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \tag{8}$$

E =Young's modulus and ν =Poisson's ratio.

Where,

$$B_3 = \begin{bmatrix} \frac{\partial N_1}{\partial \xi} & 0 & \frac{\partial N_2}{\partial \xi} & 0 & \frac{\partial N_3}{\partial \xi} & 0 & \frac{\partial N_4}{\partial \xi} & 0 \\ \frac{\partial N_1}{\partial \eta} & 0 & \frac{\partial N_2}{\partial \eta} & 0 & \frac{\partial N_3}{\partial \eta} & 0 & \frac{\partial N_4}{\partial \eta} & 0 \\ 0 & \frac{\partial N_1}{\partial \xi} & 0 & \frac{\partial N_2}{\partial \xi} & 0 & \frac{\partial N_3}{\partial \xi} & 0 & \frac{\partial N_4}{\partial \xi} \\ 0 & \frac{\partial N_1}{\partial \eta} & 0 & \frac{\partial N_2}{\partial \eta} & 0 & \frac{\partial N_3}{\partial \eta} & 0 & \frac{\partial N_4}{\partial \eta} \end{bmatrix} \tag{9}$$

The stiffness matrix is evaluated with strain displacement matrix using Gaussian two point integration.

$$K = \iint B^T D B \tag{10}$$

By using the above Formula (10) elemental stiffness matrix is calculated with two point integration at the respective positions.

$$\begin{aligned} (\xi, \eta) &= \left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right) \\ (\xi, \eta) &= \left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right) \\ (\xi, \eta) &= \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) \\ (\xi, \eta) &= \left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) \end{aligned} \tag{11}$$

The shear components present in the strain displacement matrix causes element to behave stiffer than is desired. To overcome this problem the shear components should be sampled at (0, 0).

$$\overline{B_3} = \begin{bmatrix} \frac{\partial N_1}{\partial \xi} & 0 & \frac{\partial N_2}{\partial \xi} & 0 & \frac{\partial N_3}{\partial \xi} & 0 & \frac{\partial N_4}{\partial \xi} & 0 \\ \frac{\partial N_1}{\partial \eta} & 0 & \frac{\partial N_2}{\partial \eta} & 0 & \frac{\partial N_3}{\partial \eta} & 0 & \frac{\partial N_4}{\partial \eta} & 0 \\ 0 & \frac{\partial N_1(0,0)}{\partial \xi} & 0 & \frac{\partial N_2(0,0)}{\partial \xi} & 0 & \frac{\partial N_3(0,0)}{\partial \xi} & 0 & \frac{\partial N_4(0,0)}{\partial \xi} \\ 0 & \frac{\partial N_1(0,0)}{\partial \eta} & 0 & \frac{\partial N_2(0,0)}{\partial \eta} & 0 & \frac{\partial N_3(0,0)}{\partial \eta} & 0 & \frac{\partial N_4(0,0)}{\partial \eta} \end{bmatrix} \tag{12}$$

The elemental stiffness matrix is calculated with modified strain displacement matrix.

$$\mathrm{K} = \iint \overline{\mathrm{B}^{\mathrm{T}}} \mathrm{D} \overline{\mathrm{B}} \tag{13}$$

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Now with the modified B matrix, elemental stiffness matrix for each element is evaluated and assembled.

2. Numerical Example

A tip loaded cantilever beam has been modeled with elements based on B and $\overline{\mathrm{B}}$ .The beam has length L=20, height H=1, and thickness t=1. Poisson's ratio v=0(to be able to compare with Euler-Bernoulli beam theory) and Young's modulus E=10^4. According to the beam theory, the tip deflection is caused by a load, F=1. [4]

$$\delta = \frac{Fl^3}{3EI} = 3.2 \tag{14}$$

The given beam is discredited and tested with four different meshes, as shown in Fig: 4.

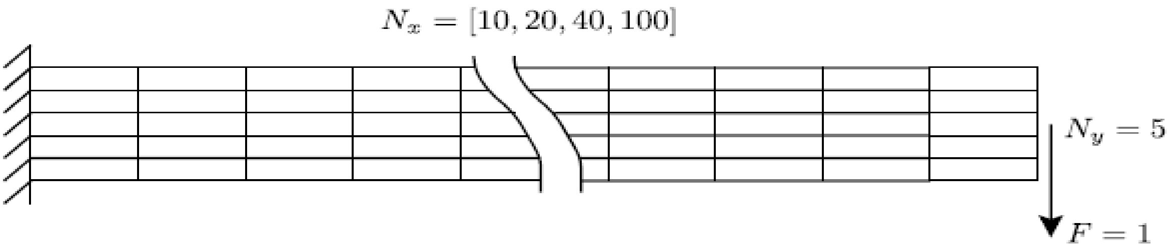

Figure 4. meshed element.

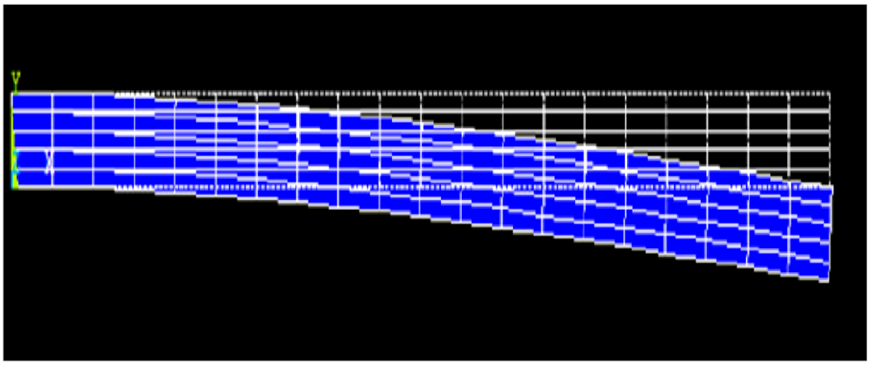

Figure 5. Deformation of Meshed element.

Four different meshes have been used to find the results. The numbers of elements are changed along x–direction and kept constant along y-direction, denoted by N_x and N_y respectively. The tip deflection is obtained both before and after modifying the strain displacement matrix and the ratio between the calculated deflections with and without modifying the B matrix to the theoretical tip deflection has been evaluated and tabulated as shown in the Table 1.

Figure 5 represent the behavior of the cantilever beam which is fixed at the left side end and force is applied at the right side tip point along y direction. The Figure 5 is only for the one particular mesh, so for different meshes different displacements and shapes can be obtained.

3. Results

Table 1. Tip displacement convergence

MESHING $N_x \times N_y$	**Normal(B) 4-noded**	**Modified ($\overline{B}$) 4-noded**	**Normal (B) 8-noded**
10 X 5	0.33603	1.04066	1.00166
20 X 5	0.67666	1.04272	1.00166
40 X 5	0.90634	1.04334	1.00166
100 X 5	1.00153	1.04369	1.00166

4. Conclusion

From the results (Table 1), it is clear that refinement of mesh gives the solution for reducing the shear locking in bending dominated problem. However, it is not possible to predict which particular mesh will help to achieve the desired results. Using higher order 8 nodded elements and selective reduced integration converges the answers closer to the theoretical deflection. The methods that are used in this paper help to overcome the problem of shear locking.

5. References

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