

# Static and dynamic stability for floating cranes

**D G Musca (Anghelache)**

”Dunarea de Jos” University of Galati, Engineering and Agronomy Faculty in Braila,  
Research Center for Mechanics of Machines and Technological Equipment,  
Calea Calarasilor 29, 810017, Braila, Romania

E-mail: danchelache@ugal.ro

**Abstract.** Dynamic processes have been considered in this research paperwork, coming up when static balance of the crane is disturbed and behave as some periodic or non-periodic motions on the way to another balance position. Most often, these motions are oscillating non-linear and attenuated and concern the time of transitory mode, period and maximum amplitude of the oscillations. For a crane, the most dangerous disturbance of static balance is caused by sudden loose of the load. The crane subject to analysis has no other connection to environment that by means of non-linear yield seats only on vertical direction. Static stability in idle or under load condition is no longer considered by checking whether the stabilization moment is greater than the overturning moment both in idle and under load conditions like for on-shore crane fixed on rigid seats. Dynamic processes come up when static balance of the crane is disturbed and behave like periodical or non-periodic motions on their way to another balance position. This process has generic name of transitory mode. For a crane, the most dangerous disturbance of static balance is caused by sudden loose of the load.

## 1. Introduction

Floating cranes may be ranged within the following constructive and functional types:

1. Cranes which are mounted on any kind of ships (passenger ships, container ships, ships for fluids shipping, bulk carriers) and serve for handling of loads with negligible weight comparing to ship's displacement. The ships are so dimensioned that their static and dynamic stability is not affected by crane and crane handled loads.
2. Cranes which are mounted on hulls built specially built to form together a floating crane itself and handling big loads comparing to hull's displacement.

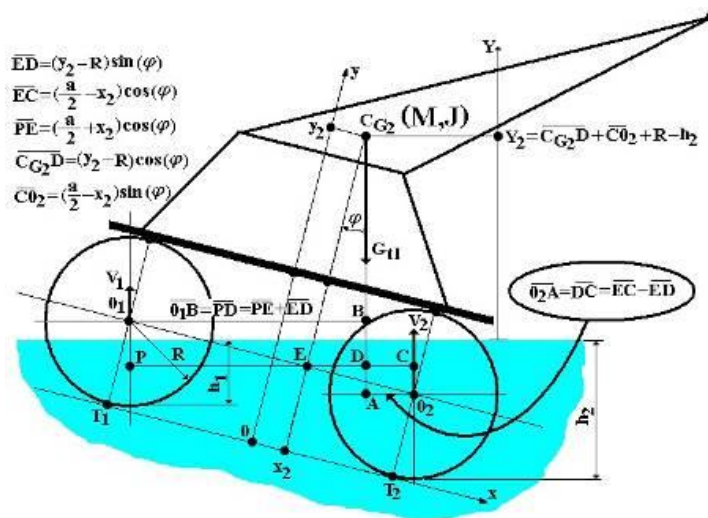
In their turn, such floating cranes are made in two constructive versions:

- Floating crane on a hull of catamaran type consisting of two floaters, ensuring special stability;
- Floating crane on classic hull ensuring fast travelling to the place of action.

The cranes on classic hull are dimensioned, as to stability, like any classic ship, the methodology being specific to shipbuilding and is based on analysis of three essential points from underwater hull geometry, respectively center of gravity, center of floatability and metacenter [9]. As being part of other specialty, static and dynamical stability calculations won't be reviewed deeper here below.

Cranes on catamaran type hull can be considered like on-shore cranes supported by non-linear yielding seats, Archimedes force occurring on floaters depending on vertical motion of the floater against water surface (immersion) producing displacement of a variable water volume. Dynamics study for such a crane belongs, therefor, to the study of a conventional crane and will be done by considering the following hypothesis:

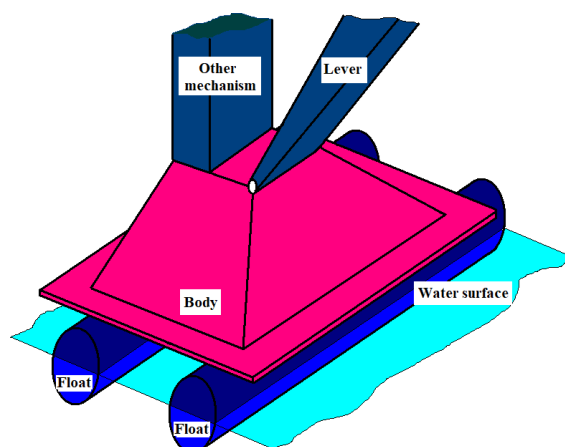




**Figure 1.** Calculation diagram for dynamic stability.

1. Water surface is calm and there are no side forces produced by weather conditions (neither on-shore cranes work during stormy weather).
2. The floaters realise only yield seating points on vertical direction, with no other connections on other directions with crane's environment. The floaters do not lock any degree of freedom of the ship-crane unit, but only produce vertical reactions with yielding feature.
3. The structure of hull-crane assembly is considered rigid. If consideration of a yield structure is wanted, nothing else should be done but overlapping the dynamics of a classic crane, considered elastic deformable, over the dynamics presented in this paperwork.
4. The crane only moves the load vertically. As floaters do not take over any degree of freedom, any horizontal movement of the load (translation or rotation on a circumference around vertical axis), according to conservation laws for barycenter and kinetic moment, will produce crane movements in contrary direction, which should be counterbalanced either by its own propulsion units or by securing it to a rigid shore in which case the on-shore dock cranes are preferable. And, however, if own propulsion units should be used, it can be considered that load horizontal movements can be achieved by crane travelling on water.
5. The floaters are cylindrical, as they achieve the biggest floatability for the same weight compared to other oblong shapes like rectangular, flap type etc.

Figure 2 shows the sketch of a basic detail of the crane, where it can be seen that it consists of a floating body on two parallel floaters and the boom and related mechanisms are crosswise oriented against the floaters.

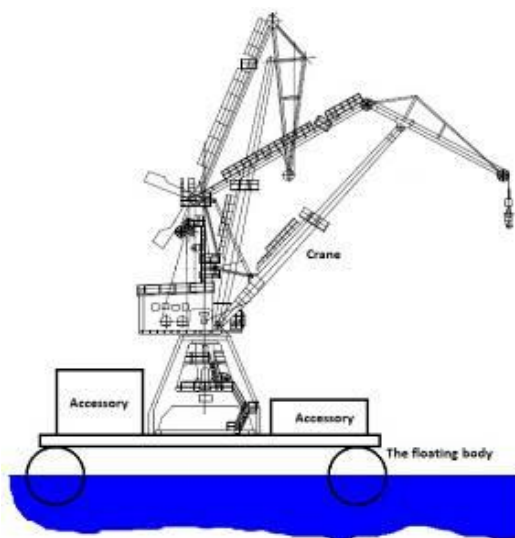


**Figure 2.** Basic constructive shape.

## 2. Static and dynamic stability of the floating crane

Research of a floating crane according to the above basic constructive shape will include two issues:

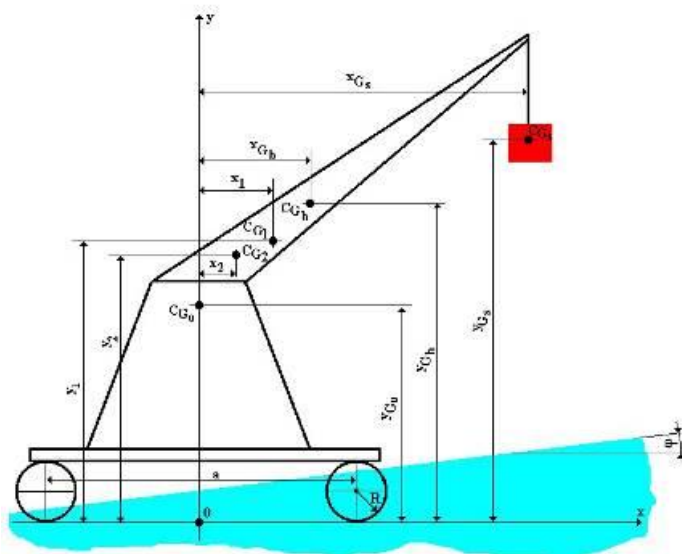
1. Static stability, similar to on-shore cranes, with a new issue in addition: stability will be provided by accepting a crosswise inclination angle (trimming angle in ships terms) due to different loading on the two floaters. Unlikely, for on-shore cranes, no inclination (declivity) of the seating base of the crane is accepted and neither yield seats.
2. Dynamic stability, researching both the risk for overturning and also the oscillation period of the crane (due to yield seats), respectively attenuation time in absence of some dynamic buffers. The ships where oscillation around longitudinal axis (rolling motion) should be stabilized and fast attenuated, are provided with underwater flaps (similar to those from aviation) which, like some oars with large surface, act contrary to rolling motion and reduce oscillations fast, being controlled by an automatic system. For the studied floating crane such a device is considered as missing.



**Figure 3.** A certain model of floating crane.

Figure 3 shows the certain model to be subject of study.

To simplify the physical models, based on which mathematical patterns will be built, the physical model in figure 4 will be worked on.



**Figure 4.** Work model.

Notations in the figure mean:

$C_{G_0}$  = centre of gravity of the floating body including floaters, platform and crane lower body;

$C_{G_b}$  = centre of gravity for the boom;

$C_{G_s}$  = centre of gravity for the load;

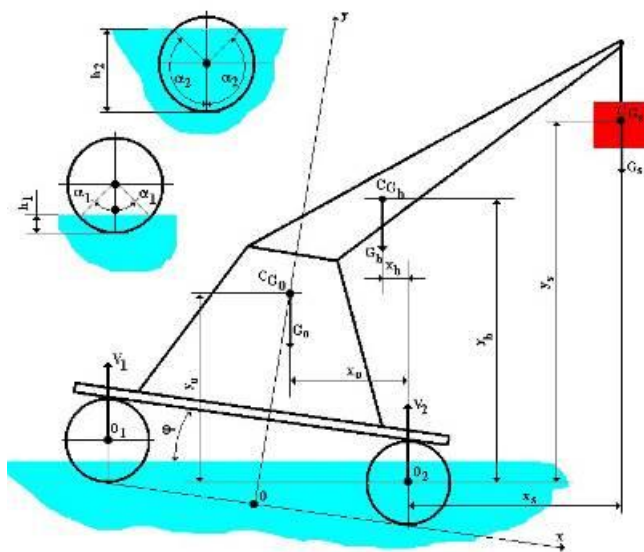
$C_{G_1}$  = total centre of gravity, for the floaters, platform, lower body, boom and load (loaded crane);

$C_{G_2}$  = like  $C_{G_1}$  but without considering the load (idle crane).

The chosen system of reference  $xOy$  is connected to the crane. As the crane is inclined towards the load by angle  $\varphi$ , this is shown by drawing the crane in vertical position and by water surface inclination.

With notations from figure 4 stability calculations as well as a dynamics model in extreme manoeuvring situations will be done.

Figure 5 shows the forces that should be taken into calculation when evaluating static stability under load. The weight force  $G_{t1}$  occurs in the centre of gravity  $C_{G_1}$ . In case of idle crane the representation stays valid, just the weight  $G_{t1}$  will be replaced by weight  $G_{t2}$  which occurs in the centre of gravity  $C_{G_2}$ .



**Figure 5.** Static stability under load.

Herebelow the equations describing static stability under load are written.

Archimedes' force per each floater is expressed by functions:

$$V_{1,2} = f(h_{1,2}) \quad (1)$$

And inclination angle  $\varphi$  is

$$\varphi = \arcsin \frac{h_2 - h_1}{a} \quad (2)$$

Furtheron it can be written:

$$\alpha_{1,2} = \begin{cases} \arccos \frac{R - h_{1,2}}{R} & \text{if } h_{1,2} < R \\ \pi - \arccos \frac{h_{1,2} - R}{R} & \text{if } h_{1,2} \geq R \end{cases} \quad (3)$$

The water volumen displaced by each floater with  $L$  length is

$$\begin{aligned} Volume_{1,2} &= L \left\{ R^2 \cdot \alpha_{1,2} - \frac{1}{2} [2R \sin(\alpha_{1,2}) \cdot R \cos(\alpha_{1,2})] \right\} = \\ &= L \cdot R^2 \left[ \alpha_{1,2} - \frac{\sin(2\alpha_{1,2})}{2} \right] \end{aligned} \quad (4)$$

And Archimedes' reaction force results

$$V_{1,2} = \rho_{apa} \cdot g \cdot Volum_{1,2} \quad (5)$$

where  $\rho_{apa}$ =water density,  $g$ =gravitational acceleration.

By replacing within (5) the volume expression given by (4) and within (4) the expressions for  $\alpha_{1,2}$  given by (3), the expressions for calculation of Archimedes' type reaction forces result depending only on immersion depths  $h_{1,2}$ , as transcendent functions, the transcendence being easily visible in (4). Similarly, easily visible is the fact that the reaction forces are of the kind of elastic forces (depending on a travel) but non-linear.

Static balance equations are written as:

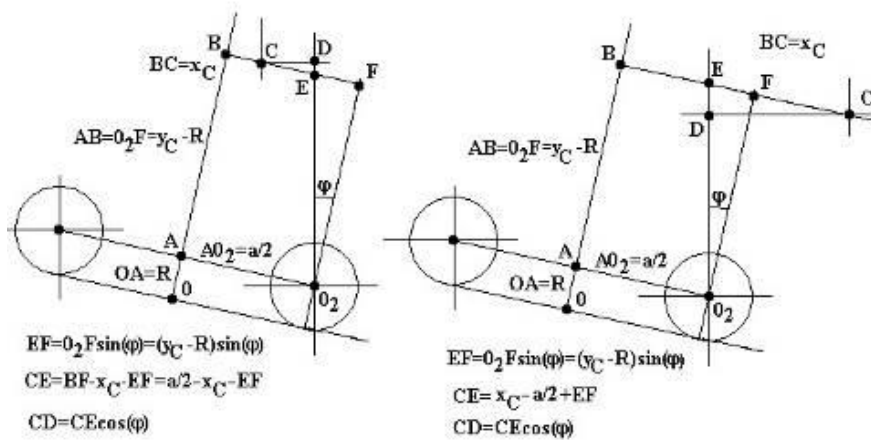
Projection on vertical:  $V_1 + V_2 = G_{r1}$

Moments against  $O_2$ :  $V_1 \cdot a \cdot \cos(\varphi) - G_o \cdot x_o - G_b \cdot x_b + G_s \cdot x_s = 0$  (6)

In (6) the force arms are found by using figure 5. Point  $C$  in figure 5 may be, one by one, the points  $C_{G_0}$ ,  $C_{G_b}$ ,  $C_{G_s}$  and distance  $CD$  is the force arm.

$$\begin{aligned} x_o &= \left[ \frac{a}{2} - (y_{Go} - R) \sin(\varphi) \right] \cos(\varphi) \\ x_b &= \left[ \frac{a}{2} - x_{Gb} - (y_{Gb} - R) \sin(\varphi) \right] \cos(\varphi) \\ x_s &= \left( x_{Gs} - \frac{a}{2} + (y_s - R) \sin(\varphi) \right) \cos(\varphi) \end{aligned} \quad (7)$$

The second equation in (6) simplifying by  $\cos(\varphi)$  can be done. By making all replacements given by expressions (2)---(5) it results that all this problem has two unknown elements,  $h_1$  and  $h_2$ , which form a system of two transcendent equations, respectively the two equations given by (6), meaning numerical settlement is a must.



**Figure 6.** Sketch for finding the force arms.

Static stability in idle or under load condition is no longer considered by checking whether the stabilization moment is greater than the overturning moment both in idle and under load conditions like for on-shore crane fixed on rigid seats, but by the value of angle  $\varphi$ , as per stability criteria for floating bodies set by registers of shipping.

### 3. Static stability of floating cranes with floaters

Static stability under load and its dependance on the crane construction and handled load is studied in this chapter, by numerical solving the system of equations described within previous chapter. As stability evaluation can only be numerically performed, certain examples with known dimensions and mass should be selected. As a first practical example, I consider a light crane with two floaters having diameter  $D=2R=2\text{ m}$  each and length  $L=15\text{ m}$ . This construction leads to a maximum theoretical **mass displacement** (floaters completely submerged) of:

$$Depl_{\max \text{ teor}} = 2 \cdot \pi \frac{D^2}{4} L \cdot \rho_{\text{apa}} = 94,2 \text{ (tonsor } m^3 \text{ water)}$$

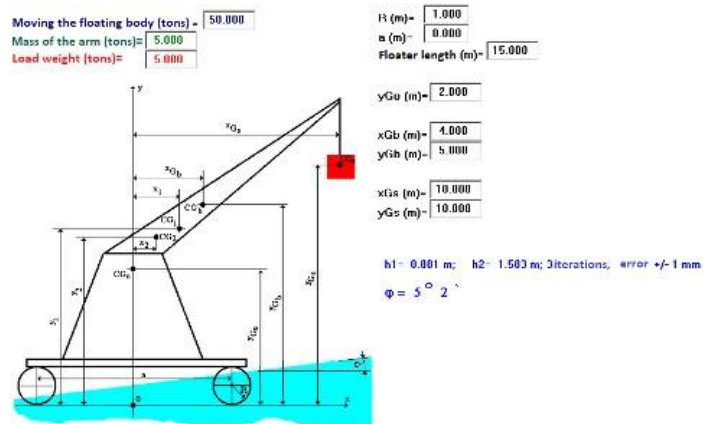
I consider that, an immersion depth without load at hook till half diameter of the floaters, can be a usual situation, which makes the idle displacement to be  $47,1\text{ tons}$ , consisting of floaters weight, solid ballast weight, crane, machinery and other equipment weights, which is perfectly possible. If we would consider only the two floaters as being manufactured from steel plate of  $10\text{ mm}$  thickness, their weight corresponds to a mass of  $15,7\text{ t}$ , remaining over  $30\text{ t}$  for crane, machinery, equipment, solid ballast, so that the idle displacement of  $47,1\text{ t}$  selected as calculation value is fully possible. I choose  $\varphi=10^\circ$  as maximum inclination angle for ensuring stability.

As the crane has a trim of  $\varphi=0^\circ$  (equal reactions on the two floaters) when it has no load to lift or during travel on the water, I will only treat static stability under load. If consideration would be given to an inclination to the opposite side of the load with the same angle  $\varphi=10^\circ$  for idle floating crane, this would suppose a crane out-of-balancing before lifting the loads, and this would need additional devices and, as it will be seen within dynamic stability, increase the moment of inertia of the mass under rolling with un favourable effects on stabilization. For on-shore cranes in idle condition, the reactions on the seats are not equal, but this has no effect on vertical position of the crane, by this reason an on-shore crane in idle condition can be unbalanced on reverse direction of the load effect, but this can no longer be assured for a floating crane.

By means of a special software for numerical solutioning of the systems of transcendent equations using Newton method [3], [4], [6], any values can be simulated for dimensions and masses of the crane or load, up to very heavy cranes which can lift loads of tenths or hundreds tons.

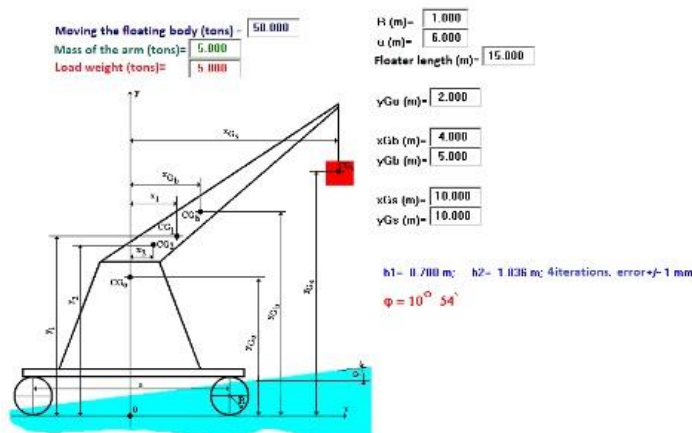


## STATIC STABILITY OF FLOATING CRANES



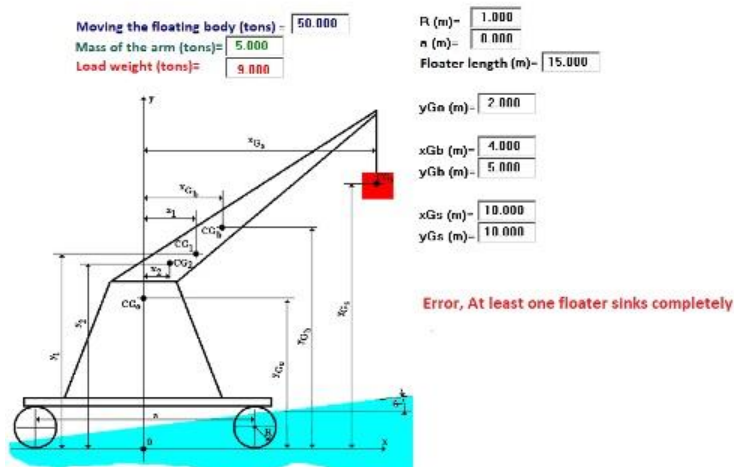
**Figure 7.**  
Static  
stability  
example.

## STATIC STABILITY OF FLOATING CRANES



**Figure 8.**  
Example of static  
instability by  
exceeding the  
maximum  
inclination angle.

## STATIC STABILITY OF FLOATING CRANES



**Figure 9.**  
Example of  
static instability  
by overturning.

Making stability calculations for different constructive versions of the crane, the following can be found:

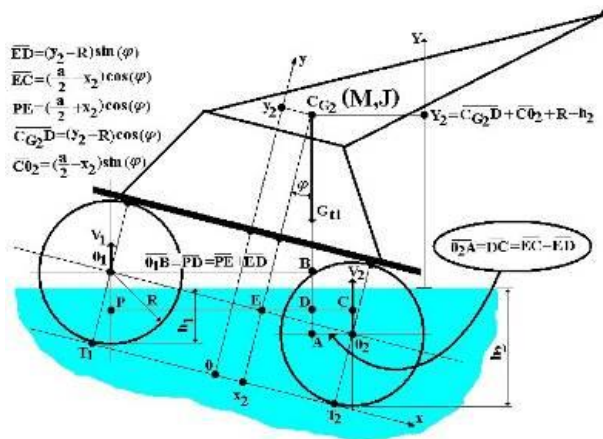
- Maximum allowed angle  $\varphi$  is not exceeded for distances between floaters centres,  $a$ , greater than a minimum value, irrespective of the load value, provided that neither floater is pushed by a force greater than it can take over by floatability. Considering to the limit that the total weight of the crane is supported on only one floater, the minimum value for the distance  $a$  is given by

$$a \geq \frac{h_{2\max} - h_{1\min}}{\sin(\varphi_{\max})} \Rightarrow a \geq \frac{2R}{\sin(\varphi_{\max})} \quad (8)$$

- To ensure static stability, the crane weight without boom and load should be greater than a minimum value depending on load size and distance this load is handled to. This value can be found by using the software presented above with required values for load, boom weight and handling distance, and testing various values for crane displacement, floaters dimensions and distance between them.

#### 4. Dynamic stability of floating cranes with floaters

Dynamic processes come up when static balance of the crane is disturbed and behave like periodical or non-periodical motions on their way to another balance position. This process has generic name of transitory mode. Most often, these motions are oscillating non-linear and attenuated and concern the time of transitory mode, period and maximum amplitude of the oscillations. For a crane, the most dangerous disturbance of statical balance is caused by sudden loose of the load. Sudden loading (similar to load falling down from hook) is not possible due to great starting time for vertical movement and crane structure flexibility (cables, booms, etc.) [3], [7].



**Figure 10.** Sketck for dynamic stability calculation.

As forces act only on vertical direction, figure 9, translation movement wil only be on this direction and based on them the dynamic balance equations are written as shown below, applying d'Alembert principle and equalizing the stresses opposing to movement with stresses which are in favor of movement. Within the stresses opposing to movement dynamic resistances are included as well as inertia opposing to linear or angular acceleration, friction and yield type forces. Friction stresses, as forces and moments, oppose to linear or angular speed, the yield type stresses are those produced by floaters due to immersion with values  $h_1$  or  $h_2$ .

When friction is missing the movement continue to infinite, by this reason, friction should be taken into consideration as viscous friction with media, respectively water and air.



Translation movement coordinate is given by position of the barycenter  $C_{G_2}$  on vertical, noted with  $Y_2$ , with origin on water surface considered as fixed referential and rotation movement coordinate is the angle  $\varphi$  between the floaters centreline and the same water surface.

$$\begin{cases} \text{Balance on vertical: } M \cdot \ddot{Y}_2 + F_{fr}(\dot{Y}_2) + V_1 + V_2 = G_H \\ \text{Balance for rotation: } J \cdot \ddot{\varphi} + M_{fr}(\dot{\varphi}) + V_2 \cdot \overline{O_2 A} - V_1 \cdot \overline{O_1 B} = 0 \end{cases} \quad (9)$$

where  $M$  is the total mass of the crane without load,  $J$  is the moment of inertia also without load determined against the barycentre  $C_{G_2}$ ,  $F_{fr}$  is the viscous friction force with water and air for translation movements,  $M_{fr}$  is the viscous friction moment with water and air for rotation movements around barycentre. The real variables depending on time in differential equations (9) are  $h_1$  and  $h_2$ , consequently  $Y_2$ ,  $\dot{Y}_2$ ,  $\ddot{Y}_2$ ,  $\varphi$ ,  $\dot{\varphi}$ ,  $\ddot{\varphi}$  which appear in (9) will have to be expressed as function of crane dimensions and  $h_1$ ,  $h_2$ . From figure 10 the expression for  $Y_2$  is deducted and becomes:

$$Y_2 = (y_2 - R) \cos(\varphi) + \left( \frac{a}{2} - x_2 \right) \sin(\varphi) + R - h_2$$

which, considering (2) is written

$$Y_2 = \frac{y_2 - R}{a} \sqrt{a^2 - (h_2 - h_1)^2} + \frac{a - 2x_2}{2a} (h_2 - h_1) + R - h_2 \quad (10)$$

Wherefrom, by derivation,  $\dot{Y}_2$  and  $\ddot{Y}_2$  are obtained. The reactions  $V_1$ ,  $V_2$  depend on  $h_1$  and  $h_2$  according to (3)---(5) and dimensions  $\overline{O_1 B}$  and  $\overline{O_2 A}$  in (9) also depend on  $h_1$  and  $h_2$ , as per figure 9. From (2) expression of angle  $\varphi$  is found, which, by derivation, leads to expressions for  $\dot{\varphi}$ ,  $\ddot{\varphi}$ . All these will be replaced in (9) which is solved by numerical methods due to transcendency of the expressions used. When performance of derivatives, the rule for derivation of composed functions will be taken into account, respectively

$$\begin{aligned} \dot{f}(h_{1,2}(t)) &= \frac{df}{dh_1} \dot{h}_1 + \frac{df}{dh_2} \dot{h}_2 \\ \ddot{f}(h_{1,2}(t)) &= \frac{d^2 f}{dh_1^2} (\dot{h}_1)^2 + \frac{df}{dh_1} \ddot{h}_1 + \frac{d^2 f}{dh_2^2} (\dot{h}_2)^2 + \frac{df}{dh_2} \ddot{h}_2 + 2 \frac{d^2 f}{dh_1 dh_2} \dot{h}_1 \dot{h}_2 \end{aligned} \quad (11)$$

By these, furtheron it is obtained:

$$\begin{aligned} \dot{Y}_2 &= \frac{(y_2 - R)}{a} \frac{h_2 - h_1}{\sqrt{a^2 - (h_2 - h_1)^2}} (\dot{h}_1 - \dot{h}_2) + \frac{a - 2x_2}{2a} (\dot{h}_2 - \dot{h}_1) - \dot{h}_2 \\ \ddot{Y}_2 &= \frac{y_2 - R}{a} \frac{a^2 (2\dot{h}_1 \dot{h}_2 - \dot{h}_1^2 - \dot{h}_2^2)}{(a^2 - (h_2 - h_1)^2) \sqrt{a^2 - (h_2 - h_1)^2}} + \frac{y_2 - R}{a} \frac{(h_2 - h_1)(\ddot{h}_1 - \ddot{h}_2)}{\sqrt{a^2 - (h_2 - h_1)^2}} - \\ &\quad \frac{a - 2x_2}{2a} \ddot{h}_1 + \left( \frac{a - 2x_2}{2a} - 1 \right) \ddot{h}_2 \\ \dot{\varphi} &= \frac{\dot{h}_2 - \dot{h}_1}{\sqrt{a^2 - (h_2 - h_1)^2}} \\ \ddot{\varphi} &= \frac{h_2 - h_1}{(a^2 - (h_2 - h_1)^2) \sqrt{a^2 - (h_2 - h_1)^2}} (\dot{h}_1^2 + \dot{h}_2^2 - 2\dot{h}_1 \dot{h}_2) + \frac{\ddot{h}_2 - \ddot{h}_1}{\sqrt{a^2 - (h_2 - h_1)^2}} \end{aligned} \quad (12)$$

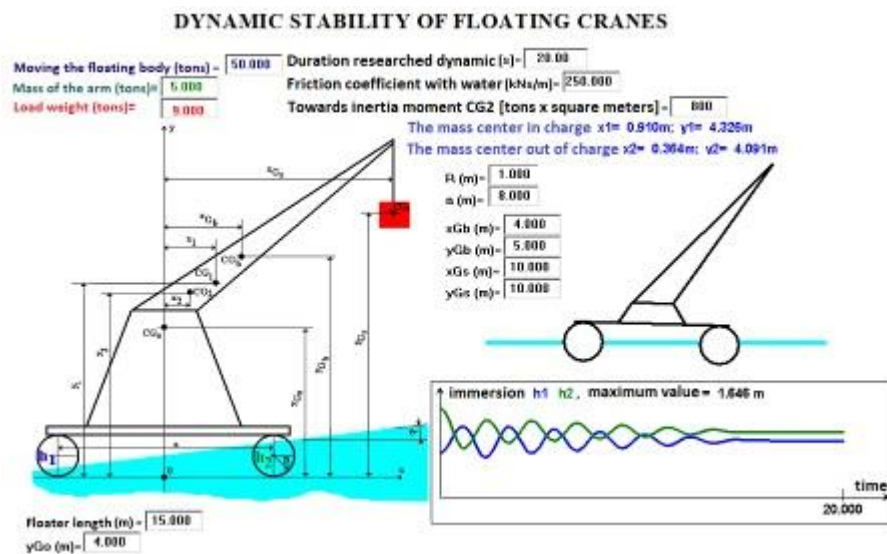
which are replaced in (9).

Due to the strong non-linearity comparing to variables  $h_1$ ,  $h_2$  and their derivatives, next step is to express such derivatives by finite differences “to the left” in order to appear as unknown only the values

at the current moment,  $h_{1,2}^{current}$  and  $h_{2,2}^{current}$ , the values at the two previous moments being known at the beginning of solving under initial conditions, then, during iterations, each current moment on an iteration becomes previous moment for the next iteration a.s.o.

$$\begin{cases} h_{1,2} = h_{1,2}^{current} \\ \dot{h}_{1,2} = \frac{h_{1,2}^{current} - h_{1,2}^{anterior}}{\delta t} \\ \ddot{h}_{1,2} = \frac{h_{1,2}^{current} - 2h_{1,2}^{anterior} + h_{1,2}^{ante-anterior}}{(\delta t)^2} \end{cases} \quad (13)$$

Which, replaced in (12) and (10) then in (9) form a system of two transcendent equations with unknown  $h_{1,2}^{current}$ , resolvable in its turn by numerical methods, preferably Newton method due to its very fast convergence.



**Figure 11.**  
Dynamic behavior  
of the crane.

Regarding frictions which occur in balance equations (9), these can be determined based on the laws of fluid mechanics. But, due to complicated configuration of the crane, I preferred to generally consider them as

$$\begin{cases} F_{fr} = c_1 \dot{Y}_2 \\ M_{fr} = c_2 \dot{\phi} \end{cases} \quad (14)$$

Where  $c_1$  and  $c_2$  have absolutely aleatory values with the only purpose to have friction to attenuate the movement. The values of this coefficient do not influence very much the value of movement period, as long as the movement is maintained periodic with at least 3---4 periods. The movement period is found by practical execution of a special software, built on the basis of the above described model, wherefrom I show a screenshot. The masses and moments of inertia used for software running are imaginary, but correspond to a practical case.

## 5. Conclusions

The floating crane has no other connections to environment than by means of some non-linear yield seats on vertical direction. Considering the reactions in the seats as external forces applied to a rigid untied body, the crane as assembly, within the transitory mode, will perform movements under influence of its own weight force and the forces in the seats. It can be said that the crane moves freely under the influence of its weight and some external forces depending on its position. A rigid solid with physical dimensions (and not a material point) in this case performs translation movements of barycentre and rotations around this centre, as per the principle of minimum action (Maupertuis principle), given the consumed power for movement excitation is minimum for rotations around barycentre.

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