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Pneumatic muscle actuated rotation modules for elbow rehabilitation equipment

G Vetrice and A Deaconescu
Department of Industrial Engineering and Management, Transilvania University of Brasov, Mihai Viteazul Street, no. 5, 500174, Brasov, Romania

E-mail: vetrice.georgiana@unitbv.ro

Abstract. The paper presents a theoretical study concerning the possibility of using a pneumatic actuation system for a piece of elbow rehabilitation equipment with two degrees of freedom. For this purpose the design and calculation of a rotation module for each movement performed by the equipment is necessary: flexion-extension and pronation-supination. Construction of the modules implies a force and torque analysis leading to the correct selection of the pneumatic muscles, able to perform the desired movements. The static analysis of the rotation module can be done in two different modalities. The first modality takes into account the influence of neuronal control quantities on the forces developed by the two pneumatic muscles, while the second one aims to determine forces based on the technical data and load pressures of the pneumatic muscles. The dynamic analysis of the rotation module starts from two equations of the dynamic model, where first taken into consideration are: J – the moment of inertia, ω – angular velocity, T – total moment in the joint and T_g – gravitational moment; then, for a simplified model, the term T_g is neglected. The exhaustive calculations carried out during the dynamic analysis yielded satisfying results. Further presented are variation graphs of pressure Δp and rotation angle θ versus time, respectively.

1. Introduction

The proposed equipment for the rehabilitation of the elbow has two rotational degrees of freedom placed perpendicularly, meaning ensures two movements: flexion-extension and pronation-supination, motions that are performed separately and consequently are addressed separately in this paper.
Each rotation module is driven by a couple of pneumatic muscles, arranged in an antagonistic connection. The neutral position is achieved when both muscles are fed the same pressure; in order for the movement to take place one muscle is compressed, while the other one is relaxed. The transmission of motion between actuators and joints is made by means of steel cables. That causes the rotation of a shaft and also of a support where the hand is placed.

2. Calculation of the rotation modules

2.1. Calculation of the rotation module for flexion-extension

A force and torque analysis is required for designing the flexion-extension rotation module. Dimensioning the force exerted by the biceps is based on the schematic shown in figure 1.

The notations refer to:

- Force F is equal to the sum of the weight of a hand-held object of *e.g.* 2 kg mass and the weight of the hand. In this case, force F = (2 + 0.46)·9.81 = 24.13 N.

- Force G is the weight of the forearm: G = 1.2 · 9.81 = 11.77 N.
- F_m is the force that the biceps must exert in order to achieve the flexion-extension movement.

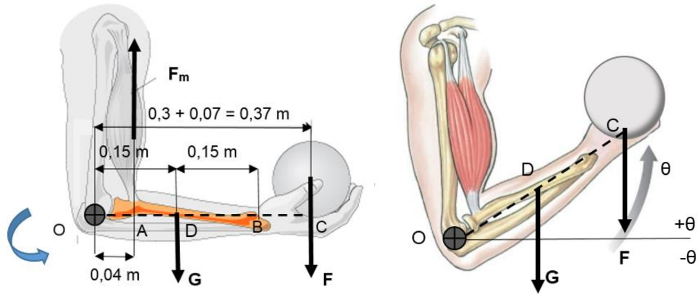

Figure 1. Display of forces.

In this case, the equilibrium equation of torques in O joint is:

$$\sum T_O = 0;\ \ F \cdot OC \cdot \cos\theta + G \cdot OD \cdot \cos\theta - F_m \cdot OA = 0 \qquad (1)$$

Hence force F_m is:

$$F_m = \frac{(F \cdot OC + G \cdot OD) \cdot \cos\theta}{OA} = \frac{(24.13 \cdot 0.37 + 11.77 \cdot 0.15) \cdot \cos\theta}{0.04} = 267.34 \cdot \cos\theta\ [\mathrm{N}] \qquad (2)$$

In the case of elbow rehabilitation equipment, the forces F and F_m are not present, therefore the value of the torque in joint O can be calculated as follows:

$$T_O = G' \cdot OD' \cdot \cos\theta = M_{O\max} \cdot \cos\theta \qquad (3)$$

where G' is the sum of the weights of the hand, of the forearm and of the hand support (G'=(0.46+1.2+3)·9.81 = 45.71 N), and OD' is the distance between the rotation axis and the point of application of force G' (OD' = 0.185 m).

Thus, for the data in Figure 1, the torque in O joint is:

$$T_O = -(45.71 \cdot 0{,}185) \cdot \cos\theta = -8.45 \cdot \cos\theta\ [\mathrm{N \cdot m}]$$

where $T_{O\max}$=-8.45 Nm. The (-) sign has been assigned conventionally, because the significant torque in the joint has to be overcome by the actuation system.

According to [1], the medium torque in the elbow joint for flexion is of approximately 7Nm and for extension 10 times smaller. For dimensioning the rotation module for flexion-extension a value of 8.45 Nm will be taken into consideration.

In this case, the rotation corresponding to the flexion-extension of the forearm is obtained by means of two pneumatic muscles in a push-pull relationship: as one muscle becomes expands its length, the other one shortens and vice versa.

Figure 2 shows the working principle of the pneumatic muscles denoted M1 and M2.

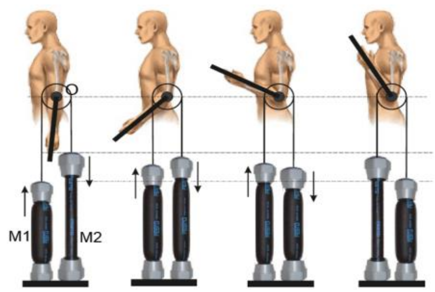

Figure 2. Flexion motion.

The upper ends of the pneumatic muscles are connected by a steel wire, drawn over a pulley and fixed in a specially designed structure, the winding angle of the wire over the pulley being 270°.

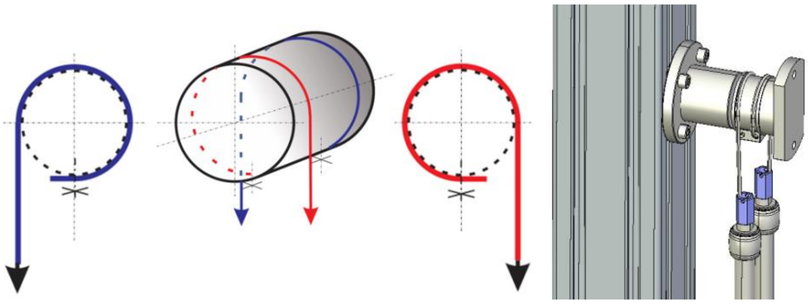

Figure 3. The winding scheme of the wires.

For flexion-extension two DMSP-20-300N-RM-CR pneumatic muscles were manufactured by Festo, Germany were selected, with an interior diameter of 20 mm and initial length of 300 mm. The maximum axial contraction of these muscles, when fed compressed air at a pressure of 6 bar, is ΔL_{max}=60 mm (20% of initial length). The MuscleSim v. 2.0.1.5 programme provided by Festo calculates the values of the forces based on to the feed pressure and specific axial contraction.

In this case, for a 3 bar pressure and a 30 mm stroke performed by the pneumatic muscle, meaning a 10% specific axial contraction, the developed force is of 287.8 N.

Initially, the feed pressure of the two pneumatic muscles is 0 bar. In order to obtain a rotation, the first step is to feed half of the maximum work pressure to both pneumatic muscles simultaneously. When $p_0=p_{max}/2$, the axial contraction of the two muscles is $\Delta L_{max}/2$, the stroke performed by the lower ends of the pneumatic muscles being limited by a fixed stop.

The specific axial contraction of a muscle, ε, is defined as follows:

$$\varepsilon = \frac{L_i - L}{L_i} \cdot 100 \ [\%] = \frac{\Delta L}{L_i} \cdot 100 \, [\%] \tag{4}$$

where L_i is the initial length of the muscle (when p=0 bar) and L is the length of the muscle fed at a certain pressure p.

The maximum specific axial contraction ε_{max} of a pneumatic muscle is:

$$\varepsilon_{\max} = \frac{\Delta L_{\max}}{L_i} \cdot 100 \, [\%] \tag{5}$$

where ΔL_{max} is the maximum stroke carried out by the free end of the muscle when fed maximum pressure.

Thus both pneumatic muscles have length L_0 and their specific axial contraction ε_0, will be:

$$\varepsilon_0 \ = \ \frac{\frac{\Delta L_{\max}}{2}}{L_i} \cdot 100 \ = \ \frac{\Delta L_{\max}}{2 \cdot L_i} \cdot 100 \quad [\%] \tag{6}$$

In order to carry out a rotation by a certain angle θ one muscle will be fed compressed air at a pressure $p_1 = p_0 + \Delta p$ and the other one will relax, the pressure being $p_2 = p_0 - \Delta p$. Thus the following modifications will take place: the fed muscle will shorten to a length $L_1 = L_0 - \Delta L_1$, and the relaxed one will extend to a length $L_2 = L_0 + \Delta L_2$.

When joint rotation reaches θ_{max}, the specific axial contraction of the two muscles becomes:

$$\varepsilon_1 = \varepsilon_0 + \frac{R \cdot \theta_{\max}}{L_i} = \frac{\Delta L_{\max}}{L_i} \cdot 100 \quad [\%] \tag{7}$$

$$\varepsilon_2 = \varepsilon_0 - \frac{R \cdot \theta_{\max}}{L_i} = 0 \quad [\%] \tag{8}$$

where R is the radius of the pulley over which the wire affixed to the free end of the pneumatic muscles is drawn.

By means of equations (6) and (7) the radius of the pulley can be calculated, knowing that the maximum angle of rotation, $\theta_{max} = \pi/2 = 90^{\circ}$. Thus,

$$R = \frac{\Delta L_{\max}}{\pi} = \frac{60}{\pi} = 19.1 \text{ mm} \tag{9}$$

2.1.1. Static analysis of the rotation module for flexion-extension

There are two approaches to the static analysis of the rotation module. The first approach considers, similarly to the biological model, the influence of neuronal control quantities on the forces developed by the two pneumatic muscles, while the second one measures the forces based on the technical data and load pressures of the pneumatic muscles.

In the first approach, the general equation that describes the evolution of the static force developed by a pneumatic muscle is:

$$F = u \cdot F_{\max} \cdot (1 - \frac{\varepsilon}{\varepsilon_{\max}}) \tag{10}$$

where F_{max} is the maximum force exerted by the muscle when the axial contraction is zero and u is a neuronal control quantity, with values between 0 si 1 ($0 \le u \le 1$) [2]. Nerve impulses (control quantities u) are generated by alpha motor neurons causing contractions of the muscle fibres and implicitly force and movement. The antagonistic relationship of the two pneumatic muscles helps developing a torque within the joint, as follows:

$$T = R \cdot (F_1 - F_2) \tag{11}$$

where F_1 and F_2 are the forces developed by the two pneumatic muscles (agonistic force and antagonistic force).

Upon applying equation (10) for each force:

$$F_1 = u_1 \cdot F_{\max} \cdot (1 - \frac{\varepsilon_1}{\varepsilon_{\max}}) \tag{12}$$

$$F_2 = u_2 \cdot F_{\max} \cdot (1 - \frac{\varepsilon 2}{\varepsilon_{\max}}) \tag{13}$$

where $u_1 + u_2 = 1$.

Introducing formulas (12) and (13) into equation (11), by means of equation (6) to (8) torque T becomes:

$$T = R \cdot F_{\max} \cdot (1 - \frac{\varepsilon_0}{\varepsilon_{\max}}) \cdot (u_1 - u_2) - \frac{F_{\max} \cdot R^2}{L_i \cdot \varepsilon_{\max}} \cdot (u_1 + u_2) \cdot \theta \tag{14}$$

According to [3], if:

$$K_1 = R \cdot F_{\max} \cdot (1 - \frac{\varepsilon_0}{\varepsilon_{\max}}) \tag{15}$$

$$K_2 = \frac{F_{\max} \cdot R^2}{L_i \cdot \varepsilon_{\max}} \tag{16}$$

then

$$T = K_1 \cdot (u_1 - u_2) - K_2 \cdot (u_1 + u_2) \cdot \theta \tag{17}$$

When T=0, the joint equilibrium position is:

$$\theta_{ech} = \frac{K_1 \cdot (u_1 - u_2)}{K_2 \cdot (u_1 + u_2)} \tag{18}$$

If an external force is exerted on the rotation module, and it reaches a new position $\theta_{eq} \pm \Delta\theta$, the system tends to return to its original position, due to a return torque T_{ret} [3]:

$$T_{ret} = -K_2 \cdot (u_1 + u_2) \cdot \Delta\theta = -k \cdot \Delta\theta \tag{19}$$

where k is the torsional rigidity of the joint.

$$k = \frac{dT}{d\theta} = -K_2 \cdot (u_1 + u_{2)} \tag{20}$$

The maximum angle of rotation θ is:

$$\theta_{\max} = \pm \frac{(\varepsilon_{\max} - \varepsilon_0) \cdot L_i}{R} \tag{21}$$

In the case discussed in this paper the data required for calculations are: ε_{max}=20%; ΔL_{max}=60 mm; ε_0=10%; F_{max}=1552.9N. The maximum angle of rotation is $\theta_{max} = \pm \pi/2$.

Further, by means of equations (15) and (16) there follows:

$$K_1 = R \cdot F_{\max} \cdot (1 - \frac{\varepsilon_0}{\varepsilon_{\max}}) = 0.0191 \cdot 1552.9 \cdot \left(1 - \frac{0.1}{0.2}\right) = 14.83$$

$$K_2 = \frac{F_{\max} \cdot R^2}{L_i \cdot \varepsilon_{\max}} = \frac{1552.9 \cdot 0.0191^2}{0.3 \cdot 0.2} = 9.44$$

When $u_1 = 1$ and $u_2 = 0$ or u1 $= 0$ and $u_2 = 1$, *i.e.* when one muscle is maximum contracted and the other one is maximum relaxed, the torque reaches its maximum values too:

$$T_{\max}(\theta) = K_1 \cdot (1-0) - K_2 \cdot (1+0) \cdot \theta = K_1 - K_2 \cdot \theta \tag{22}$$

Further on, certain values of $T_{max}(\theta)$ are calculated:

$$T_{\max}(-\frac{\pi}{2}) = K_1 - K_2 \cdot (-\frac{\pi}{2}) = 14.83 + 9.44 \cdot \frac{\pi}{2} = 29.65 \text{ Nm}$$

$$T_{\max}(0) = K_1 - K_2 \cdot 0 = 14.83 \text{Nm}$$

$$T_{\max}(\frac{\pi}{2}) = K_1 - K_2 \cdot \frac{\pi}{2} = 14.83 - 9.44 \cdot \frac{\pi}{2} = 0.009 \text{ Nm}$$

These values are presented in figure 4.

Another important aspect to be included by the study of this joint is the "gravitational test", which takes into account the influence of weight of the moving masses. Figure 5 shows the schematic underlying the calculations, wherein G is the sum of the weights of the hand, of the forearm and of the hand support (G=(0.46+1.2+3)·9.81 = 45.71 N), and a is the distance between the rotation axis and the point of application of force G (a = 0.185 m).

In this case, the gravitational moment is:

$$T_g = (45.71 \cdot 0.185) \cdot \cos\theta = 8.45 \cdot \cos\theta \text{ [Nm]}$$

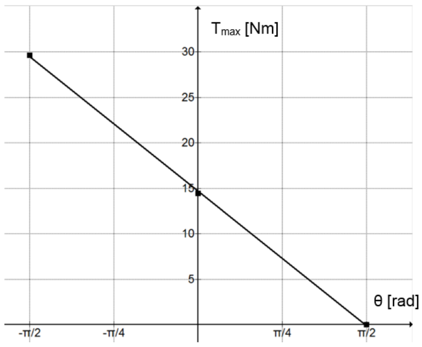

Figure 4. Maximum torque variation depending on the rotation angle.

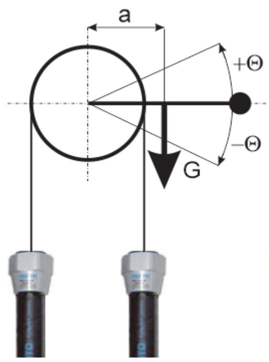

Figure 5. The effect of the weight of the moving masses on the rotation module.

Figure 6 shows the variation of the total torque in the rotation module.

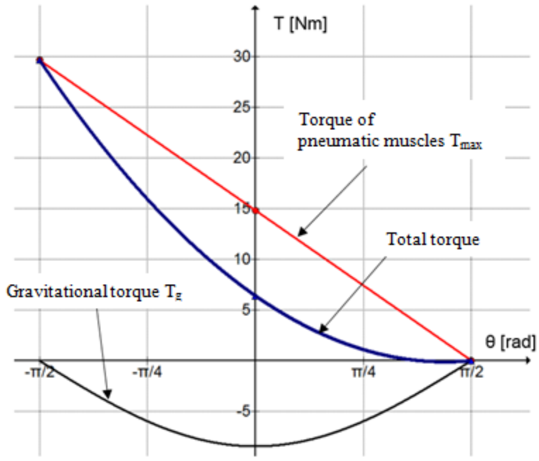

Figure 6. The variation of the total torque in rotation module.

The second approach takes into consideration a series of technical data of the pneumatic muscle, the static force, according to [4] and [5] being:

$$F = p \cdot \frac{\pi}{4} \cdot d^2 \cdot [a \cdot (1 - c \cdot \varepsilon)^2 - b] \tag{23}$$

where

$$a = \frac{3}{(\tan \alpha_{\min})^2} \tag{24}$$

$$b = \frac{1}{(\sin \alpha_{\min})^2} \tag{25}$$

- p is the supply pressure of the pneumatic muscle;
- d is the interior diameter of the pneumatic muscle, when relaxed;
- α is the angle of the pneumatic muscle's tissue;
- c is an experimental coefficient, c=1.5.

According to [6]:

$$F = p \cdot \frac{\pi}{4} \cdot d^2 \cdot \left[\frac{3 \cdot (\cos \alpha)^2 - 1}{1 - (\cos \alpha)^2} \right] \tag{26}$$

The limit values of angle α angle are:

- α_{max}, when the force is equal to 0 → $\alpha_{max} = 54.7^o$.
- α_{min}, when the force is maximum; for DMSP-20-300N muscle the maximum force is 1552.9 N, → $\alpha_{min} = 24.94^o$.

The values of a and b coefficients can be calculated by means of formulas (24) and (26) above: a = 13.87 and b = 5.62. Subsequently the force can be calculated:

$$F = p \cdot \pi \cdot [13.87 \cdot (1 - 1.5 \cdot \varepsilon)^2 - 5.62] \tag{27}$$

The torque developed in the rotation module joint is:

$$T = 24.52 \cdot (p_1 - p_2) - 15.83 \cdot (p_1 + p_2) \cdot \theta \tag{28}$$

or, upon replacing $K_1 = 24.52$ and $K_2 = 15.83$:

$$T = K_1 \cdot (p_1 - p_2) - K_2 \cdot (p_1 + p_2) \cdot \theta \tag{29}$$

When T = 0 the equilibrium position is:

$$\theta_{eq} = \frac{K_1 \cdot (p_1 - p_2)}{K_2 \cdot (p_1 + p_2)} \tag{30}$$

The torsional rigidity of the joint, *k*, is:

$$k = \frac{dT}{d\theta} = -K_2 \cdot (p_1 + p_2) \tag{31}$$

The maximum angle of rotation is $\theta_{max} = \pm \pi/2$.

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The maximum values of the torque are obtained when the feed pressures are either p_1 = *6 bar* and p_2 = *0 bar* or p1 = *0 bar* and p_2 = *6 bar*, meaning a maximum contraction of one muscle and a complete relaxation of the other.

In this case the maximum torque is:

$$T_{\max}(\theta) = [K_1 \cdot (6-0) - K_2 \cdot (6+0) \cdot \theta] \cdot 10^{-1} = 6 \cdot (K_1 - K_2 \cdot \theta) \cdot 10^{-1} \tag{32}$$

AS pressure is measured in bar and the two coefficients K_1 and K_2 in cm^3, it was necessary to introduce the factor 10^{-1}.

Further the maximum torque $T_{max}(\theta)$ is calculated for different values of the angle of rotation.

$$T_{\max}(-\frac{\pi}{2}) = [6 \cdot (K_1 - K_2 \cdot \theta) \cdot 10^{-1} = \left\{6 \cdot \left[24.52 - 15.83 \cdot \left(-\frac{\pi}{2}\right)\right]\right\} \cdot 10^{-1} = 29.62\,\text{Nm}$$

$$T_{\max}(0) = \left[6 \cdot (K_1 - K_2 \cdot 0)\right] \cdot 10^{-1} = 14.71\,\text{Nm}$$

$$T_{\max}(\frac{\pi}{2}) = [6 \cdot (K_1 - K_2 \cdot \theta)] \cdot 10^{-1} = \left\{6 \cdot \left[24.52 - 15.83 \cdot \left(\frac{\pi}{2}\right)\right]\right\} \cdot 10^{-1} = -0.199\,\text{Nm}$$

It should be noted that the values are almost identical with the ones obtained by the first method, thus confirming the correctness of the data and also of the graphs presented in figure 4 and figure 6.

2.1.2. Dynamic analysis of the rotation module for flexion-extension

Starting from the notations used in figure 5, the equations of the dynamic model are:

$$\begin{cases} \dot{\theta} = \omega \\ J \cdot \dfrac{d\omega}{dt} = T - T_g \end{cases} \tag{33}$$

where J – moment of inertia, ω – angular velocity, T – total torque in the joint and T_g – gravitational moment.

The last equation can be expanded as follows:

$$J \cdot \ddot{\theta} = 2 \cdot K_1 \cdot \Delta p - 2 \cdot K_2 \cdot p_0 \cdot \theta - 8.45 \cdot \cos\theta \tag{34}$$

where $\ddot{\theta} = \dfrac{d^2\theta}{dt^2}$.

According to the parallel axis theorem (Huygens-Steiner theorem), the moment of inertia of moving masses with respect to the axis of the pulley of the rotation module is:

$$J = J_{cm} + m \cdot a^2 = \frac{1}{12} \cdot m \cdot (2 \cdot a)^2 + m \cdot a^2 \tag{35}$$

where J_{cm} is the moment of inertia of the system with respect to an axis that passes through its centre of mass; *m* is the mass of the assembly and *a* is the distance between the centre of mass and the axis of the pulley.

For *m* = 4.66 kg and a = 0.185 m, the moments of inertia are J_{cm}=0.05316 kg·m^2 and *J*=0.2126 kg·m^2, respectively.

Starting from (34), further on a simplified dynamic model is shown, where T_g is neglected:

$$0.2126 \cdot \ddot{\theta} = 2 \cdot 24.52 \cdot \Delta p - 2 \cdot 15.83 \cdot 3 \cdot \theta$$

or

$$0.2126 \cdot \ddot{\theta} + 94.98 \cdot \theta = f(\Delta p)$$

with $f(\Delta p) = 49.04 \cdot \Delta p$

The relationship above is an inhomogeneous differential equation, whose overall solution is the sum of the solution of the homogeneous equation and a particular solution.

The solution of the homogeneous differential equation is:

$$0.2126 \cdot \ddot{\theta} + 94.98 \cdot \theta = 0$$

$$\theta(t) = C_1 \cdot \cos(21.13 \cdot t) + C_2 \cdot \sin(21.13 \cdot t)$$

The particular solution:

$$f(\Delta p) = a \cdot \Delta p + b$$

$$\dot{f}(\Delta p) = a\,; \ \ddot{f}(\Delta p) = 0$$

Upon replacing these terms in the first equation there results:

$$0.2126 \cdot 0 + 94.98 \cdot (a \cdot \Delta p + b) = 49.04 \cdot \Delta p$$

where a=0.516 and b=0.

Thus, the overall solution of the differential equation is:

$$\theta(t) = C_1 \cdot \cos(21.13 \cdot t) + C_2 \cdot \sin(21.13 \cdot t) + 0.516 \cdot \Delta p$$

Δp is the symmetrical pressure by that the muscles are charged and released, respectively and can be expressed as a linear function of time:

$$\Delta p = A \cdot t \qquad (36)$$

where A can have different values. For A = 3; A = 1; A = 0.5 and A = -1, figures 7 and 8 show the variation graphs of the pressure Δp and of the rotation angle θ versus time, respectively.

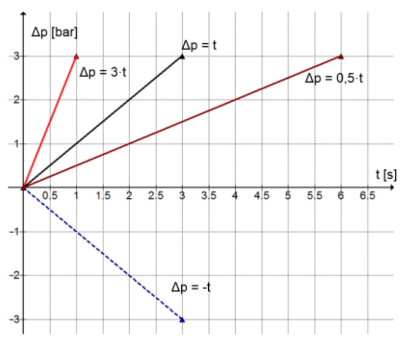

Figure 7. Variation of pressure Δp vs time.

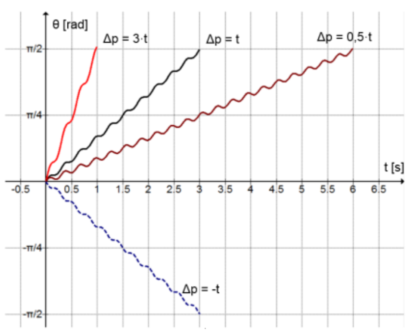

Figure 8. Variation of rotation angle θ vs time.

2.2. Calculation of the rotation module for pronation-supination movements

Identical to flexion-extension, pronation-supination is achieved by means of two pneumatic muscles working one against the other, in this case the interior diameter of the muscle being 10 mm.

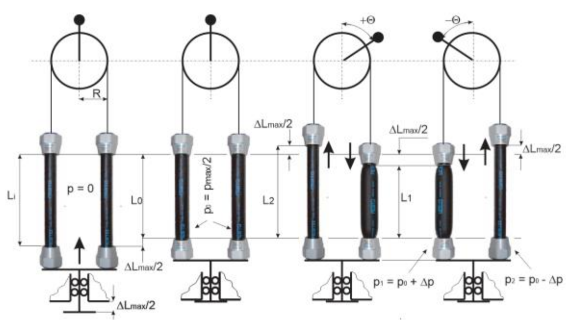

Figure 9. Working principle of the pronation-supination module.

According to figure 9, the pneumatic muscles are initially pre-charged to $p_0 = p_{max}/2$ (p_0 = 3 bar). Therefore, the stroke performed is of 30 mm, their specific contractions are ε_0= 10% and the force developed by each of them is of 25.1 N. The rotation takes place when their pressures vary antagonistically.

Using equations (7) and (8) the axial contractions can be calculated and as above, and the radius of the pulley over which the wire affixed to the free ends of pneumatic muscles is drawn is 19.1 mm (according to (9)).

2.2.1. Static analysis of the rotation module for pronation-supination

The data required for static analysis are: ε_{max} = 20%; ΔL_{max} = 60 mm; ε_0 = 10%; F_{max} = 475.8 N. Thus the maximum angle of rotation, compared to the equilibrium position, is $\theta_{max}=\pm\pi/2$.

The two coefficients, K_1 and K_2, can be calculated as follows:

$$K_1 = R \cdot F_{\max} \cdot (1 - \frac{\varepsilon_0}{\varepsilon_{\max}}) = 0.0191 \cdot 475.8 \cdot \left(1 - \frac{0.1}{0.2}\right) = 4.54$$

$$K_2 = \frac{F_{\max} \cdot R^2 \cdot}{L_i \cdot \varepsilon_{\max}} = \frac{475.8 \cdot 0.0191^2}{0.3 \cdot 0.2} = 2.89$$

The maximum moment in the joint occurs when $u_1 = 1$ and $u_2 = 0$ or u1 = *0* and $u_2 = 1$. For the first situation, the maximum torque is:

$$T_{\max}(\theta) = K_1 \cdot (1-0) - K_2 \cdot (1+0) \cdot \theta = K_1 - K_2 \cdot \theta = 4.54 - 2.89 \cdot \theta \tag{37}$$

Different values of maximum torques $T_{max}(\theta)$ are calculated below:

$$T_{\max}\left(-\frac{\pi}{2}\right) = K_1 - K_2 \cdot \left(-\frac{\pi}{2}\right) = 4.54 + 2.89 \cdot \frac{\pi}{2} = 9.07\,\text{Nm}$$

$$T_{\max}(0) = K_1 - K_2 \cdot 0 = 4.54\,\text{Nm}$$

$$T_{\max}\left(\frac{\pi}{2}\right) = K_1 - K_2 \cdot \frac{\pi}{2} = 4.54 - 2.89 \cdot \frac{\pi}{2} = 0.0027\,\text{Nm}$$

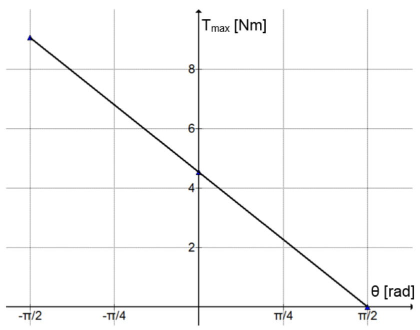

Figure 10. Maximum torque variation versus the rotation angle.

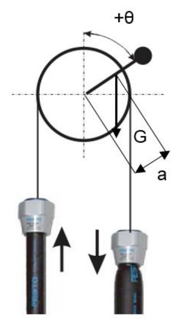

Figure 11. The effect of the weight of the moving masses on the rotation module.

The moving masses are those of the hand and forearm (0.46+1.2=1.66 kg); the distance between the point of application of the force to the rotation axis is 6 cm. The gravitational moment will be:

$$T_g = (1.66 \cdot 9.81 \cdot 0.06) \cdot |\sin\theta| = 0.977 \cdot |\sin\theta| \quad [\text{N·m}]$$

Figure 12 shows the variation of total torque in the rotation module joint.

The static analysis also entails expressing the force of pneumatic muscles as in eqs. (2) to (26). Hence,

- α_{max} is achieved when the force developed by the muscle is zero → $\alpha_{max} = 54.7°$.
- α_{min} is achieved when the force developed by the muscle is maximum. In this case the maximum force is 475.8 N → $\alpha_{min} = 23°$. For this value, coefficients a and b are: a = 16.65 and b = 6.55.

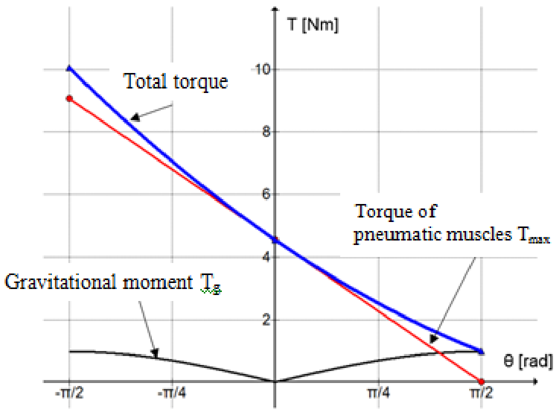

Figure 12. Variation of the total torque in the rotation module.

The force and torque are expressed below:

$$F = p \cdot \frac{\pi \cdot d^2}{4} \cdot [16.65 \cdot (1 - 1.6 \cdot \varepsilon)^2 - 6.55] \tag{38}$$

$$T = R \cdot (F_1 - F_2) = R \cdot \pi \cdot \frac{d^2}{4} \{p_1 \cdot [16.65 \cdot (1 - 1.6 \cdot \varepsilon_1)^2 - 6.55] - p_2 \cdot [16.65 \cdot (1 - 1.6 \cdot \varepsilon_2)^2 - 6.55]\} \tag{39}$$

Neglecting the terms ε^2 and introducing the formulas for ε_1 and ε_2 there results:

$T = 7.15 \cdot (p_1 - p_2) - 5.07 \cdot (p_1 + p_2) \cdot \theta$

If $K_1 = 7.15$ and $K_2 = 5.07$, then: $T = K_1 \cdot (p_1 - p_2) - K_2 \cdot (p_1 + p_2) \cdot \theta$

The equilibrium position of the joint is achieved when T = 0:

$$\theta_{eq} = \frac{K_1 \cdot (p_1 - p_2)}{K_2 \cdot (p_1 + p_2)}$$

Calculations of the maximum torque $T_{max}(\theta)$ are as follows:

$$T_{\max}\left(-\frac{\pi}{2}\right) = [6 \cdot (K_1 - K_2 \cdot \theta)] \cdot 10^{-1} = \left\{6 \cdot \left[7.15 - 5.07 \cdot \left(-\frac{\pi}{2}\right)\right]\right\} \cdot 10^{-1} = 9.06\,\text{Nm}$$

$$T_{\max}(0) = [6 \cdot (K_1 - K_2 \cdot 0)] \cdot 10^{-1} = 4.29\,\text{Nm}$$

$$T_{\max}\left(\frac{\pi}{2}\right)=[6\cdot(K_1-K_2\cdot\theta)]\cdot 10^{-1}=\left\{6\cdot\left[7.15-5.07\cdot\left(\frac{\pi}{2}\right)\right]\right\}\cdot 10^{-1}=-0.486$$

These values are very close to the ones obtained in the first approach. After converting the MIMO system into SISO, the torque in the joint becomes:

$$T=2\cdot K_1\cdot\Delta p-2\cdot K_2\cdot p_0\cdot\theta \tag{40}$$

and the equilibrium position (when T=0) is:

$$\theta_{eq}=\frac{K_1\cdot\Delta p}{K_2\cdot p_0}=0.47\cdot\Delta p \tag{41}$$

The torsional rigidity:

$$k=\frac{dT}{d\theta}=-2\cdot K_2\cdot p_0\cdot 10^{-1}=-3.042\,\mathrm{N{\cdot}m/rad}$$

2.2.2. Dynamic analysis of the rotation module for pronation-supination

Starting from equation (40) and from the value of the gravitational moment $T_g=0.977\cdot|\sin\theta|$, the equations of the dynamic model are:

$$\begin{cases}\dot{\theta}=\omega\\ J\cdot\ddot{\theta}=2\cdot K_1\cdot\Delta p-2\cdot K_2\cdot p_0\cdot\theta-0.977\cdot|\sin\theta|\end{cases} \tag{42}$$

The last equations can also be written as:

$$J\cdot\ddot{\theta}+2\cdot K_2\cdot p_0\cdot\theta+0.977\cdot|\sin\theta|-2\cdot K_1\cdot\Delta p=0 \tag{43}$$

With the notations in figure 11 the moment of inertia of the moving masses with respect to the axis of the pulley of the rotation module is:

$$J=J_{cm}+m\cdot a^2=\frac{1}{12}\cdot m\cdot(2\cdot a)^2+m\cdot a^2 \tag{44}$$

For m = 1.66 kg and a = 0.06 m, the moment of inertia is J=0.007968 kg·m^2

The equation of the simplified dynamic model, neglecting T_g is:

$$0.007968\cdot\ddot{\vartheta}+2\cdot 5.07\cdot p_0\cdot\theta-2\cdot 7.15\cdot\Delta p=0$$

or $0.007968\cdot\ddot{\theta}+30.42\cdot\theta=f(\Delta p)$

where $f(\Delta p)$=14.3·Δp

The relationship above is an inhomogeneous differential equation, whose overall solution is the sum of the solution of the homogeneous equation and a particular solution.

The solution of the homogeneous differential equation is: $0.007968\cdot\ddot{\theta}+30.42\cdot\theta=0$

$$\theta(t)=C_1\cdot\cos(61.78\cdot t)+C_2\cdot\sin(61.78\cdot t)$$

The particular solution is: $f(\Delta p)=a\cdot\Delta p+b$; $\dot{f}(\Delta p)=a$; $\ddot{f}(\Delta p)=0$

The main equation will be: $0.007968\cdot 0+30.42\cdot(\alpha\cdot\Delta p+b)=14.3\cdot\Delta p$

meaning a = 0.47 and b = 0.

The overall solution of differential equation is:

$$\theta(t)=C_1\cdot\cos(61.78\cdot t)+C_2\cdot\sin(61.78\cdot t)+0.47\cdot\Delta p$$

Δp is the symmetrical pressure that charges or releases the muscles and can be expressed as a linear function of time:

$$\Delta p = A \cdot t \tag{45}$$

where A can have different values. For A = 3; A = 1; A = 0.5 and A = -1, graphs showing the variation of pressure Δp versus time (figure 7) and the variation of rotation angle θ versus time (figure 13) are presented.

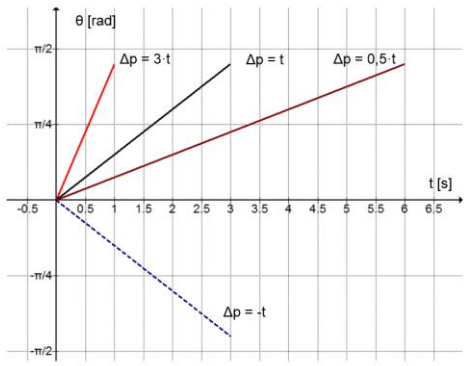

Figure 13. Variation of rotation angle θ versus time.

3. Conclusions

This paper discusses a theoretical study of two rotation modules of an elbow rehabilitation device equipped with a pneumatic actuation system. As the working principle of the pneumatic muscles is the same in both cases, the same methods were used in calculations.

The static analysis was conducted by two different approaches; the results were almost identical, which justifies using the two pneumatic muscles, of 20 mm interior diameter and 300 mm initial length for flexion extension- and of 10 mm interior diameter and 300 mm initial length for pronation-supination. The actuation system of each rotation module implies two feed pressures p_1 and p_2 as inputs, the outputs being the static moment T, the angle of rotation θ and the torsional rigidity k. For an accurate value of the rotation angle θ, the following quantities were used: Δp as input (the pressure by that symmetrically one muscle is charged and the other one released) and the rotation angle θ as output.

The dynamic analysis implied solving a differential equation in both cases, what led to the fact that the symmetrical pressure by that the muscles are charged and released, respectively can be expressed as a linear function of time and variation graphs of the pressure Δp and of the rotation angle θ versus time can be plotted.

In conclusion, the selected muscles are able to perform the desired movements, and consequently will be used in the construction of the equipment.

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