

Influence of the deviations caused by radial and axial runout of couplings on the positioning accuracy of the industrial robots

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Abstract: In most of the industrial domains, the workmanship has been replaced by industrial robots, due to their superior technical features and higher productivity. In order to continuously increase the production rates and performance characteristics required by the more and more complex technologies, the research and implementation of new methods to optimize and improve the performances of the industrial robots are necessary. According to such researches, most of the positioning and follow-up errors of the moving elements are caused by various factors belonging to the mechanical structure of the industrial robot. The deviations caused by radial and frontal runout of the bearings/couplings are an important factor that affects the positioning accuracy of the industrial robots, further to the friction and wear of the bearings in the kinematical linkages. This work presents a new method for determining and decreasing the positioning errors generated by the values of the axial and radial runout of the kinematic couplings, as well as a mathematical method based on the matrix of the direct kinematic. This mathematical model will also be used for simulating the positioning errors generated by the radial and frontal runout. The systematic characteristic of the axial and radial runout deviation rates of each coupling reflects on the positioning errors too, so that correction values may be inserted that eventually lead to increasing the positioning accuracy of the industrial robots.

1. Introduction

The structure of an industrial robot, due to the overall dimensions/working space ratio, mostly includes rotation couplings. The positioning accuracy of the industrial robots is also affected by the rigidity of the robotic arm, arm geometry and by the kinematic errors of each coupling in the structure of the robotic arm. The influence of the positioning accuracy produced by the arm geometry considers the geometry of each coupling of the arm, as well as the mutual location of the couplings in the design structure of the industrial robot. The geometry of the couplings of an industrial robot requires the moving element of each type of kinematic coupling to comply with several admissible deviations, as follows:

- for the rotation couplings, the moving (output) element has to comply with admissible values of the frontal runout, as well as of the radial runout;
- for the translation couplings, the moving element has to comply with admissible values of straightness in two perpendicular planes.



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At the end of the assemblage of a robotic arm there will be admissible deviations from the required geometry, resulting from the specific tolerances of the constitutive elements that eventually will negatively affect the positioning accuracy of the industrial robot. The determination, by measurements, of the main factors (radial and frontal runout of each rotation coupling, mutual position of the rotation axes of the couplings settled through their parallelism and perpendicularity, as well as the straightness deviations in case of the translation couplings) makes possible their insertion to a compensatory circuit in order to increase the positioning accuracy of the industrial robot.

2. Determining the relations of dependence of the axial and radial runout on a rotation coupling

A rotation coupling has as a basis, through its structure, a journal and a bushing/bearing. Each composing element has dimensional and shape deviations as well as deviations from the mutual location of the surfaces. After the assemblage of the components within the subassembly named rotation coupling, all admissible deviations mentioned above will “contribute” on the frontal runout figure (1a) and radial runout figure (1b) of the subassembly.

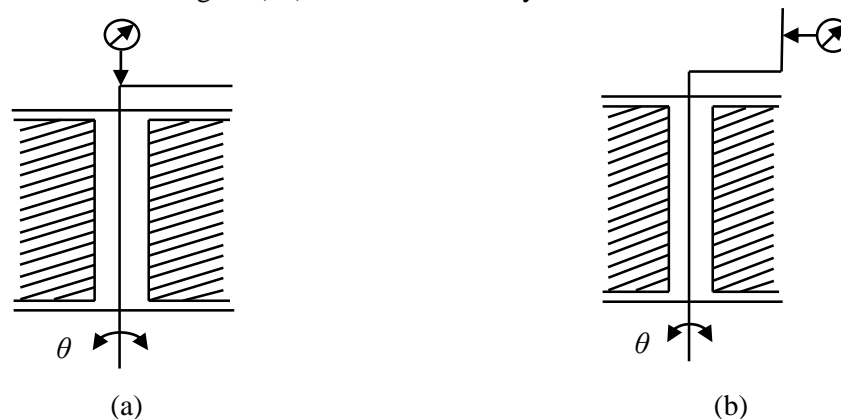


Figure 1 Axial runout and radial runout of a rotation coupling: (a) axial; (b) radial.

During the rotation stroke of a coupling, the angle θ , the two deviations will have variable values and, in order to be inserted to the analysis of their influence on the positioning accuracy of the industrial robot, they will be expressed through a relation of dependence, as close as possible to the real variation. The dependence relation of the axial and radial runout related to the rotation angle θ of the coupling may have a linear, exponential, hyperbolic or combined form.

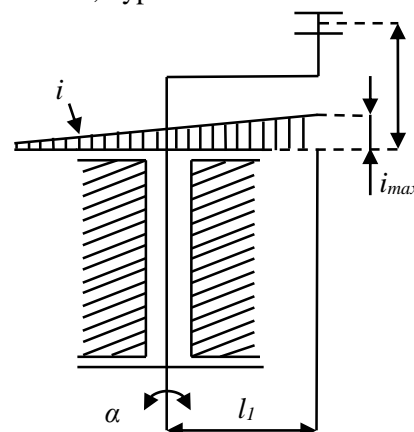


Figure 2 Linear variation of the axial deviation.

The relation of linear dependence of the axial deviation shown in figure 2 may be expressed through a relation of the form:

$$i = a \cdot \theta + c \quad (1)$$

where i is the axial deviation; a and c are constants to be determined in function of the maximum value i_{max} , effective, of the coupling after assemblage; θ is the rotation angle of the coupling. In order to settle the constants a and c the measured values of the axial deviation of the coupling will be used: V_{min} – axial deviation at $\theta = 0^\circ$; V_{max} – axial deviation at $\theta = t^\circ$.

By replacing the conditions below in the relation (1):

$$\begin{cases} \theta = 0^\circ \\ i = V_{min} \end{cases} \rightarrow c = V_{min} \quad (2)$$

$$\begin{cases} \theta = t^\circ \\ i = V_{max} \end{cases} \rightarrow V_{max} = a \cdot t + V_{min} \rightarrow a = \frac{V_{max} - V_{min}}{t} \quad (3)$$

As such, the dependence relation of the axial deviation related to the rotation angle θ of the coupling will be:

$$i = \frac{(V_{max} - V_{min}) \cdot \theta}{t} + V_{min} \quad (4)$$

The value of the axial deviation being obtained, expressed through the relation of linear dependence (4), will be found in the dimensional variation of the first element axially directed after the rotation coupling, so that in the case shown at figure 2 there will be “ $l+i$ ”. At a positioning of the terminal organ (end element) of the industrial robot, various values of the articular coordinates will correspond. Based on these articular coordinates, for each rotation coupling apart, in function of the value of its rotation angle and the dependence relation (4), the value of the frontal runout will be settled. Similarly, the dependence relation of the axial deviation related to the rotation angle θ will be determined, when the variation has parabolic, hyperbolic exponential or combined forms.

After the assemblage completion of the rotation couplings, effective measurements are to be done for settling the values of V_{min} at an angle $\theta = 0^\circ$ then V_{max} at $\theta = t^\circ$; afterwards the relation of dependence (4) of the frontal runout of the rotation coupling will be established.

In case of the radial runout of a rotation coupling, the causes of its occurrence consist mostly of the shape deviations (especially roundness) and eccentricity of the revolution surfaces of the parts composing the rotation couplings. Same as in the case of the frontal runout, at the radial runout of a rotation coupling the variation of the values may be linear in shape, with one or two slopes, or may be in the shape of a curve (parabolic, hyperbolic, exponential etc.).

If the rotation angle of the coupling is wide ($\theta > 90^\circ$), case that often happens, the presence of a linear variation with two slopes figure 3(a) or under the shape of a curve figure 3(b) will be frequently met on the rotation couplings.

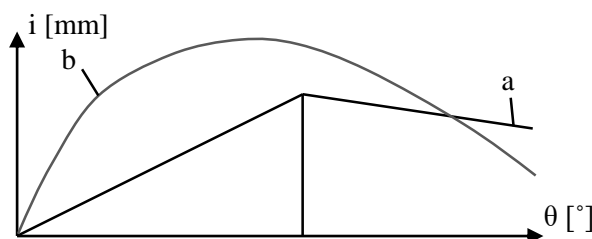


Figure 3 Variation shapes of the values of the radial deviation of a rotation coupling: a - linear, with two slopes; b - parabolic.

In case when the radial runout has a linear variation with two slopes, relations of dependence will be settled separately for each one of the slopes, based on the method presented at the frontal runout, following their validity to be settled by the interval of the rotation angle θ of the coupling. If the radial runout has a variation of parabolic shape, the equation of the 2nd degree has to be settled that could render at the best fidelity the experimental model of variation of the radial runout of the respective rotation coupling. In this case as well, based on the effective values of the radial runout related to the rotation angle θ , especially the points will be measured where the radial runout value is maximum and minimum and, subsequently, the constants of the dependence relation will be determined. The radial

deviation value of a rotation coupling that is expressed through the parabolic dependence relation, will be found in the dimensional variation of the first element radially directed, located after the rotation coupling, so that in the case shown at figure 2 it will be “ l_1+r ”, where r means the radial runout.

3. Experimental model on the influence of axial and radial runout of the rotation couplings on the positioning accuracy of an industrial robot

In order to increase the positioning accuracy of an industrial robot, the deviation values of the axial and radial runout of each rotation coupling in the robot structure will be inserted to the matrix model of homogenous transfer, thus obtaining the corrected position of the terminal organ (end element) in relation to the basic reference system. Experiments have been carried out on a palletizing robot. This robot belongs to the mechanic's laboratory of the university; its structure is presented at figure. 4.

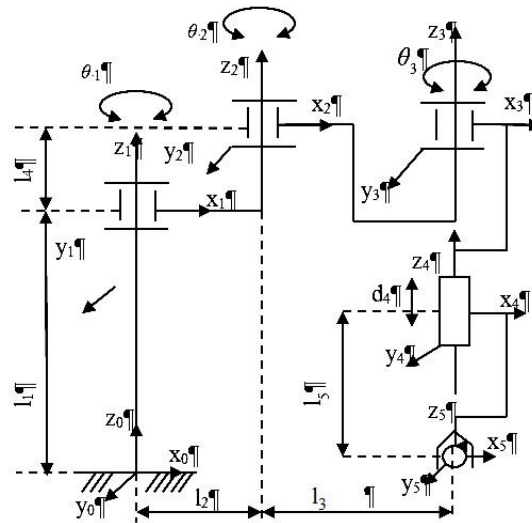


Figure 4 Schematic structure and reference systems attached to the palletizing robot.

The robot submitted to this expertise has three rotation couplings whose axial and radial runout will be determined experimentally for each one of them; afterwards the relations of dependence that will be found in the matrix model of homogenous passage will be settled. The reference system attached to the terminal organ is noted R_4 and the basic reference system is noted R_0 . The maximum values of the strokes will be: $\theta_{1max} = 85^\circ$, $\theta_{2max} = 95^\circ$, $\theta_{3max} = 110^\circ$; $l_{4max} = 180$ mm and the constructive dimensions are $l_2 = 300$ mm and $l_3 = 360$ mm. The matrix of homogenous transfer T_{01} that makes the passage from the reference system (x_0, y_0, z_0) to (x_1, y_1, z_1) is accompanied by the rotation θ_1 and the translations i_1 and r_1 that represent the axial and radial runouts of the rotation coupling 1. Where r_1 and i_1 are the dependence relations of the axial and radial runouts that are to be settled based on the effective diagram of variation related to the rotation angle θ_1 of the first coupling. Similarly, the homogenous transfer matrixes will be established for the other two rotation couplings T_{12} and T_{23} , resulting:

$$T_{01} = \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 & 0 & r_1 \\ \sin \theta_1 & \cos \theta_1 & 0 & 0 \\ 0 & 0 & 1 & i_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} T_{12} = w \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 & 0 & l_2 + r_2 \\ \sin \theta_2 & \cos \theta_2 & 0 & 0 \\ 0 & 0 & 1 & i_2 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$$T_{23} = \begin{bmatrix} \cos \theta_3 & -\sin \theta_3 & 0 & l_3 + r_3 \\ \sin \theta_3 & \cos \theta_3 & 0 & 0 \\ 0 & 0 & 1 & i_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

Where r_2 and r_3 are the dependence relations of the radial runout of the second and the third rotation coupling and i_2 and i_3 are the dependence relations of the frontal runout of the rotation couplings 2 and

3. By applying the calculation method that is known, only for the position parameters (X, Y, Z) of the reference system attached to the terminal organ, as well as based on the articular coordinates ($\theta_1, \theta_2, \theta_3$ and l_4) and on the fixed geometrical constructive parameters (l_2, l_3), it will result:

$$\begin{aligned} X &= r_1 + (l_3 + r_3) \cdot [\cos(\theta_1) \cdot \cos(\theta_2) - \sin(\theta_1) \cdot \sin(\theta_2)] + \cos(\theta_1) \cdot (l_2 + r_2) \\ Y &= (l_3 + r_3) \cdot [\cos(\theta_1) \cdot \sin(\theta_2) + \sin(\theta_1) \cdot \cos(\theta_2)] + \sin(\theta_1) \cdot (l_2 + r_2) \\ Z &= l_4 + i_1 + i_2 + i_3 \end{aligned} \quad (6)$$

The position parameters being calculated show the influence of the frontal runout i and radial runout r on the positioning accuracy. In an analog manner the orientation parameters (directing cosines) of the terminal organ can be determined, where the axial and radial runouts of the three rotation couplings have an important role. The diagrams at figure 5 show the effective axial and radial runouts of the three rotation couplings, 1, 2 and 3.

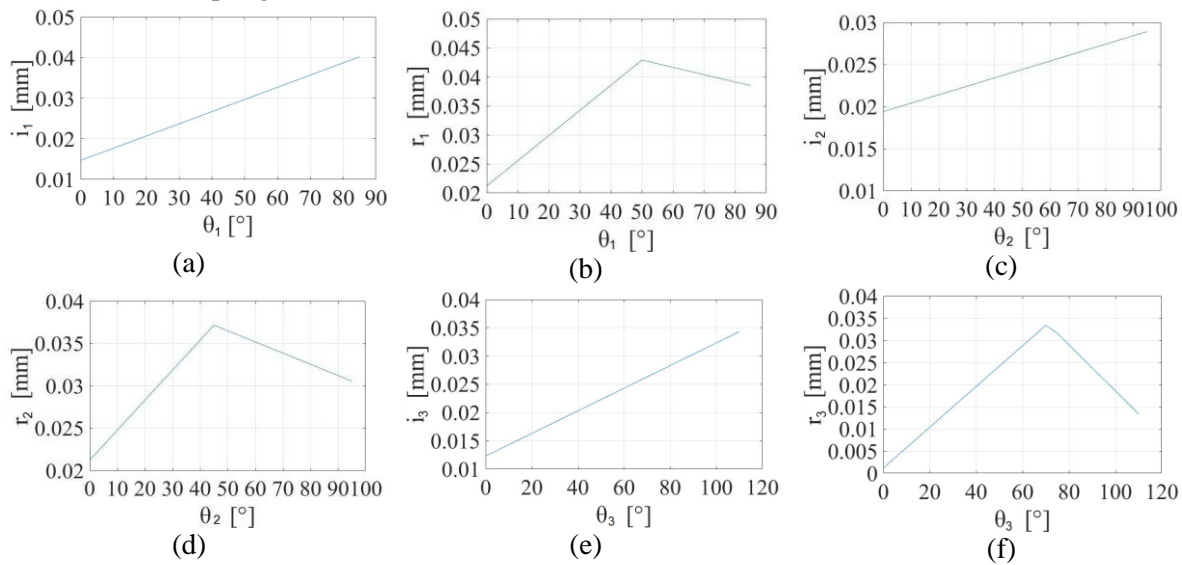


Figure 5. Effective frontal and radial runouts of the rotation couplings axial runout of coupling 1; b) radial runout of coupling 1 axial runout of coupling 2; d) radial runout of coupling 2 e) axial runout of coupling 3; f) radial runout of coupling 3.

On the basis of the diagrams at figure 5 the dependence relations of the axial runout i and radial runout r can be determined for the three rotation couplings. The dependence relation of the frontal runout i_1 is obtained from the straightening, as close as possible, of the diagram in figure 5(a):

$$i_1 = \frac{(V_{max} - V_{min})\theta_1}{t} + V_{min} = 0.0003 \cdot \theta_1 + 0.0147 \quad (7)$$

The dependence relation of the radial runout r_1 is obtained from the straightening of the diagram at figure 5(b) that has two slopes, an ascending one and a descending one:

$$r_1 = 0.0004 \cdot \theta_1 + 0.0212 \quad \text{for } \theta_1 \text{ between } 0^\circ \text{ and } 48^\circ \quad (8)$$

$$r_1 = -0.0001 \cdot \theta_1 + 0.0492 \quad \text{for } \theta_1 \text{ between } 49^\circ \text{ and } 85^\circ \quad (9)$$

For the second rotation coupling: $[\theta_2]$

$$i_2 = 0.00005 \cdot \theta_2 + 0.0217 \quad (10)$$

$$r_2 = 0.0004 \cdot \theta_2 + 0.0212 \quad \text{for } \theta_2 \text{ between } 0^\circ \text{ and } 45^\circ \quad (11)$$

$$r_2 = -0.0001 \cdot \theta_2 + 0.0431 \quad \text{for } \theta_2 \text{ between } 46^\circ \text{ and } 85^\circ \quad (12)$$

For the third rotation coupling:

$$i_3 = 0.0002 \cdot \theta_3 + 0.0123 \quad (13)$$

$$r_3 = 0.0005 \cdot \theta_3 + 0.0012 \quad \text{for } \theta_3 \text{ between } 0^\circ \text{ to } 70^\circ \quad (14)$$

$$r_3 = -0.0005 \cdot \theta_3 + 0.0706 \quad \text{for } \theta_3 \text{ between } 71^\circ \text{ to } 110^\circ \quad (15)$$

In order to make a comparison of the positioning accuracy on the industrial robot being researched, several values of the articulate angles $\theta_1, \theta_2, \theta_3$ will be considered, to which the position parameters $x,$

y, z are corresponding, in two versions: with correction values of the axial and radial runout and without corrections, see table 1.

Table 1 Comparative values of the positioning accuracy.

With axial and radial runout values						Without axial and radial runout values					
θ_1	θ_2	θ_3	X[mm]	Y [mm]	Z [mm]	θ_1	θ_2	θ_3	X[mm]	Y [mm]	Z [mm]
30°	45°	60°	393.714	117.912	-417.95	30°	45°	60°	393.66	117.857	-417.94
20°	35°	70°	-390.89	-102.52	-426.32	20°	35°	70°	-390.8	-102.52	-426.28
25°	45°	80°	230.061	230.074	230.096	25°	45°	80°	230	230	230

From the analysis of table 1 it results a difference of the values of the position parameters X, Y and Z that leads to an increase of the positioning accuracy of the industrial robot. Based on the dependence relations of the axial and radial runout of each rotation coupling, the control unit of the industrial robot will automatically offset the programmed parameters so that the positioning accuracy of the robot should not be affected by such deviations.

4. Conclusions

The method for increasing the positioning accuracy of an industrial robot by automatic offset/compensation of the axial and radial runout of the rotation couplings provides the increase of its performances. The method presented in this work may be implemented by the industrial robot builders, even by the industrial robot users; thus through relatively low efforts, the positioning accuracy and the path error of an industrial robot can be improved.

5. References

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