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Influence of the interference of bounce and pitch vibrations upon the dynamic behaviour in the bogie of a railway vehicle

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Abstract. The dynamic behaviour of the bogie of the railway vehicle is generally studied by capitalizing on the advantage of the symmetrical construction, which allows the vibrations in the vertical be considered decoupled from the ones in the horizontal plan. Likewise, the bounce and pitch vertical vibrations of the bogie are decoupled, due to the fact that elastic and damping elements with identical characteristics are normally used in the suspension corresponding to each axle of the bogie. The paper studies a particular situation, correlative with the failure of a damper in the primary suspension of a two-axle bogie. As a consequence of the damping reduction, an imbalance in the system occurs that will prompt dynamic interferences between the bounce and pitch vibrations of the bogie. The level of vibrations will rise, a fact focused on in the paper as based on the results from the numerical simulations, which represent the frequency response functions of the bogie calculated in the three reference points of the bogie, for different cases of reduction in the damping constant, compared with the reference value. The increase of the level of vibrations has an impact on the dynamic behaviour of the bogie, evaluated on the basis of the root mean square of the vertical acceleration.

1. Introduction

The railway vehicle represents a complex oscillating system, which features a behaviour of vibrations with specific characteristics, mainly generated by the vehicle-track interaction phenomena [1-4]. Under certain circumstances, the vibrations of the railway vehicle can have damaging effects upon the ride quality, safety, comfort of the passengers or integrity of merchandise being transported [5-10].

The oscillating movements of the railway vehicle develop both in the vertical and horizontal plans, in the shape of translation and rotation moves, independent or coupled among them. The construction of the railway vehicle usually comply with the rules of geometric symmetry, inertial and elastic, hence the moves in the vertical plan can be regarded as decoupled from the ones in the horizontal plan and, for that reason, separately dealt with.

The oscillating movements of the railway vehicle are made up of simple vibration modes of the suspended masses of the vehicle – the rigid modes [11], to which the complex modes of vibrations global or local, due to the carbody elasticity characteristics, such as bending and distortion, the modes of diagonal torsion and the local deformations of the floor, walls or ceiling [12].

The paper will examine the rigid vibration modes of the bogie in the vertical plan – bounce and pitch. These vibrations are decoupled if the bogie is symmetric geometrically and mass-related by comparison with the vertical – transversal plan and if the primary suspension corresponding to each axle of the bogie uses elastic and damping components with identical characteristics. While running, changes in the suspension parameters can occur and they originate, for instance, in the failure of a damping that is the critical element of suspension from the feasibility perspective [13]. As a consequence of a reduction in damping, there occurs an imbalance in the system resulting into dynamic interferences between the bounce and pitch vibrations of the bogie [14, 15]. The level of vibrations in the bogie will increase, a fact that is pointed out at herein, based on the results derived from numerical simulations. To this end, the frequency response functions are calculated in three reference points of a two-axle bogie – at the centre and against the suspension corresponding to the axles, for different cases of lowering the damping constant compared with the reference value. The increase in the level of vibrations affects the dynamic behaviour of the bogie, which is evaluated on the basis of root mean square of the vertical acceleration at various velocities.

2. The model of the bogie and the equations of motion

Figure 1 shows the mechanical model of a two-axle bogie travelling at a constant velocity V on a track with vertical irregularities described against each axle via the functions $\eta_{1,2}$. The model of the bogie includes three rigid bodies by which the bogie chassis and the two axles connected by Kelvin-Voigt type systems are modelled; on their turn, such systems help with the modelling the suspension of each axle. The elastic element of the suspension has the constant $2k_b$, while the damping element has the constant $2c_{b1}$ and $2c_{b2}$, respectively. The damping constants of the suspension in the two axles are equal ($2c_{b1} = 2c_{b2}$) when neither of the dampers is faulty. The bogie parameters are m_b – mass of the bogie, $2a_b$ – wheelbase of the bogie, J_b – inertia moment.

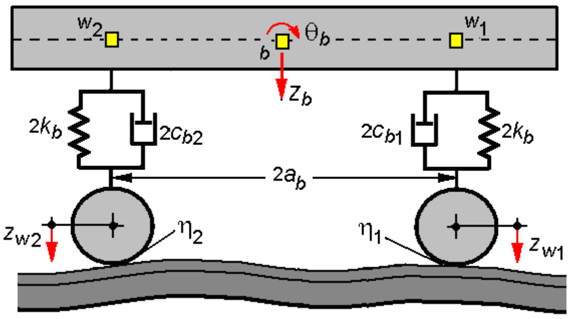

Figure 1. The model of the bogie.

The hypothesis of a perfectly rigid track is considered, which means that the axles closely follow the vertical irregularities of the track; the vertical displacements of the axles, noted with $z_{w1,2}$, are equal with these irregularities, namely $z_{w1} = \eta_1$ and $z_{w2} = \eta_2$.

The plan of the axles will have a translation motion – bounce z_w (figure 2, (a)), and a rotation motion – pitch θ_w (figure 2, (b)). The position of each axle in the bogie, compared to the referential OXZ (figure 2, (c)) located in the rotation axis of the plan of the axles, is the result of the overlapping of the bounce and pitch movements of this plan, such as below:

$$z_{w1} = z_{pw}^{+} + z_{pw}^{-}\,;\; z_{w2} = z_{pw}^{+} - z_{pw}^{-}\,, \tag{1}$$

where $z_{pw}^{+} = z_w$ is the displacement of the plan of the axles due to bounce and $z_{pw}^{-} = a_b\theta_w$ the displacement from the pitch movement.

We will then have

$$z_{pw}^{+} = \frac{1}{2}(z_{w1} + z_{w2})\,;\;\; z_{pw}^{-} = \frac{1}{2}(z_{w1} - z_{w2})\,. \tag{2}$$

Should the two axles are moving in phase ($z_{w1} = z_{w2}$), the pitch of the plan of the axles is noticed as not being excited. The plan of the axles will only have a bounce motion.

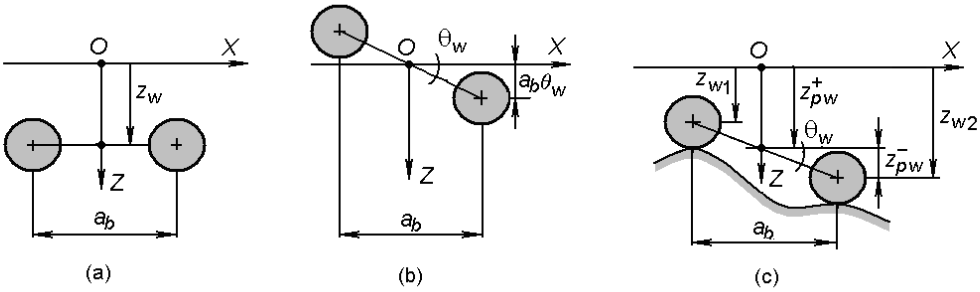

Figure 2. The bounce and pitch of the axles plane.

Figure 3 features the displacements of the plan of the bogie axles due to bounce and pitch. The motions of bounce and pitch in this plan are conveyed via the suspension to the chassis of the bogie, thus triggering its bounce z_b and pitch θ_b. Figure 4 presents the bounce and pitch motions of the bogie.

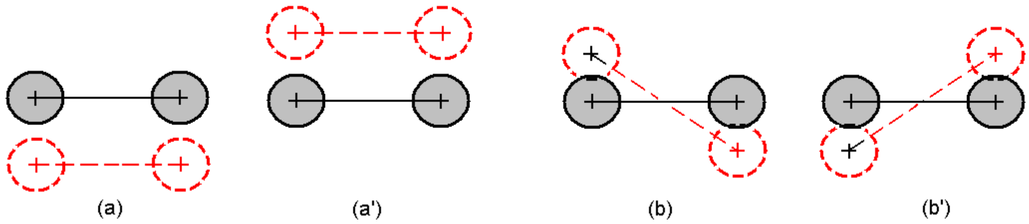

Figure 3. The motion of the plan of the axles: (a) and (a') bounce; (b) and (b') pitch.

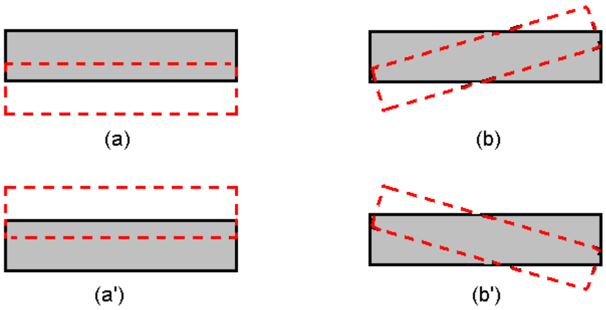

Figure 4. The bogie motions: (a) and (a') bounce; (b) and (b') pitch.

Supposing that $2c_{b1} \neq 2c_{b2}$, the bogie equations of motion are written as:

$$m_b\ddot{z}_b + 2c_{b1}(\dot{z}_b + a_b\dot{\theta} - \dot{z}_{w1}) + 2c_{b2}(\dot{z}_b - a_b\dot{\theta} - \dot{z}_{w2}) + 2k_b[2z_b - (z_{w1} + z_{w2})] = 0\,, \tag{3}$$

$$J_b\ddot{\theta}_b + 2c_{b1}a_b(a_b\dot{\theta}_b + \dot{z}_b - \dot{z}_{w1}) + 2c_{b2}a_b(a_b\dot{\theta}_b - \dot{z}_b + \dot{z}_{w2}) + 2k_b a_b[2a_b\theta_b - (z_{w1} - z_{w2})] = 0\,. \tag{4}$$

The coupled equations (3) and (4) show that the interaction between the bogie vibrations of bounce and pitch are due to the failure of the dampers.

Further on, we have the situation when the axles are moving in phase ($z_{w1} = z_{w2} = z_w$) – the plan of the axles has only a bounce motion. Nevertheless, the bogie pitch is excited, as seen in relation (5):

$$J_b\ddot{\theta}_b + 2c_{b1}a_b(a_b\dot{\theta}_b + \dot{z}_b - \dot{z}_w) + 2c_{b2}a_b(a_b\dot{\theta}_b - \dot{z}_b - \dot{z}_w) + 4k_b a_b^2\theta_b = 0 \tag{5}$$

When neither of the dampers is faulty, as seen earlier, the damping constants are equal ($2c_{b1} = 2c_{b2} = 2c_b$), the bogie equations of motion are decoupled and written as

$$m_b\ddot{z}_b + 2c_b[2\dot{z}_b - (\dot{z}_{w1} + \dot{z}_{w2})] + 2k_b[2z_b - (z_{w1} + z_{w2})] = 0 \tag{6}$$

$$J_b\ddot{\theta}_b + 2c_b a_b[2a_b\dot{\theta}_b - (\dot{z}_{w1} - \dot{z}_{w2})] + 2k_b a_b[2a_b\theta_b - (z_{w1} - z_{w2})] = 0 \quad (7)$$

while for $z_{w1} = z_{w2} = z_w$ the bogie has only a bounce motion.

3. The frequency response functions of the bogie

To underline the dynamic interferences occurring between the vibrations of bounce and pitch of the bogie upon the failure of the damper, the phase shifting in the track vertical irregularities will not be taken into account against the two axles, due to the wheelbase of the bogie. The track vertical irregularities are considered to be in a harmonic shape with the wavelength Λ and amplitude η_0,

$$\eta_1(t) = \eta_2(t) = \eta_0 \cos\omega t \quad (8)$$

where $\omega = 2\pi V/\Lambda$ is the pulsation induced by the track excitation.

Should the bogie response is believed harmonic, with the same frequency as the one induced by the track excitation, the coordinates describing the bogie motions can be under the generic form of

$$p_{1,2}(t) = P_{1,2} \cos\omega t \quad (9)$$

where $P_1 = z_b$, $P_2 = a_b\theta_b$ are the amplitudes of the displacements corresponding to the bogie bounce and pitch.

Thenceforth, the complex associated to the real quantities will be introduced into the equations (3) and (4)

$$\bar{z}_{w1,2}(t) = \bar{\eta}_{1,2}e^{\mathrm{i}\omega t}, \; \bar{p}_{1,2}(t) = \bar{P}_{1,2}e^{\mathrm{i}\omega t} \quad (10)$$

where $z_{w1}(t) = \eta_1(t)$ and $z_{w2}(t) = \eta_2(t)$. A linear system of two non-homogeneous algebraic equations is obtained, whose solution allows the determination of the frequency response functions of the bogie.

The frequency response functions of the bogie are calculated in three reference points, shown in figure 1 as b – at the bogie centre, $w_{1,2}$ – against the suspension corresponding to each axle.

The acceleration response function at the centre of the bogie derives from

$$\bar{H}_{ab}(\omega) = \omega^2 \bar{H}_{z_b}(\omega) \quad (11)$$

and against the suspension of the two axles, the relations to apply are

$$\bar{H}_{aw1,2}(\omega) = \omega^2[\bar{H}_{z_b}(\omega) \pm a_b \bar{H}_{\theta_b}(\omega)] \quad (12)$$

where $\bar{H}_{z_b}(\omega)$ is the response function for the bogie bounce and $\bar{H}_{\theta_b}(\omega)$ is the response function for the bogie pitch.

Further on, the track vertical irregularities are regarded as a stochastic stationary process, which can be described via the power spectral density from relation [16]

$$S(\Omega) = \frac{A\Omega_c^2}{(\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_c^2)} \quad (13)$$

where Ω is the wave number, $\Omega_c = 0.8246$ rad/m, $\Omega_r = 0.0206$ rad/m; $A = 4.032 \cdot 10^{-7}$ radm – for a good quality track; $A = 1.080 \cdot 10^{-6}$ radm – for a low quality track.

Depending on the angular frequency $\omega = V\Omega$, the power spectral density of the track irregularities can be expressed as in the general relation

$$G(\omega) = S(\omega/V)/V \quad (14)$$

From equations (13) and (14), we have the power spectral density of the track vertical irregularities

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$$G(\omega) = \frac{A\Omega_c^2 V^3}{[\omega^2 + (V\Omega_c)^2][\omega^2 + (V\Omega_r)^2]} \tag{15}$$

When starting from the response functions of the bogie acceleration, the equations (11) – (12), the spectrum of the track irregularities and from the relation (15), the power spectral density of the vertical acceleration at the centre of the bogie can be calculated,

$$G_{ab}(\omega) = G(\omega)\left|\overline{H}_{ab}(\omega)\right|^2 = G(\omega)\left|\omega^2 \overline{H}_{z_b}(\omega)\right| \tag{16}$$

and against the suspension of the two axles,

$$G_{aw1,2}(\omega) = G(\omega)\left|\overline{H}_{aw1,2}(\omega)\right|^2 = G(\omega)\left|\omega^2[\overline{H}_{z_b}(\omega) \pm a_b \overline{H}_{\theta_b}(\omega)\right|^2 \tag{17}$$

4. The root mean square of the bogie acceleration

The root mean square of the vertical acceleration in the bogie reference points is calculated based on the dynamic response of the bogie expressed under the form of power spectral density of the acceleration, as follows:

- at the centre of the bogies,

$$a_b = \sqrt{\frac{1}{\pi}\int_0^\infty G_{ab}(\omega)\mathrm{d}\omega} \tag{18}$$

- against the suspension of each axle,

$$a_{w1,2} = \sqrt{\frac{1}{\pi}\int_0^\infty G_{aw1,2}(\omega)\mathrm{d}\omega} \tag{19}$$

5. The numerical study

This section deals with the results from the numerical simulations regarding the influence of the interference of the bounce and pitch vibrations coming from the failure of a damper in the suspension of an axle over the dynamic behaviour of the bogie in the railway vehicle. The dynamic behaviour of the bogie is evaluated within the frequency response functions and of the acceleration root mean square, calculated in the bogie reference points, for different cases of reducing the damping constant of the suspension in axle 1 (c_{b1}) versus the reference value (c_b).

The parameters of the bogie used in the numerical simulations are shown in table 1.

Table 1. Parameters of numerical simulation.

Bogie mass	m_b = 3200 kg
Bogie wheelbase	$2a_b$ = 2.56 m
Inertia moment	$J_b = 2.05 \cdot 10^3$ kg·m^2
Elastic constant of the suspension	k_b = 1.10 MN/m
Damping constant of the suspension	c_b = 13.05 kNs/m

Figure 5 displays the response functions of the acceleration corresponding to the bogie bounce and pitch, calculated as below

$$\overline{H}_{az_b}(\omega) = \omega^2 \overline{H}_{z_b}(\omega), \ \overline{H}_{a\theta_b}(\omega) = \omega^2 \overline{H}_{\theta_b}(\omega). \tag{20}$$

It is marked out that the bogie pitch is not excited (figure 5, (b)) when the damping constants are equal, but the bogie has only a bounce motion (figure 5, (a)). The response function of the acceleration correlated with the bogie bounce is maximum at the resonance frequency, i.e. 5.9 Hz.

The damping constant is reduced to half in relation to the reference value ($c_{b1} = c_b/2$) due to the failure in the damper of the suspension of the front axle. In this context, the bogie pitch is excited, as seen in figure 5, (b). The response function of the acceleration corresponding to the bogie has the highest value at the resonance frequency, which is 9.4 Hz. The maximum influence of the reduction in the damping constant upon the response function matching the bogie bounce is visible at the resonance frequency by an increase in $\overline{H}_{az_b}$. At sub-critical frequencies, the influence is negligible, yet there is a decrease in the response function of the acceleration correlated with the bogie bounce for $c_{b1} = c_b/2$ that manifests for frequencies higher than 8.7 Hz.

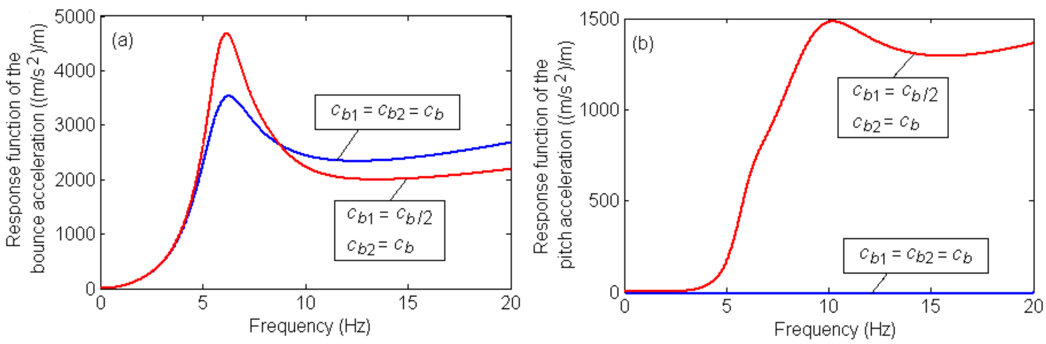

Figure 5. The response functions of the acceleration: (a) bogie bounce; (b) bogie pitch.

Figure 6 shows the response functions of the acceleration in the bogie reference points. Should the dampers have equal damping constants (figure 6, (a)), the bogie has only a bounce motion, both at its centre and against the axles. A reduction in the damping constant in the suspension of the front axle, the bogie pitch is excited, which can be visible in the bogie response against the two axles that is not symmetrical (figure 6, (a)). The level of vibrations in the bogie rises, mainly at the resonance frequencies of the bounce and pitch vibrations.

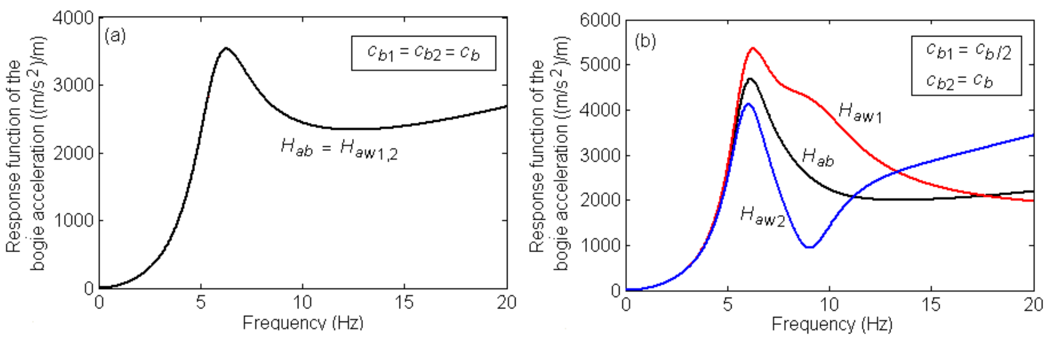

Figure 6. The acceleration response functions in the bogie reference points.

Figure 7 presents the power spectral density of the vertical acceleration in the bogie calculated at the bounce resonance frequency (5.9 Hz) in the reference points located against the suspension correlative with each of the two axles. The reduction of the damping constant in the suspension of the front axle is noticed to lead to the amplification of the bogie response above both axles. The increase in the power spectral density of the acceleration is significant, mainly at high velocities. For instance, this power spectral density of the acceleration goes up from 0.11 $(m/s^2)^2/(1/s)$ to 0.25 $(m/s^2)^2/(1/s)$ – for $c_{b1} = c_b/2$, and to 0.72 $(m/s^2)^2/(1/s)$ – for $c_{b1} = 0$, in the reference point w_1, at velocity of 200 km/h.

In the reference point w_2, the power spectral density of the acceleration rises from 0.11 $(m/s^2)^2/(1/s)$ to 0.16 $(m/s^2)^2/(1/s)$ – for $c_{b1} = c_b/2$, and to 0.40 $(m/s^2)^2/(1/s)$ – for $c_{b1} = 0$.

Figure 8 features the power spectral density of the bogie vertical acceleration calculated at the pitch resonance frequency (9.4 Hz) in the reference points located against the suspension corresponding to each of the two axles. In the reference point w_1, the power spectral density of the acceleration has a uniform increase along with the decrease of c_{b1}. Alternatively, for the reference point w_2, the power spectral density of the acceleration goes down by lowering the c_{b1} to a certain value. Once this value of the damping constant is exceeded, G_{aw2} starts going up. As an example, at velocity of 200 km/h, G_{aw2} decreases from 0.015 $(m/s^2)^2/(1/s)$ – for $c_{b1} = c_b = 13.05$ kNs/m, to 0.0018 $(m/s^2)^2/(1/s)$ – for $c_{b1} =$ 5 kNs/m, then increases to 0.029 $(m/s^2)^2/(1/s)$ – for $c_{b1} = 0$.

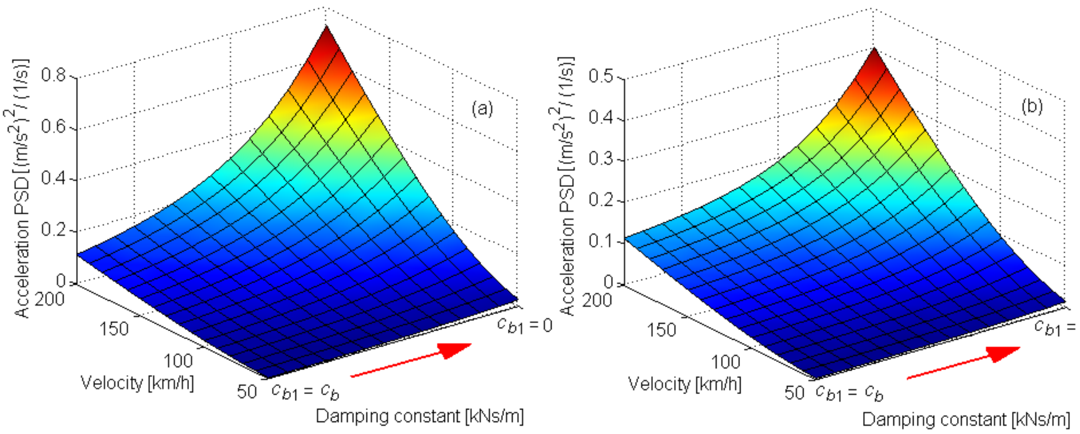

Figure 7. Power spectral density of the bogie acceleration at the bounce resonance frequency: (a) in the reference point w_1; (b) in the reference point w_2.

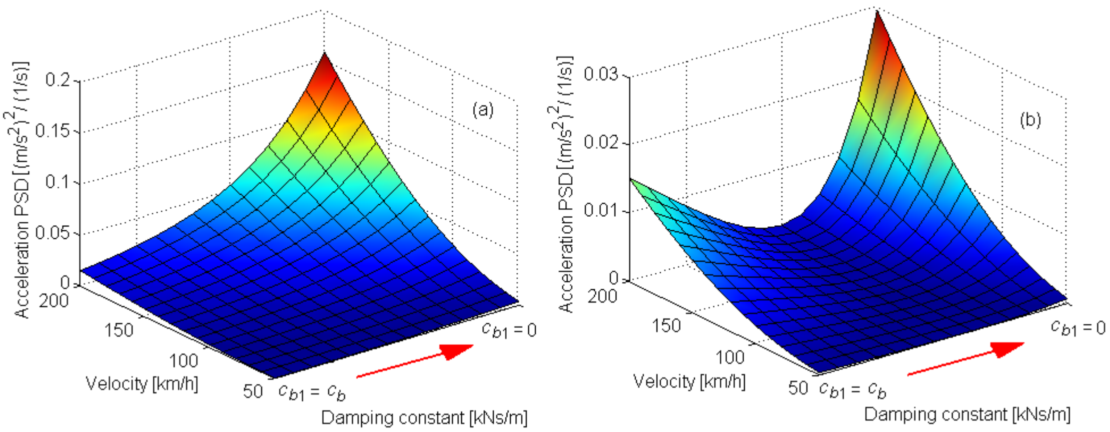

Figure 8. Power spectral density of the bogie acceleration at the pitch resonance frequency: (a) in the reference point w_1; (b) in the reference point w_2.

Figure 9 shows the root mean square of the vertical acceleration in the three reference points of the bogie for velocities ranging from 60 to 200 km/h and different values of the damping constant of the suspension in the front axle. The reduction of the damping constant c_{b1} is noticed to determine a general increase in the level of vibrations in the bogie; the acceleration rises in all the reference points of the bogie. The highest growth is visible in the reference point located above the suspension in the front axle and the lowest in the suspension in the rear axle. For instance, a_b rises by 62%, a_{w1} by 134%, and a_{w2} by 49% at the velocity of 200 km/h, should the damping constant decreases from the reference value ($c_{b1} = c_b = 13.05$ kNs/m) to $c_{b1} = 0$.

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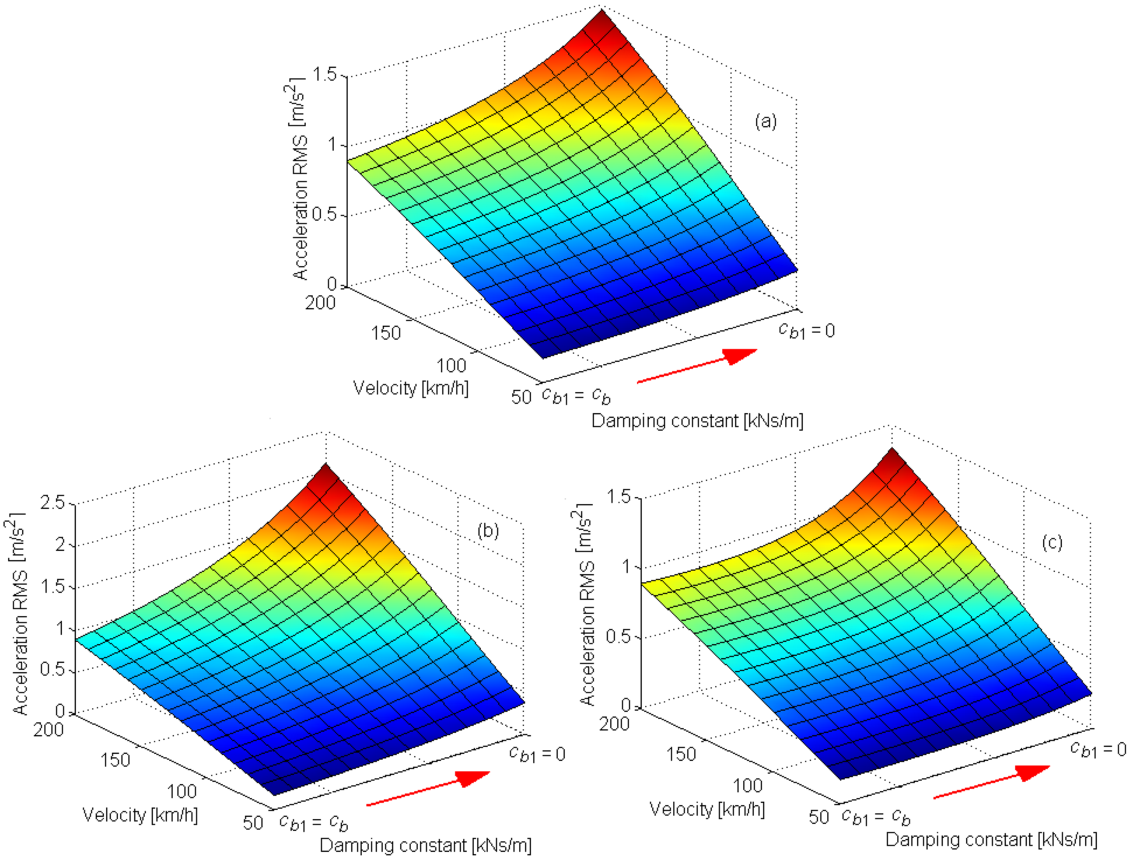

Figure 9. Root mean square of the vertical acceleration:
(a) in the reference point b; (b) in the reference point w_1; (c) in the reference point w_2.

6. Conclusions

This paper examines the dynamic behaviour of a two-axle bogie during running in a track with vertical irregularities, when considering the particular situation of a failure in a damper in the suspension of one of the axles.

The damper failure is simulated by the reduction in the damping constant in relation to the reference value. Based on both the analytical results and the ones derived from numerical simulations, the bounce and pitch vibrations in the bogie have been shown to be coupled herein. The dynamic interferences between the two vibrations will trigger a higher level of the vibrations in the bogie, a fact that is visible in the frequency response functions calculated in three reference points of the bogie – at the centre and against the suspensions corresponding to the two axles. It has been thus shown that the reduction in the damping constant has a significant influence on the bogie response in all the reference points, mainly at the resonance frequencies of the bounce and the pitch. Similarly, the results concerning the power spectral density of the vertical acceleration have proven that the higher the velocity, the more important the amplification of the regime of vibrations in the bogie. The highest vertical accelerations of the bogie are recorded at high velocity, against the suspension with the failed damper.

Acknowledgments
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