

Upon the relationship between rolling friction and sliding friction

S Alaci¹, F C Ciornei¹, I C Romanu¹ and M C Ciornei²

¹Stefan cel Mare University of Suceava, Department of Mechanics and Technologies, Universitatii Street, No. 13, 720229 Suceava, Romania

²Carol Davila University of Medicine and Pharmacy, Bucharest-Romania, Department Clinic 2, Blvd. Eroii Sanitari, No. 8, 050474, Bucharest, Romania

E-mail: alaci@fim.usv.ro

Abstract. Whereas for sliding friction the Amonton-Coulomb law clearly states the proportionality between the friction force and the normal force, the rolling friction torque and normal force dependency is assumed linear in some references and nonlinear in others. The coefficient of rolling friction can be obtained using various pendula, based on the assumption of linear dependence between the rolling friction torque and normal force. The theoretical models lead to a linear decrease of angular amplitude (confirmed experimentally for the dry sliding friction case) while the experimental damping of amplitude is nonlinear for any of the employed pendula. The basis for finding the rolling friction coefficient is the equality between the theoretical and experimental slopes of the decreasing angular amplitude of the pendulum. Due to this nonlinearity, the value of the coefficient of rolling friction depends on the launching amplitude of the pendulum. In the present paper the equation of motion of the evolvent pendulum is obtained based on the hypothesis that the dependency of resistant torque on normal force is a power law. Experimental tests performed with the new heavy pendulum have confirmed a nonlinear, exponential attenuation of angular amplitude. Thus, finding the coefficient of rolling friction for the least favourable situation was aimed. This happens when the launching amplitude of the pendulum has the highest possible value and occurs when the ratio between the tangential force and the normal force equals the coefficient of sliding friction. A technique is proposed for obtaining the critical value of the coefficient of rolling friction.

1. Introduction

The problem of accurate estimation of the rolling friction (RF) characteristics is particularly important in the situation when the relative angular velocity between contacting elements has considerable values since the lost mechanical power is proportional to this velocity. The issue is stringent when the assembly has a considerable number of parts. A specific example is met in the gearbox of the wind turbines (WT); from studies it resulted statistically that this is the less feasible assembly of the WT [1-3]. But, RF is present both in the teeth of the gears and in the rolling bearings met in WT. The study of the RF is more difficult than the one regarding the sliding friction (SF). If for the SF the Amonton-Coulomb law [4] undoubtedly states that the friction force is proportional to the normal force, the dependency of the RF torque on the normal force is supposed linear in some references [5] or nonlinear in others [6]. Another difference between the two types of friction consists in the fact that while the SF happens in any type of contact, the RF occurs only in nonconforming contacts. In a



recent paper [7] a device and method for finding the coefficient of rolling friction (C_{RF}) using an evolvent pendulum were proposed. The fundament of the scheme consists in imposing the demand that the theoretical angular amplitude, obtained after integrating the differential equation of motion of the pendulum, must decrease in time likewise the experimental amplitude, found using a device designed and assembled in the laboratory. The construction of the test-rig is extremely simple as it consists in an aluminum rod with square cross-section brought into contact to the side surface of a steel disc. The rod is initially in horizontal position and afterward is set into oscillations. Assuming that pure rolling is in the contact point between the rod and the cylinder, the points from the rod describe a family of evolvents [8] and so it is given reason for the name of the method. The researches were carried on and different contact curves between the body of the pendulum and the support, resulting from combinations of internal or external cylindrical surfaces and plane surfaces were used. There were intended methods for obtaining the C_{RF} based on oscillatory motions such as cycloidal [9], pericycloidal [10] or hypocycloidal [11]. From the above works, it was identified a discrepancy between the experimental data and theoretical results consisting in the fact that, for dry friction conditions, the theoretical analysis [12] stipulate a linear attenuation of oscillations amplitude with time while the experimental records deviate less or more from this form. As example, in figure 1 there are presented the experimental data obtained in [11] for the hypocycloidal oscillations executed by a bearing ball inside a steel cylinder.

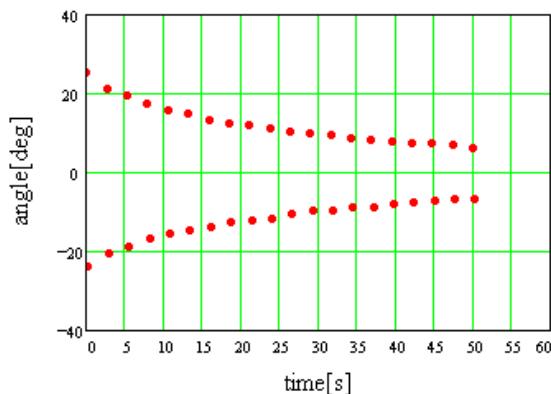


Figure 1. Angular amplitude variation with time for hypocycloidal oscillations performed by a bearing ball inside a steel cylinder [11].

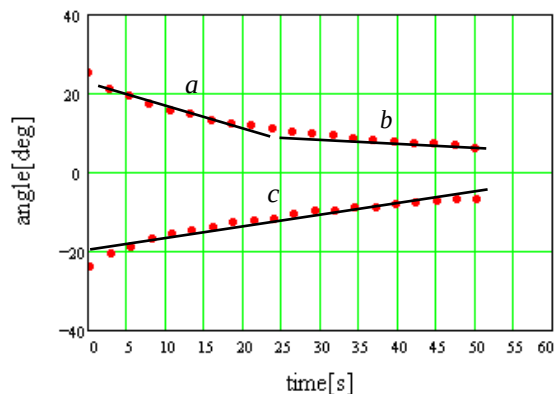


Figure 2. Possible cases of choosing the characteristic experimental data.

The quasi-exponential shape of the variation of swinging angular amplitude creates difficulties when the interpolation with the theoretical amplitude decrease is intended. It is obvious that the result depends on the time period of the experimental test because, for short intervals, it can be considered a quasilinear decrease of amplitude. For a long time period, case met especially for hard metals that require a large number of oscillations to proof the damping phenomenon, the experimental data shape is similar to the one from figure 1 and the interpolation manner is the subject. Referring to the data from figure 1, in figure 2 there are presented the interpolations with theoretical results for three hypothesis: a) the results from the beginning of the motion are the ones to be considered for interpolation; b) the data from the ending of the oscillations are relevant; c) all data are equally important and first, the straight line that best interpolates the data is found and afterward it is imposed that the angle of this line is matching the tilt of theoretical model. It is obvious that the values of the C_{RF} are different for the three situations presented in figure 2. Considering the aspects revealed above, the identification of the cause producing the shape of the experimental curve was attempted. Two possibilities were identified:

- a first source could be the use of the premise that, similar to SF, the RF torque is proportional to the normal force, assumption that is challenged by a series of recent papers [6], [13];

- the second cause to be considered is the effect of the friction between pendulum and air; for a pendulum with small mass, this friction may play the role of viscous damping with an exponential decrease of amplitude. Consequently, the manner these two hypotheses are verified is presented next.

2. The equation of motion of the nonlinear evolvent pendulum

An evolvent pendulum is presented in figure 3 where the contact curves are the immobile base circle of radius r_b and the mobile straight line Δ and the contact point is C characterized by pure rolling. The center of mass (CM) is placed on the axis of symmetry of the pendulum beneath the contact point at the distance ξ . The theorem of motion of the CM and the moment of momentum theorem with respect to the CM are applied in order to obtain the equation of motion.

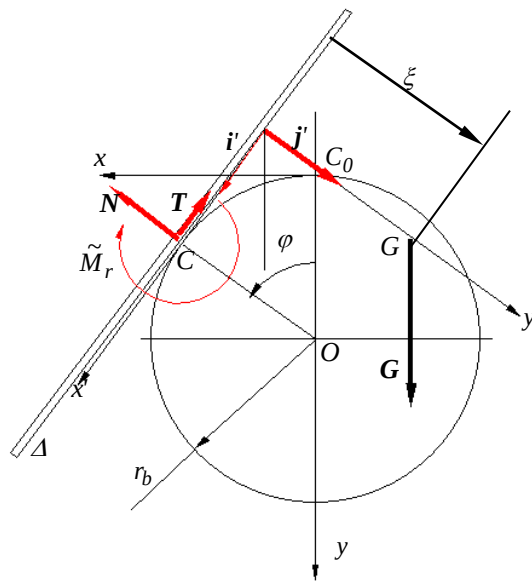


Figure 3. Schematic of the evolvent pendulum.

To this end, two frames are chosen: the fixed system $Oxyz$ with versors $\mathbf{i}, \mathbf{j}, \mathbf{k}$ and a mobile reference system, with versors $\mathbf{i}', \mathbf{j}', \mathbf{k}' \equiv \mathbf{k}$ with the origin in the initial contact point C_0 . The following relations are applicable:

$$\mathbf{i}' = \mathbf{i} \cos \varphi + \mathbf{j} \sin \varphi \quad (1)$$

$$\mathbf{j}' = -\mathbf{i} \sin \varphi + \mathbf{j} \cos \varphi$$

The position vector of the CM of the pendulum, expressed in the fixed frame is obtained by projecting the vector relation:

$$\mathbf{r}_G = \overline{OC} + \overline{CC_0} + \overline{C_0G} \quad (2)$$

and the respective coordinates are:

$$x_G = r_b \sin \varphi - r_b \varphi \cos \varphi - \xi \sin \varphi \quad (3)$$

$$y_G = -r_b \cos \varphi - r_b \varphi \sin \varphi + \xi \cos \varphi$$

In the relations (3) the pure rolling condition from the ball-plane contact was considered, expressed as:

$$C_0C = r_b \varphi \quad (4)$$

By differentiating the relations (3) twice with respect to time the components of the acceleration of the CM are obtained:

$$a_{Gx} = (-\xi \cos \varphi + r_b \varphi \sin \varphi) \varepsilon + (r_b \sin \varphi + \xi \sin \varphi + r_b \varphi \cos \varphi) \omega^2 \quad (5)$$

$$a_{Gy} = (-\xi \sin \varphi - r_b \varphi \cos \varphi) \varepsilon + (-r_b \cos \varphi + \xi \cos \varphi + r_b \varphi \sin \varphi) \omega^2$$

where:

$$\omega = \dot{\varphi}, \quad \varepsilon = \ddot{\varphi} \quad (6)$$

represent the angular velocity and the angular acceleration of the pendulum, respectively. The forces acting upon the pendulum are the weight G , the normal reaction N and the force T tangent to the base circle. The theorem of motion of the CM is written under the form:

$$M \ddot{\mathbf{r}}_G = -N\mathbf{j}' - T\mathbf{i}' + Mg\mathbf{j} \quad (7)$$

where M is the mass of the pendulum and g is gravitational acceleration. The equation (7) projected on the axes of the mobile frame give the magnitudes of N and T reactions from the contact point:

$$N = M(\varepsilon r_b \varphi + \omega^2 \xi + g \cos \varphi) \quad (8)$$

$$T = M(\varepsilon \xi - r_b \varphi \omega^2 + g \sin \varphi)$$

Obviously, these reactions depend merely on the law of motion of the pendulum, $\varphi = \varphi(t)$.

The moment of momentum theorem applied with respect to the CM of the pendulum leads the following vector equation:

$$J_z \varepsilon \mathbf{k} = [\overline{GC} \times (-N\mathbf{j}' - T\mathbf{i}') - M_r \operatorname{sgn}(\omega) \mathbf{k}] \quad (9)$$

The RF moment is assumed as in [3] to obey a power law of the form:

$$M_r = s_\beta N^\beta \quad (10)$$

where s_β is a coefficient (since the product is a torque, dimension of s_β depends on the exponent β).

$$J_z \varepsilon + \left[\xi^2 + r_b^2 \varphi^2 \right] \varepsilon + (\xi \sin \varphi + r_b \varphi \cos \varphi) M + s_\beta \left\{ \left[\varepsilon \varphi r_b + (r_b + \xi) \omega^2 + g \cos \varphi \right] M \right\}^\beta \operatorname{sgn}(\omega) = 0 \quad (11)$$

The equation (11) is not cannot be solved with respect to ε . Considering that the amplitudes of the oscillations are small, the parenthesis from the last term is expanded in a power series with respect to ε and only the first two terms are kept. A linear equation is therefore obtained and the unknown ε is found under the form:

$$\varepsilon = - \frac{(\xi \sin \varphi + r_b \varphi \cos \varphi) g + r_b^2 \varphi \omega^2 + s_\beta M^{\beta-1} \left[\omega^2 (r + \xi) + g \cos \varphi \right]^\beta \operatorname{sgn} \omega}{\frac{J_z}{M} + (r_b^2 \varphi^2 + \xi^2) + s_\beta \beta r_b \varphi M^{\beta-1} \left[\omega^2 (r + \xi) + g \cos \varphi \right]^{\beta-1} \operatorname{sgn} \omega} \quad (12)$$

The equation (12) is the differential equation of motion of the evolvent pendulum with power law damping. To solve this nonlinear differential equations [16] a numerical algorithm is required and the Runge-Kutta 4 methodology [17] was applied here. The equation (12) can be evidently applied for the particular case when the RF moment M_r and the normal force are related by a linear dependency expressed by means of the $C_{RF} = s_r$ and takes the particular form:

$$\varepsilon = - \frac{(\xi \sin \varphi + r \varphi \cos \varphi)g + r_b^2 \varphi \omega^2 + s_r \left| \omega^2 (r + \xi) + g \cos \varphi \right| \operatorname{sgn} \omega}{\frac{J_z}{M} + (r_b^2 \varphi^2 + \xi^2) + s_\beta \beta r_b \varphi \operatorname{sgn} \omega} \quad (13)$$

The equation (13) was integrated for $M = 2 \text{ Kg}$, $s_r = 10^{-4} \text{ m}$, $J_z = 0.03 \text{ Kg} \cdot \text{m}^2$ and $\theta_0 = 10^\circ$ - the initial amplitude. The variations of the angular displacement and normal reaction are represented in figure 4 and figure 5 respectively.

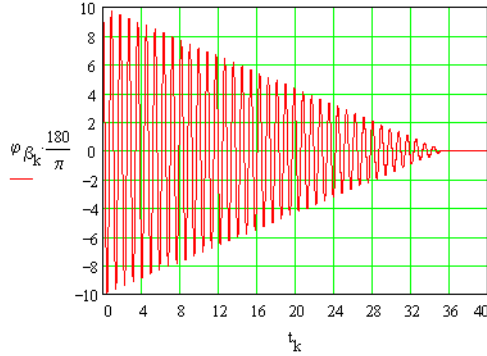


Figure 4. Variation of the angular displacement of the pendulum.

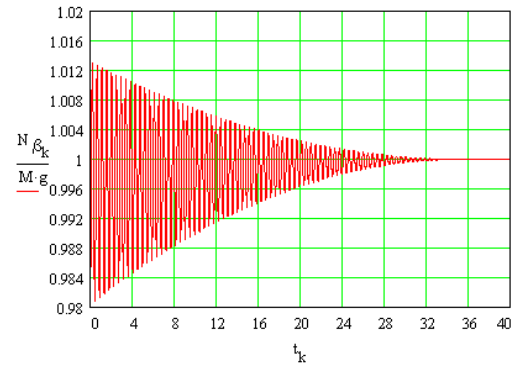


Figure 5. Variation of normal reaction.

From figure 4 it is confirmed that for a nonlinear dependency expressed as power law between the RF torque and the normal force the amplitude damping is linear, the shape of the experimental curve cannot be explained by the power law relationship. Another observation resulting from figure 4 is that for initial amplitude values smaller than 10° the normal force and the pendulum weight are vaguely different, so the normal force can be considered constant. This assumption allows for finding the correlation between the $C_{RF} = s_r$ and the coefficient s_β from relation (10).

$$s_r M g = s_\beta (M g)^\beta \quad (14)$$

The equations of motion (12) and (13) were integrated applying the condition (14) and it was found that all characteristic parameters have quasi-identical values. The displacements of the pendulum for the two hypothesis, $\varphi(t)$ for linear and $\varphi_\beta(t)$ for nonlinear dependency of the moment of RF are represented comparatively in figure 6. The relative error of the moment of RF for linear law versus the moment for the resting pendulum is presented in figure 7.

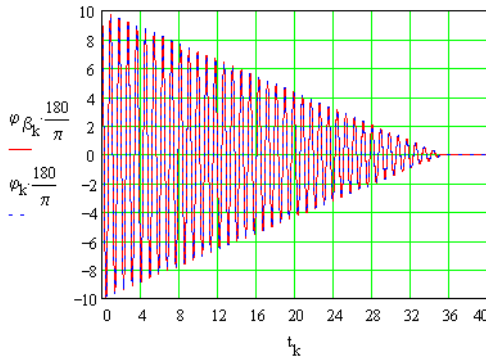


Figure 6. Comparison between the angular displacement of the pendulum for linear and non-linear law.

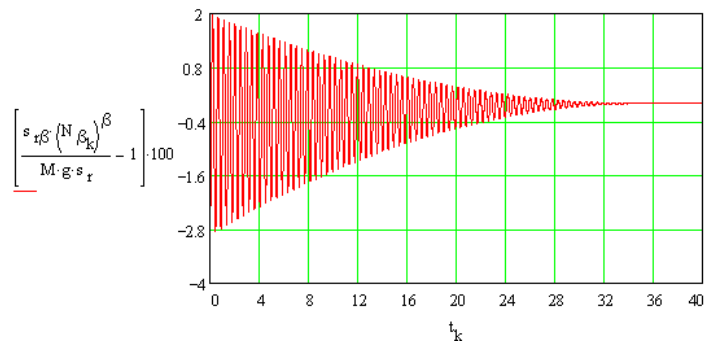


Figure 7. Variation of the RF torque for non-linear law compared to the static case.

3. The experimental set-up

A pendulum with large mass was used in order to verify if the friction with air affects the deviation from theoretical curve of the angular amplitude damping.

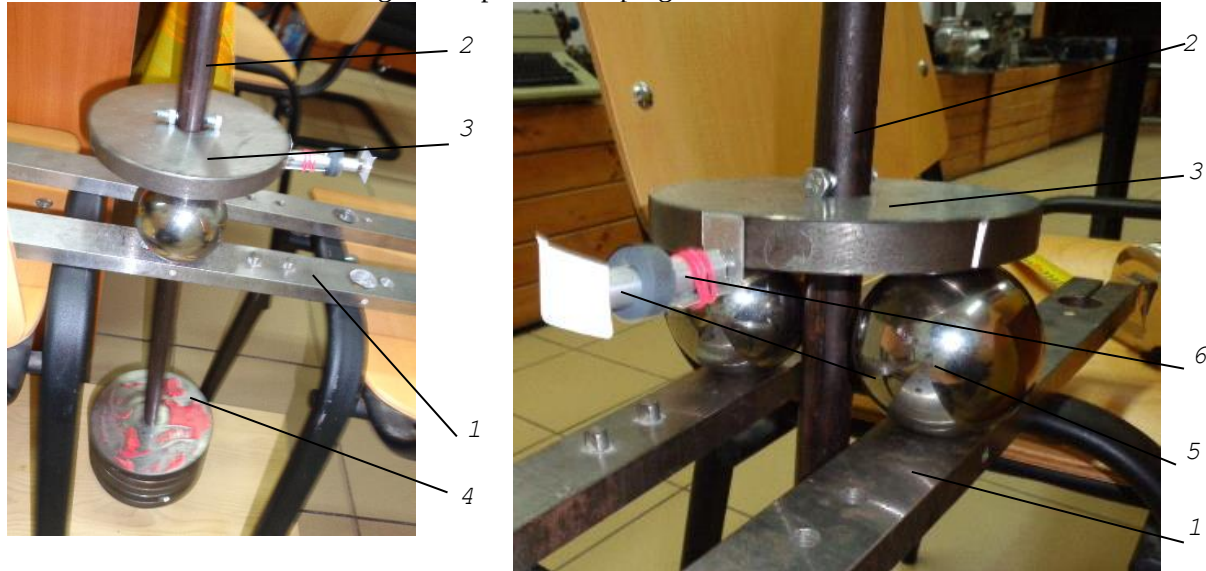


Figure 8. Experimental set-up.

The experimental device consists in two fixed metallic rods, 1. The pendulum is constructed from a metallic bar with five identical cylinders assembled at the lower end. The cylinders have the following characteristics: diameter $D=0.21m$; height $H=0.016m$ and mass $M_D=1.85kg$. Two holes are made in the rods 1 and bearing balls 5 of radius $r_b=0.03m$ are positioned in these; so, the base cylinder of the evolvent pendulum is obtained. At the upper end of the bar a cylinder 3 is attached, identical to the ones from the lower end that will come into contact to the balls with the inferior front face that materializes the mobile plane of the pendulum. The pendulum has the total mass $M=12,133kg$ and the moment of inertia $J_z=0.379kg \cdot m^2$. A laser stick is fixed on the contact face of the contact cylinder. The displacement of the laser beam on a ruler is correlated to the motion of the pendulum. As in previous works [9-10] the study of the motion of the pendulum was made by recording with a camera and subsequently splitting the film into frames for image analysis. The experimental data are presented in figure 9. Both the balls and the face of the contact cylinder are heat treated surfaces and also very well finished and thus the RF is especially reduced. For the results from figure 3 the oscillations were recorded for an interval of 15 minutes, the period of oscillation of the pendulum being $0.6s$.

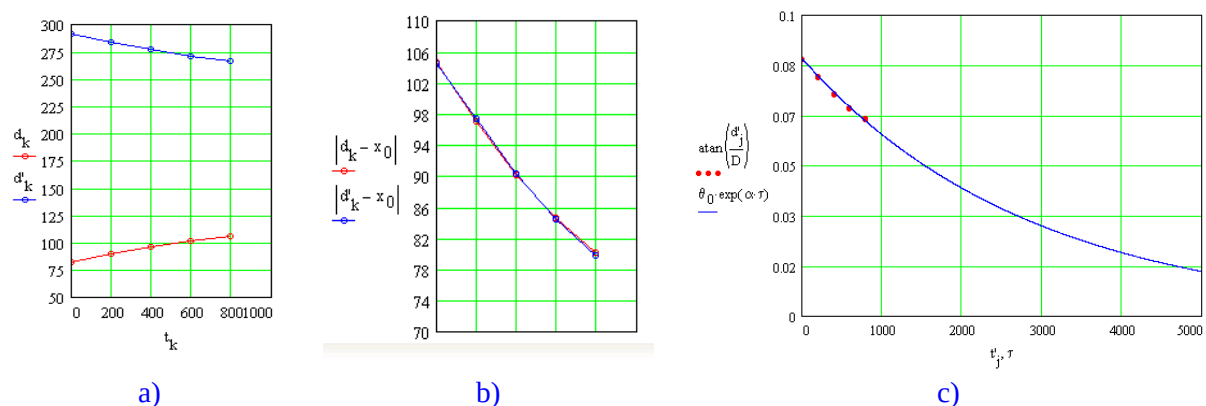


Figure 9. Finding from experimental data the law of damping of the oscillation amplitude.

The displacements of the laser spot on a screen, placed at a distance $D = 1220$ mm from the initial point of contact ball-cylinder, are presented in figure 9a. The position x_0 of the center of oscillation presented in figure 9b is found by imposing the minimization condition for the function:

$$H(x_0) = \sum_k (d_k - x_0)^2 \quad (15)$$

where k is the total number of experimental data points. Assuming for theoretical amplitude decrease an exponential law having the form:

$$\theta(t) = \theta_0 \exp(\alpha t) \quad (16)$$

and by applying the least squares method [15] the value of logarithmic decrement α is obtained. For the present case, it results:

$$\alpha = -3.498 \cdot 10^{-4} \text{ sec}^{-1} \quad (17)$$

and this value was utilized in plotting the interpolation curves of experimental amplitudes from figure 9c.

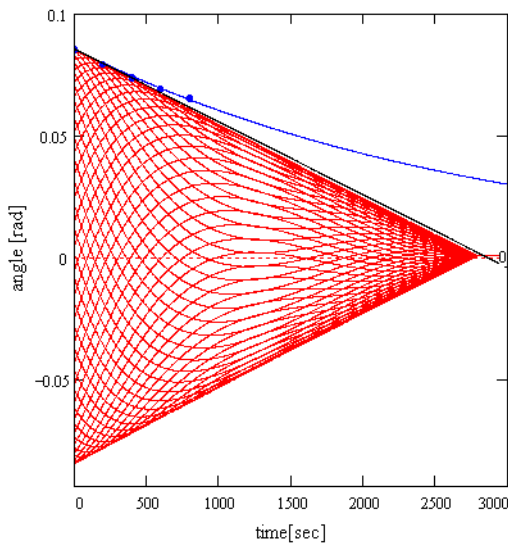


Figure 10. Variation of experimental amplitude, relation (16) and theoretical displacement of pendulum according to equation (12).

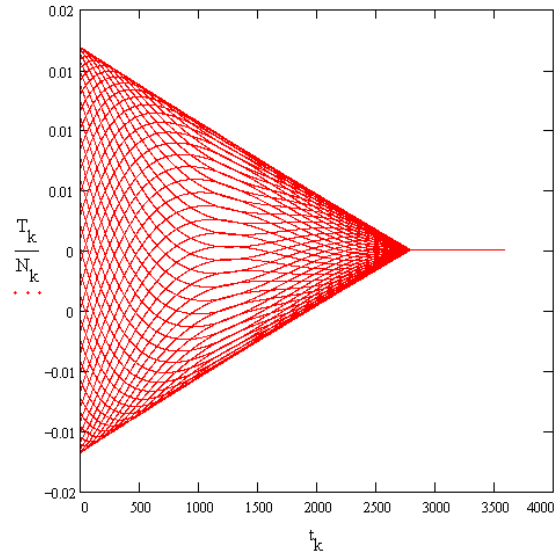


Figure 11. Testing the condition of pure rolling.

The experimental amplitude expressed by relation (16) is plotted in figure 10 alongside the theoretical displacement according to relation (12). It can be observed that the theoretical curve decreases according to the slope of the tangent to the experimental curve traced for the initial moment of the motion. Obviously, this situation leads to the greatest value of the C_{RF} since for any consequent instant the slope of the experimental curve is smaller. From here it results the conclusion that the initiation amplitude will control the value of the C_{RF} obtained based on the principle from figure 10. The magnitude of the maximum starting amplitude can be found imposing the pure rolling condition:

$$\frac{|T|}{N} \leq \mu \quad (18)$$

where μ is the coefficient of dynamic friction between the ball and the cylinder. From figure 11 it can be remarked that the condition (18) is satisfied and significantly exists the possibility of increasing the

initial amplitude θ_0 , taking into account that for the contact between two parts made of hardened steel the coefficient of SF has the value $\mu \cong 0.6$. Evidently, if the launching is made with a different starting amplitude the damping of the experimental amplitude will have the same logarithmic decrement α . The conclusion resulting is that *for an evolvent pendulum the coefficient of sliding friction μ directly establishes the value of the initial amplitude θ_0 of an oscillation performed under pure rolling conditions. On its turn, the initial value of the oscillation amplitude θ_0 determines the value of the maximum coefficient of rolling friction $s_{r\max}$ that may exist between the two surfaces.*

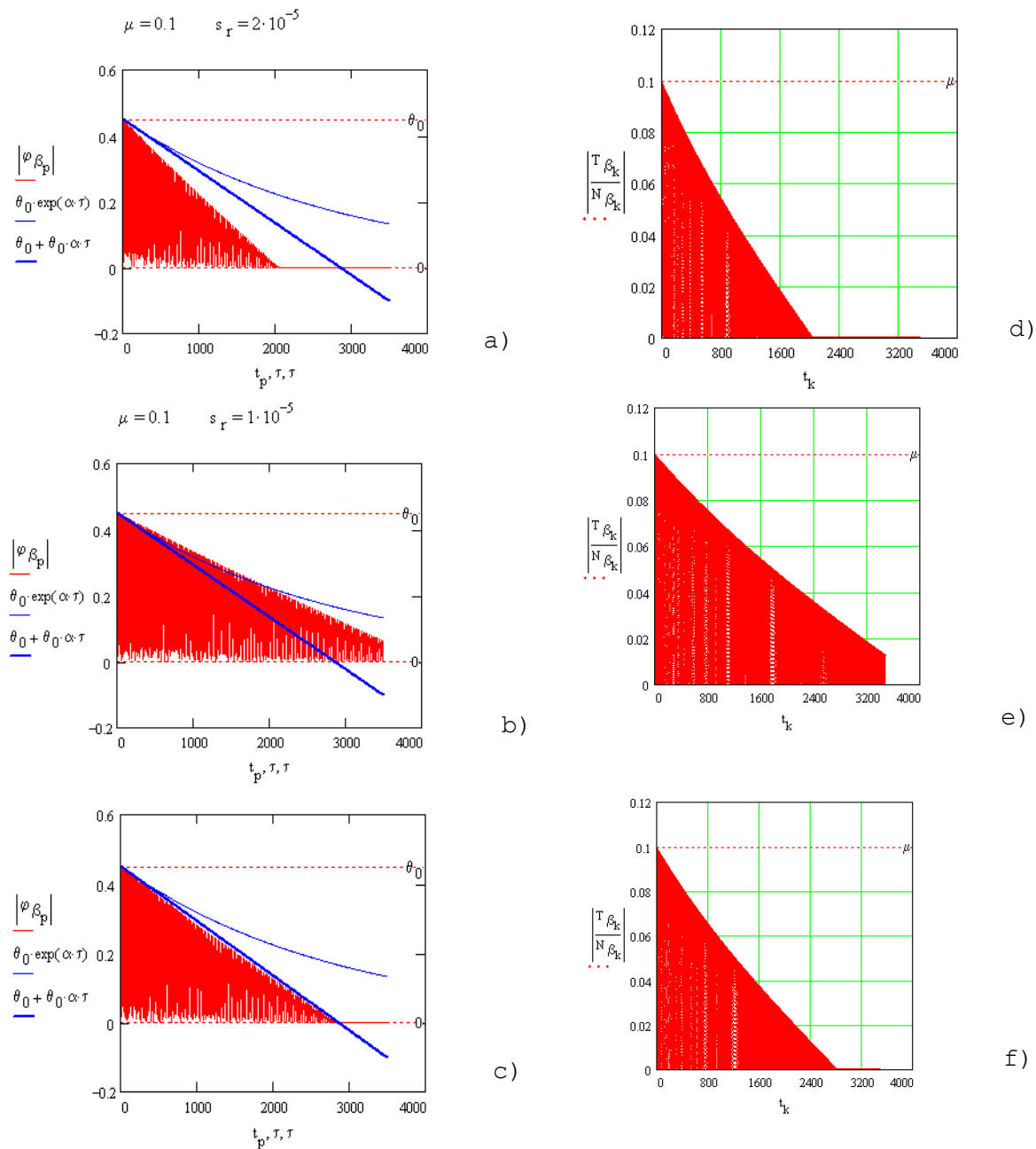


Figure 12. Finding the optimum value of s_r from a set of values for imposed μ (SFC).

For an actual case the following steps have to be made: beginning angular amplitudes θ_{0slide} are set as to facilitate sliding occurrence; after integration of the equation (13) the moment when the condition

(18) is satisfied can be found and therefore the amplitude θ_0 corresponding to the installing of pure-rolling regime for the ball-plane contact; the graph from figure 10 is traced and most probable, the tangent in the origin to the experimental curve will not envelope the theoretical curve of pendulum displacement variation. The algorithm is repeated for a series of values, of the C_{RF} (as in figure 12) until the value ensuring the condition of superposition between the tangent and the envelope of the curve is fulfilled. Considering a set of values for the coefficient of rolling friction s_r (figure 12 a, b, c) there are presented all possible situations: for a coefficient greater than actual (a), smaller (b) and fitted for $s_r = 1.45 \cdot 10^{-5} m$ in (c). For these values, in figure 12 (d,e,f) the pure rolling condition is verified at limit, at the beginning of the motion (the value $\mu = 0.1$ of the ratio T / N).

4. Conclusions

In a recent paper the C_{RF} was found using the evolvent pendulum with the supposition of linear dependence between the RF torque and the normal force. Other types of pendula (cycloidal, pericycloidal) used in related works led to the idea that the theoretical models conduct to linear decrease of angular amplitude but the experimental tests proved nonlinear amplitude decrease for any of the employed pendulum.

The fundament for all the proposed methods for finding the C_{RF} is the equality between theoretical and experimental slopes of angular amplitude diminishment. Due to the nonlinear variation of amplitude damping the value of the C_{RF} depends on the launching amplitude of the pendulum.

Taking into account that the damping torque of the pendulum is the result of the complex interaction between RF, adhesion forces and hysteresis from contact to which the friction with air is added, in the present work the equation of motion of the evolvent pendulum was obtained assuming power law dependence between the resistant torque and the normal force. In order to reduce the effect of friction with air that conducts to a nonlinear attenuation of angular amplitude due to fluid friction characteristics, a pendulum with large mass ($\approx 14kg$) was used.

All test performed with the new pendulum confirmed the nonlinear amplitude damping, moreover, an exponential shape of amplitude attenuation. In this situation, the problem of finding the C_{RF} for the most unfavorable case came about. This happens when the initial amplitude of the pendulum takes the maximum possible value. And this happens when the ratio between the tangential force from contact and the normal force is equal to the CSF, $T / N = \mu$. Establishing from the above condition the value of the critical beginning amplitude and assuming a constant logarithmic decrement of damping, equal to the one found by experimental means, the critical value of the C_{RF} is obtained by equalling the slope of the exponential damping for the moment of launching with critical amplitude, to the slope of the theoretical linear decrease of amplitude.

Acknowledgment

This work was supported by a grant of the Romanian Ministry of Research and Innovation, CCCDI – UEFISCDI, project number PN-III-P1-1.2-PCCDI-2017-0404 / 31PCCDI/2018, within PNCDI III

5. References

- [1] M D Reder, E Gonzalez and J J Melero 2016 Wind Turbine Failures - Tackling current Problems in Failure Data Analysis The Science of Making Torque from Wind (TORQUE 2016) *IOP Publishing Journal of Physics: Conference Series* **753** 072027
- [2] Yagi S 2004 Bearings for Wind Turbine *NTN Technical Review* **71** 1-8
- [3] Oyague F 2009 Gearbox Modeling and Load Simulation of a Baseline 750-kW Wind Turbine Using State-of-the-Art Simulation Codes *Technical Report National Renewable Energy Laboratory NREL/TP-500-41160* Colorado
- [4] Marques F, Flores P, Pimenta Claro J C and Lankarani H M 2016 A survey and comparison of several friction force models for dynamic analysis of multibody mechanical systems *Nonlinear Dynamics* **86** 1407–1443

- [5] Voinea R, Ceausu D and Voiculescu V 1983 *Mecanica*, EDP Publishing House, Bucharest
- [6] Cherepanov G P 2014 Theory of roling: Solution of the Coulomb problem *J Appl Mech Tech Phys* **55**(1) 182–9
- [7] S Alaci S, Ciornei F C, Ciogole A and Ciornei M C 2017 Estimation of coefficient of rolling friction by the evolvent pendulum method, Innovative Ideas in Science 2016, *IOP Conf. Series: Materials Science and Engineering* **200** 012005
- [8] Handra-Luca V and Stoica I A 1982 *Introducere in teoria mecanismelor* Vol I, (Dacia Publishing House, Cluj-Napoca)
- [9] Ciornei MC, Alaci S, Ciornei F C and Romanu IC 2017 A method for the determination of the coefficient of rolling friction using cycloidal pendulum, *IOP Conf. Series: Materials Science and Engineering* **227** 012027
- [10] Alaci S, Ciornei F C, Bujoreanu C Ciornei M C and Acinte I L Finding the coefficient of rolling friction using a pericycloidal pendulum, accepted for *ACME 2018 International Conference, Iasi*
- [11] Siretean S T, Musca I, Alaci S, Ciornei F C 2017 Use of Hypocycloidal Motion in the Study of Rolling Friction, *Syrom 2017 International Symposium IFTOMM Iasi* (in press)
- [12] Rouillard V 2003-2013 *Mechanical Vibrations – Single Degree-of-Freedom systems* (Victoria University Melbourne Australia)
- [13] Cherepanov G P 2015 The contact problem of the mathematical theory of elasticity with stick and slip areas. The theory of roling and tribology *Journal of Applied Mathematics and Mechanics* **79** 81–101
- [14] Dzhilavdari I Z, Riznookaya N N 2008 Studies of the Dynamics of Free Microoscillations of a Pendulum Supported by Two Balls *J. Frict Wear* **29**(1) 1-6
- [15] Dzhilavdari Z, Riznookaya N N 2007 Features of Small Oscillations of Pendulum on Elastic Surface with Hysteresis Friction *Journal of Friction and Wear* **28** (2) 145–153.
- [16] Jordan D W and Smith P 2007 *Nonlinear ordinary differential equation problems and solution* (Oxford University Press)
- [17] Gear C W 1971 *Numerical Initial Value Problems in Ordinary Differential Equations* (Englewood Cliffs NJ Prentice-Hall)
- [18] Hamming RW 1987 *Numerical Methods for Scientists and Engineers* 2-nd Ed (Dover Publications)