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Packing circle items in an arbitrary marble slab

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Abstract. Nature marble is widely used in architecture, but generally it is irregular in shape. It is necessary to pack regular shapes such as circular and rectangular into irregular marble slabs, with an optimistic way to save material. However, when the arbitrary region is a concave set, it is difficult to recognize its characteristics if the circular item is in the region. An angle bisector method was proposed to avoid overlapping inside circles and ensure the circle is contained in the marble slab. Meanwhile, the mathematical model for packing circle items in an arbitrary region was established based on the heuristic algorithm. The experimental results show that the approach is effective and it can be used not only in a convex region, but also in a concave one.

1. Introduction

Packing questions are a class of optimization problems in mathematics that involve attempting to pack objects together into containers. Many of these problems can be related to real life packaging, storage and transportation issues, for example, packing circle items in an arbitrary marble slab.

Many scholars have studied packing circular items, Szabó, P. G [1] outlined the history and wide application of the circular layout problem. George, J. A [2] used a method that combines quasi-random and genetic algorithms to solve the problem of arranging different size circles in a rectangle. López, C. O [3] presented a heuristic for the problem based upon formulation space search, to solve the problem of packing different size circles in a circular container. According to Newton–Raphson, to get nonlinear equations, Birgin E. G [4] solved the problem of arranging circles of the same size in circles, triangles, and rectangles. Galiev S. I [5] et al. transformed the problem of packaging circles of the same size into a 0-1 linear programming problem, and proposed a heuristic algorithm based on a linear model to solve the problem. In the optimal layout of circular parts, X X Song [6] et al. proposed the placement algorithm Arc Search Algorithm (ASA), and used the hybrid inheritance algorithm as a search strategy which reduces the calculation time and increases the material utilization rate to be arranged effectively. The maximum hole degree strategy was proposed by Huang et al [7], to solve the problem of arranging circles of different sizes in the garden. Akeb et al. [8] proposed an algorithm that embeds the beam search in a binary local search with a minimum container radius. The basis of the algorithm search is that the largest hole metric branches out of a node. Hifi M [9] et al. proposed a heuristic algorithm based on simulated annealing for both constrained and unconstrained circular cutting problems. They defined an energy function, the minimum value of this function representing the good results. In Grosso et al. [10] an algorithm was proposed based on monotonic basin hopping [11], which solved the problem of round and round circles of equal packing in a round container with the smallest radius. An algorithm that combines local search and nested partitioning of feasible space was proposed [12]. The search process of this algorithm is tabu search, and the tabu list is used to find the best of the arrangement of circles; then, the feasible set is explored by dividing it. A solution method based on a combination of the branch-and-bound algorithm and the reduced gradient method were proposed by Stoyan Y. G [13] ,it arranged different size circles in the rectangle. Mladenović N [14] et al. compared some of the round layout issues and developed a general reformulation descent heuristic algorithm to solve the problem of the arrangement of circles of the same size. Castillo I [15] et al. presented illustrative numerical results using generic global optimization software packages, Birgin E. G [16] confirmed the relevance of global optimization in solving circle packing problems.

The above algorithm is mainly aimed at studying and solving the optimization of the circle layout in the regular graphics, but the study of packing circles in an irregular shape is less represented.

This paper focuses on the problem of packing circular elements in an irregular shape. The circles cannot be overlapped on each other, and the circles must be contained in the region as a constraint condition. A mathematical model was proposed based on the angle bisector method which involved the constraint condition and overlapped condition. Meanwhile, the limited branch tree search algorithm, as one of the heuristic algorithms, was used to solve the mathematical model.

2. Build mathematical model

P_i, P_j are defined as circular items ($i{\neq}j$) which will lay out in the region P. Then the mathematical description of the constraints of packing circular items in an irregular region is as follows $P_i \cap P_j = \Phi$, P_i, $P_j \in P$

2.1. Non-overlapping

There are n pieces of circle items with the same size which will be put into the irregular region. Set r as the radius of the circle, $O(x_o,y_o)$ is the centre of the circle. The distance between any two centres is greater or equal to double radius to ensure there is no overlap.

$$\left((x_{oi}-x_{oj})^2+(y_{oi}-y_{oj})^2\right)^{1/2} \geq 2r \tag{1}$$

Formula (1) can be expressed as:

$$\min\left[0,(x_{oi}-x_{oj})^2+(y_{oi}-y_{oj})^2-4r^2\right]^2=0 \tag{2}$$

So the non-overlapping formula of all the circle items is:

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\min\left[0,(x_{oi}-x_{oj})^2+(y_{oi}-y_{oj})^2-4r^2\right]^2=0 \tag{3}$$

2.2. Inside the region

2.2.1. Angle bisector method. The article proposes a new angle bisector method to determine the condition of circular items inside the irregular region. Because the irregular region can be divided into convex and concave, the condition can be described in two cases.

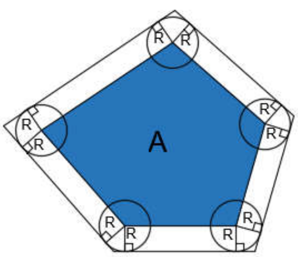

Figure 1. The feasible region.

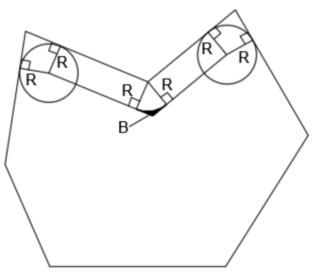

Figure 2. The ignored area B.

(1) The initial plate is a convex polygon：When the circle is tangent to the two sides of the polygon, the centre of the circle is the point where *r* is from both sides and on the angle bisector of the top corner. These circle centre points are connected in turn to form the feasible region A. As shown in figure 1, the A region represents the centre of the circle inside the region. It can be used to test whether the circle is inside the region or not.

(2) The initial plate is a concave polygon: Compared to the convex polygon, an additional condition needs to be considered at the concave vertex. The path of the circle centre is the fan shape in the concave vertex so the feasible side involves straight lines and curves. The area of shaded B is very small compared to the circular items and the irregular region, as shown in figure 2, so the feasible region will be linked with the straight line, and the shaded area B was ignored to simplify the calculation process.

2.2.2. Formulation of circular parts in sheet interior conditions. Setting any coordinate of the vertex of the irregular region is (x_m,y_m), the corresponding centre point on the vertex is $q(x_{qm},y_{qm})$. The distance from this centre point to the side line is equal to radius.

$$\begin{cases} \dfrac{\left|(y_m-y_{m-1})x_{qm}-(x_m-x_{m-1})y_{qm}+(x_m-x_{m-1})y_m-(y_m-y_{m-1})x_m\right|}{\left[(y_m-y_{m-1})^2+(-(x_m-x_{m-1}))^2\right]^{1/2}}=r \\ \dfrac{\left|(y_m-y_{m+1})x_{qm}-(x_m-x_{m+1})y_{qm}+(x_m-x_{m+1})y_m-(y_m-y_{m+1})x_m\right|}{\left[(y_m-y_{m+1})^2+(-(x_m-x_{m+1}))^2\right]^{1/2}}=r \end{cases} \tag{4}$$

There are many algorithms for judging the position between the points and region A, a simple and effective algorithm is the Angle summation method. Suppose the polygon region A has M edges, and the vertices are $P_1,\ldots,P_M$. The coordinate of the point that will be tested if it is inside $P(x, y)$. There is a function

$$f(x,y)=\sum_{k=1}^{M-1}\angle P_kPP_{k+1}+\angle P_MPP_1 \tag{5}$$

If $f(x, y)=2\pi$, P is the point inside the polygon, if $f(x, y)=\pi$, P is the point on the edge of the polygon, if $f(x, y)=0$, P is the point outside the polygon. The□P_iPP_{i+1} can be deduced as follows:

$$\begin{aligned}\angle P_iPP_{i+1}=\arccos&\frac{(x_i-x)\times(x_{i+1}-x)+(y_i-y)\times(y_{i+1}-y)}{\left[(x_i-x)^2+(y_i-y)^2\right]^{1/2}\times\left[(x_{i+1}-x)^2+(y_{i+1}-y)^2\right]^{1/2}}\\ &\times\operatorname{sgn}[(x_i-x)\times(y_{i+1}-y)-(x_{i+1}-x)\times(y_i-y)]\end{aligned} \tag{6}$$

Plug formula (6) into (5)

$$f(x,y)=\sum_{k=1}^{M}\left\{\begin{aligned}&\arccos\frac{(x_k-x)\times(x_{k+1}-x)+(y_k-y)\times(y_{k+1}-y)}{\sqrt{(x_k-x)^2+(y_k-y)^2}\times\sqrt{(x_{k+1}-x)^2+(y_{k+1}-y)^2}}\\&\times\operatorname{sgn}[(x_k-x)\times(y_{k+1}-y)-(x_{k+1}-x)\times(y_k-y)]\end{aligned}\right\} \tag{7}$$

According to whether the centre is in the feasible region, we determine whether the circle is in the irregular area. We substitute the centre coordinates $O(x_o,y_o)$ into the formula (7), and the mathematical formula for determining whether a single circle item is inside the irregular region can be express as

$$\min\left\{\sum_{m=1}^{M}\left\{\begin{aligned}&\arccos\frac{(x_{qm}-x_o)\times(x_{q(m+1)}-x_o)+(y_{qm}-y_o)\times(y_{q(m+1)}-y_o)}{\left[(x_{qm}-x_o)^2+(y_{qm}-y_o)^2\right]^{1/2}\times\left[(x_{q(m+1)}-x_o)^2+(y_{q(m+1)}-y_o)^2\right]^{1/2}}\\&\times\operatorname{sgn}[(x_{qm}-x_o)\times(y_{q(m+1)}-y_o)-(x_{q(m+1)}-x_o)\times(y_{qm}-y_o)]\end{aligned}\right\}-\pi,0\right\}^2=0 \tag{8}$$

When the number of circles is n, the formula (8) can be express as:

$$\sum_{i=1}^{n}\min\left\{\sum_{m=1}^{M}\left\{\begin{array}{l}\arccos\dfrac{\left(x_{qm}-x_{oi}\right)\times\left(x_{q(m+1)}-x_{oi}\right)+\left(y_{qm}-y_{oi}\right)\times\left(y_{q(m+1)}-y_{oi}\right)}{\left[\left(x_{qm}-x_{oi}\right)^2+\left(y_{qm}-y_{oi}\right)^2\right]^{1/2}\times\left[\left(x_{q(m+1)}-x_{oi}\right)^2+\left(y_{q(m+1)}-y_{oi}\right)^2\right]^{1/2}}\\ \times\operatorname{sgn}[\left(x_{qm}-x_{oi}\right)\times\left(y_{q(m+1)}-y_{oi}\right)-\left(x_{q(m+1)}-x_{oi}\right)\times\left(y_{qm}-y_{oi}\right)]\end{array}\right\}-\pi,0\right\}^2=0 \tag{9}$$

2.3. *Mathematical model*

Set $O_1,\ldots,O_n$ is the coordinate of the centre point of the circular items. From the above two constraints (3) and (9), the mathematical model for arranging circular elements of the same size in irregular plates can be express as:

$$f(O_1,\cdots O_n)=\sum_{i=1}^{n}\sum_{j=1}^{n}\min\left\{0,[(x_{oi}-x_{oj})^2+(y_{oi}-y_{oj})^2-4r^2]\right\}$$

$$+\sum_{i=1}^{n}\min\left\{\sum_{m=1}^{M}\left\{\begin{array}{l}\arccos\dfrac{\left(x_{qm}-x_{oi}\right)\times\left(x_{q(m+1)}-x_{oi}\right)+\left(y_{qm}-y_{oi}\right)\times\left(y_{q(m+1)}-y_{oi}\right)}{\left[\left(x_{qm}-x_{oi}\right)^2+\left(y_{qm}-y_{oi}\right)^2\right]^{\frac{1}{2}}\times\left[\left(x_{q(m+1)}-x_{oi}\right)^2+\left(y_{q(m+1)}-y_{oi}\right)^2\right]^{\frac{1}{2}}}\\ \times\operatorname{sgn}[\left(x_{qm}-x_{oi}\right)\times\left(y_{q(m+1)}-y_{oi}\right)-\left(x_{q(m+1)}-x_{oi}\right)\times\left(y_{qm}-y_{oi}\right)]\end{array}\right\}-\pi,0\right\}^2=0 \tag{10}$$

3. The solving algorithm

The search tree algorithm, one of the important heuristic algorithms, is widely used in the packing problem. In this paper, the principle of the algorithm is to arrange the next circular item on the left and bottom on the basis of the arranged circular items.

3.1. *Layout strategy*

The distance between the two circular parts when they do not overlap is at least 2R. The critical point of the circle item is shown in figure 3. The small circle in the middle of the figure is the circle P_j that has been arranged, the circle C is the trajectory of the centre of the circle P_i ready to be placed.

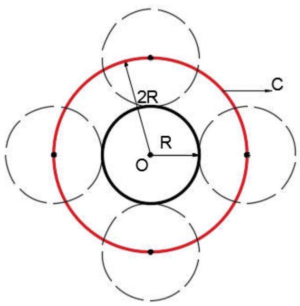

Figure 3. The critical point of the circle item.

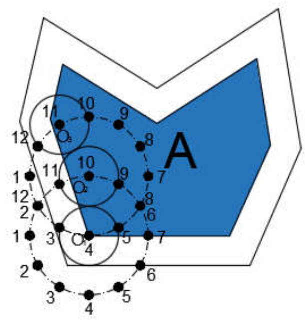

Figure 4. The layout strategy of the circle.

In order to calculate faster, path C should be separated into discrete points. The greater the number of points, the more accurately the result is. This article sets the number of equally divided into 12, the final experimental results prove that the value is more appropriate. In order to reduce the complexity of the calculation, the final optimization result is not affected. This article uses the bottom-left placement strategy to determine the unique position of a newly-arranged circle part. The first centre point is placed at the bottom left position of the polygon, and each new centre point that is laid out takes precedence over the leftmost position, only in the same left situation, the lower position is given priority. Figure 4 shows how the circle is laid out in a polygon. The polygonal shaded area in the figure is the feasible area of the centre of circle. Firstly, the first centre point O_1 is selected in the bottom leftmost position of the centre of the feasible region A, the centre O_2 is pushed out by the centre O_1， the centre O_1, O_2 launches O_3 position. This process is repeated until the feasible region A cannot arrange a new centre point.

3.2. Finite branch tree heuristic algorithm

This paper uses the branch tree structure algorithm. In the search tree, the corresponding coordinate of the root node is the lowest leftmost points of the feasible region, the parent node represents the best position of the circle centre, the child nodes represent all centre positions of the next circular part to be laid out. Each parent node can generate 12 child nodes. The number of layers in the tree represents the number of circle parts that the polygon can arrange. Figure 5 describes the growth of the branch tree.

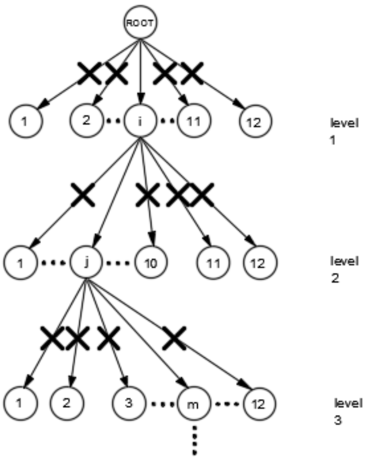

Figure 5. The growth process of the finite branch tree.

4. Numerical simulation

Example data is shown in table 1.

Table 1. Experimental date

Question	Vertex coordinates	Polygonal area (mm^2)	Round radius (mm)	Part area (mm^2)	Utilization rate
1	$\{(0,0)(19,0)(10,19)\}$	180.5	1	135.1	74.8%
2	$\{(0,0)(23,0)(23,13)(5,14)\}$	278	1.5	204.9	73.7%
3	$\{(0,0)(26,0)(12,8)(8,16)\}$	168	1.5	127.2	75.7%
4	$\{(0,0)(14,0)(18,5)(11,19)(-10,13)\}$	345	2	251.3	72.8%
5	$\{(0,0)(18,0)(20,18)(8,13)(-1,19)\}$	302	2	213.6	70.7%

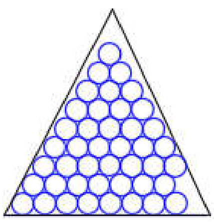

Figure 6. Problem 1 layout result.

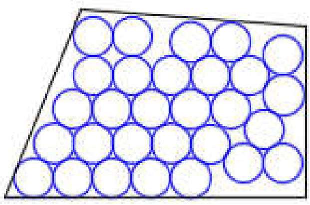

Figure 7. Problem 2 layout result.

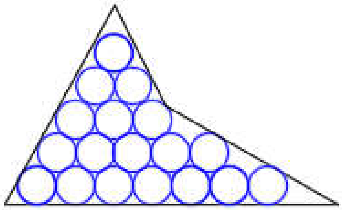

Figure 8. Problem 3 layout result.

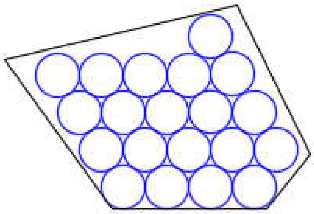

Figure 9. Problem 4 layout result.

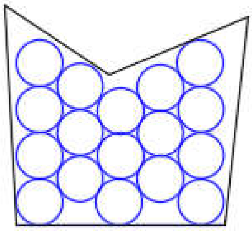

Figure 10. Problem 5 layout result.

As shown in the data of the example in table 1, the utilization of the plates is effectively improved, as displayed in figures 6 to 10.

5. Conclusion

(1) This paper proposes the angle bisector heuristic algorithm for the irregular graphics layout and solves the problem of packing circular items in irregular regions.
(2) The paper proves that the algorithm can be used successfully not only for convex set graphics, but also for concave set graphics.
(3) The numerical simulation proves that the approach can pack circle items in irregular marble slab effectively.

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