A Novel Method for Mechanical Constitutive Parameters Estimation of Human Left Ventricle

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Abstract. In this paper, a new framework for estimation of myocardium constitutive parameters is established. A more realistic, cardiac magnetic resonance image based realistic human left ventricular finite element analysis model is introduced for analyzing the deformation of left ventricle during diastole firstly, the material behavior is described by the anisotropic nonlinear Holzapfel-Ogden constitutive model; then a novel hybrid simplex and quantum behavioral particle swarm optimization algorithm which is proposed to estimate the constitutive model parameters of myocardium as the inverse problem of left ventricle deformation. Numerical results show that finite element analysis results and the estimated parameters are in good agreement with the experimental data reported in literature, which demonstrate that parameter estimation framework is valid. Comparing with current optimization algorithm, the presented hybrid optimal algorithm can estimate the mechanical parameters more efficiently.

1. Introduction

In worldwide, heart disease seriously affects the quality of human beings' life, early diagnosis and treatment can significantly reduce the incidence and fatality rate of heart disease [1]. Mechanical modelling and simulation of cardiac functionality is the main method for people to understand the mechanism of heart disease. Recent studies have shown that organ damage of human heart will cause the changes of myocardial constitutive parameters [2], so estimations of myocardial constitutive parameters have a great meaning for disease diagnosis.

The development of modern medical imaging technology, such as Cardiac Magnetic Resonance (CMR) [1,2], can provide high-quality, high-resolution medical images that accurately describe the anatomical structure and functionality of human heart, it become an important auxiliary method for the diagnosis of heart disease. Based on these real-time medical image data, how to extract the effective measure of heart failure, such as the changes of myocardial parameters, is an important part of the diagnosis of heart disease [2]. Myocardial material is generally considered as a fibre reinforced hyper-elastic material, which has a strong nonlinear stress response. Many constitutive models of myocardium, such as Fung-type, Neo-Hookean, Saint Venant-Kirchhoff [3], and Holzapfel-Ogden constitutive model [4], were developed to simulate the mechanical behaviour of myocardial. Holzapfel-Ogden constitutive model can consider structure of myocardial fibers and lamellar, it can describe mechanical behaviour of myocardial more accurately. Meng et al [2] used data obtained by medical image analysis and Kalman filter method to estimate the two-dimensional linear elastic constitutive myocardial parameters; Guchhait et al. Used Saint Venant-Kirchhoff constitutive model to estimate the mechanical parameters of a two-dimensional model, Aggarwal et al. used the Fung-type constitutive model [5], and estimated the mechanical parameters of the heart valve. These studies are based on simplified mechanical constitutive models or geometric models. Unlike these studies, a three-dimensional finite element model of human left ventricular which is constructed by medical images analysis is used to estimate the mechanical constitutive parameters in this paper.

Given the mechanical parameters of myocardial tissue, studies of the mechanical behaviour of the myocardium under certain mechanical conditions are recognized as a forward problem. While mechanical states of the myocardium are known, parameter identification problem of material is an inverse problem, which is based on the minimization of misfit functional between the observation data and the responses of the computational model under certain material parameters. In order to obtain the parameter estimation, an optimization problem must be solved. The traditional optimization algorithm includes gradient-based optimization algorithm and gradient-free optimization algorithm [6]. Gradient based methods, such as Newton algorithm, LM algorithm, are effective tools to solve optimization problems, these methods usually convergent fast, but these methods does not convergent globally, and the convergence of these methods depends on the selection of initial value. Another drawback of these methods is that calculation of gradient matrix costs huge time, to calculating the gradient, finite difference methods is usually used, it require the evaluate the misfit function many times which require huge amount calculation, especially for the problems whose constitutive model have many material parameters. Gradient-free optimization methods, such as the simplex method [6], is another kind of optimization methods, these methods does not need to the gradient matrix, but they does not convergent globally. In recent years, intelligent based optimization algorithms, such as simulated annealing algorithm, particle swarm algorithm, quantum particle swarm algorithm, genetic algorithm [7,8] are developed. Unlike the traditional algorithms, this kind of optimization algorithms has global convergence, but the speeds of such algorithms are relatively slow [7,8]. In recent years, people are focusing on the studies of hybrid algorithm [4], which can combine the advantages of both intelligent based algorithms and traditional algorithms. In this paper, a novel hybrid simplex and quantum particle swarm algorithm is developed for estimate the parameters efficiently.

The main work of paper is organized as follows: Firstly, a framework of myocardial parameter estimation is established by CMR medical image based finite element analysis model of left ventricular and optimal parameter estimation algorithm. In this framework, the mechanical behaviour of human left ventricular is described by Holzapfel-Ogden constitutive model. Then, a novel hybrid simplex and quantum particle swarm algorithm is introduced to solve the parameter estimation problem. In the end, a numerical example is given to demonstrate the effectiveness and efficiency of the proposed algorithm.

2. Framework of constitutive parameter estimation

2.1. Left ventricular diastolic finite element simulation model

In this section, CMR medical image based finite element analysis model of mechanical behaviours of left ventricle of human heart is described firstly as the forward problem, then the inverse problem for estimating the material parameters is introduced.

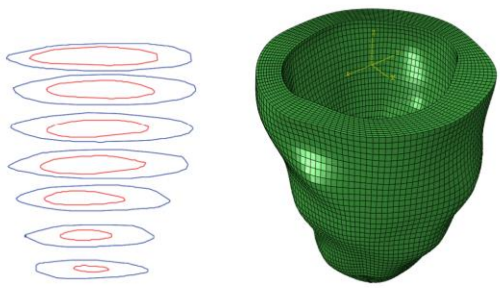

Figure 1. Image segmentation by level set method and the FE mesh of LV [1].

We first establish the geometry of the left ventricle of human heart. Based on early diastolic stage CMR image of human heart, an improved level set image segmentation method is used to construct the inner and outer walls of left ventricle, by using this series of segmentation results, the three-dimensional geometry of left ventricular is reconstructed [1], and then the finite element mesh is generated, as shown in Figure. 1.

Assuming that the deformation of left ventricle during diastole is a quasi-static process, the myocardium is considered as an anisotropic incompressible material. Let $\Omega \subset R^3$ be an open and bounded domain with coordinates X and boundary $\partial\Omega$, occupied by an incompressible hyperplastic body, such as the whole left ventricle. The deformation of left ventricle can be analysed by finite element discretize the virtual work principle, which can be written as, find displacement **u** and hydrostatic pressure *p*, such as

$$\Phi\left(\mathbf{u}, p;\delta\mathbf{u},\delta p\right)=\int_{\Omega}\left(\frac{\partial\varphi\left(\mathbf{C},\boldsymbol{\theta}\right)}{\partial\mathbf{F}}+p\mathrm{J}\mathbf{F}^{-T}\right):\delta\mathbf{u}+p\left(J-1\right)\delta pd\Omega-\int_{\partial\Omega_{\text{in}}}p_b J\mathbf{F}^{-T}\cdot\delta\mathbf{u}\mathrm{d}\Omega=0, \tag{1}$$

over the space of admissible displacements and pressures satisfying any given Dirichlet boundary conditions. In equation (1) , J = det **F**, where $\mathbf{F}=\nabla\boldsymbol{x}=Grad\,\boldsymbol{u}+\boldsymbol{I}$ is deformation gradient tensor, $\boldsymbol{I}$ is the identity tensor in R^3 , $C=J^{-\frac{2}{3}}\mathbf{F}^{-\mathbf{T}}\mathbf{F}$ is the volume-preserving right Cauchy–Green deformation tensor, φ is the strain energy density function , $\boldsymbol{\theta}=\left[a,b,a_f,b_f,a_s,b_f,a_{\text{fs}},b_{\text{fs}}\right]$ is the vector of constitutive parameters of cardiac material, p_b is the pressure on the inner wall of left ventricle.

The incompressible Holzapfel-Ogden constitutive relationship of myocardial tissue is described by the following strain energy density function

$$\varphi\left(C,\theta\right)=\frac{a}{2b}e^{b}\left(I_1-3\right)+\sum_{i=f,s}\frac{a_i}{2b_i}\left(e^{\left(b_i\left(I_{4i}-1\right)^2\right)-1}\right)+\frac{a_{\text{fs}}}{2b_{\text{fs}}}\left(e^{\left(b_{\text{fs}}\left(I_{8fs}\right)^2\right)}-1\right), \tag{2}$$

where $I_1=\text{tr}\left(C\right), I_{4f}=f_0\cdot\left(Cf_0\right), I_{4s}=s_0\cdot\left(Cs_0\right), I_{8\text{fs}}=f_0\cdot\left(Cs_0\right)$ are the first, fibre, sheet, and fibre sheet invariants of **C** respectively, f_0, s_0 are fibre and sheet directions. After linearizing equation (1) by a finite element discretization, then Newton-Raphson method is used to solve the discretized nonlinear equation. In our numerical experiments, the open sourced package FEniCS [9] is used to implement the finite element analysis procedure.

2.2. Parameter estimation by inverse approach

As the constitutive parameters of the myocardial material cannot be measured directly, the estimation of material parameters are an inverse approach of quasi-static nonlinear hyper elasticity deformation by using a given set of partially measured data, at one or more loading situations. A common approach to solve this inverse problem is to minimizing the men squared error between measured and computed responses under constrained or unconstrained optimization framework. Such as the following constraint optimization problem:

$$\begin{gathered}\boldsymbol{\theta}=\arg\min_{\theta} f(w(\mathbf{u},p;\boldsymbol{\theta}),\overline{w})=\|w(\mathbf{u},p;\boldsymbol{\theta})-\overline{w}\|^2,\\ s.t.\begin{cases}\Phi(\mathbf{u},p;\delta\mathbf{u},\delta p;\boldsymbol{\theta})=0;\forall\delta\mathbf{u}\in\mathrm{U},\delta p\in P,\\ \qquad\boldsymbol{\theta}_L\le\boldsymbol{\theta}\le\boldsymbol{\theta}_H,\end{cases}\end{gathered} \tag{3}$$

where f is the error functional between measured data w and computed responses, the equality constrain Φ is the virtual work principle described in (1), inequality of constrain represents $\boldsymbol{\theta}$ is bounded by a lower bound $\boldsymbol{\theta}_L$ and a higher bound $\boldsymbol{\theta}_H$. For notational convenience, the reduced formulation of the misfit functional is used, which is only depends on the material parameter $\boldsymbol{\theta}$. In particular, we introduce the following reduced optimization problem to estimate the parameter $\boldsymbol{\theta}$,

$$\boldsymbol{\theta}=\arg\min_{\boldsymbol{\theta}}\hat{f}(w(\mathbf{u}(\boldsymbol{\theta}),p(\boldsymbol{\theta});\boldsymbol{\theta}),\overline{w}). \tag{4}$$

3. Simplex-QPSO algorithm for parameter estimation

Many optimal algorithms can be used to solving the inverse parameters estimate problem (4), in this work, a novel hybrid simplex and Quantum-Behaved particle swarm algorithm is developed for estimating the parameters. Take parameters $\boldsymbol{\theta}$, as the coordinates of particles, if we choose *m* particle whose coordinates are denoted as $\theta^L, L=1,2,\cdots,m$, the best position of *m* particles is $P^L(k)$, the global best position is denoted as $P^g(k)$,then algorithm 1 is introduced for updating the positions of *m* particles.

Algorithm 1. (Position updating by quantum-behaved swarm algorithms)
Step 1. Initializing the position of particles;
Step 2. Calculating the average optimal positions of *m* particles;

$$C = \frac{1}{m}\sum_{L=1}^{m} P^L(k), \tag{5}$$

Step 3. Calculating the fitness of M particles, compare with fitness of last iteration, if fitness $f(\theta^{i+1}) < f(P_i)$, then update the current best position of this particle;

Step 4. Calculating the current global best position of current M particles, so as to find

$$P_g(k+1) = \arg\min\left\{ f\left(P^L\right)\right\}.$$

Comparing with the global best position of last iterations, if current global position is better, the global best position is updated.
Step 5. Updating the new positions of particle by using following equations

$$\boldsymbol{p}_d^L(k) = \phi_d(k)\boldsymbol{P}_d^L(k) + \left(1-\phi_d(k)\right)\boldsymbol{P}_d^g(k), \tag{6}$$

$$\boldsymbol{\theta}_d^L(k) = \boldsymbol{p}_{\mathrm{d}}^{\mathrm{L}} \pm \alpha(k)\left|\boldsymbol{C}_d(k) - \boldsymbol{\theta}_d^L(k)\right| \times \ln\left(1/u_d^L(t)\right) \tag{7}$$

where, ϕ_d, u_d^L are uniform distributed random number on (0,1)， the subscript *d* is denote the *d* th component of a vector.

Step 6. If the given convergence condition is satisfied, stop this algorithm, otherwise return to step 2.
Algorithm 2: (The hybrid simplex quantum-behaved particle swarm algorithm)
Step 1. Initializing the solver: setting the bounds of parameters, optimal target function and prescribed error limits ε, the positions of initial 3*m*+1 particles and current iteration number *n* =1 ;
Step 2. Calculating value of function $f_{obj}(\boldsymbol{\theta})$ with at 3*m*+1 particles, and sort it by ascending ordering,

$$f_{obj}(\boldsymbol{\theta}_1) \le f_{obj}(\boldsymbol{\theta}_2) \le \cdots \le f_{obj}(\boldsymbol{\theta}_{3m+1}).$$

Step 3. Using simplex algorithm to do iteration with first *m*+1 optimal particle, update the positions of those particles;
Step 4. Using algorithm 1 to update rest 2m particles' positions;
Step 5. If the given convergence condition is satisfied, terminate this algorithm, otherwise let *n* = *n*+1 and return to step 2.

4. Numerical examples

In this section, a numerical example is presented for validating the proposed algorithm, which is used to estimate constitutive parameters of human left Ventricle. The constitutive parameters of the myocardial material which is used in Ref [10], is taken as the true value of our numerical example, as shown in Table II. The geometry and finite element mesh of human left ventricular is generated based on the CMR medical images and level set method, which is described in this article. The ventricular inner wall pressure is taken as 8 mmHg. By using the finite element formulation of deformations of

left ventricular in section 2, a convergence result is obtained. The results of the finite element analysis are used as the observed data for parameter estimation to verify the correctness of the proposed algorithm to estimate material parameters.
To estimation of myocardial parameters, the objective functional is constructed by using the measured strain data and the ventricular volume data, which can be written as following,

$$f_{\mathrm{obj}}(\varepsilon,V)=\sum_k\sum_{i,j}\left|1-\varepsilon_{\mathrm{ij}}^{\ k}/\bar{\varepsilon}_{ij}^{\ k}\right|+\left|1-V/\bar{V}\right|, \quad (8)$$

where $\bar{\varepsilon}_{ij}^{\ k}$ are the measured strains at the *k*th observation point measurements, $\bar{V}$ is the measured ventricular volume. To obtain the estimations parameters estimates the optimization problem (4) is solved by the proposed optimal algorithm. The Levenberg-Marquardt (LM) algorithm [4] is used to solve problem (4) as comparison of the proposed algorithm. The parameter estimation results are shown in Table 1, the parameters a,b,a_f,b_{fs} obtained by the LM algorithm are less than the true value, the parameters b_f,a_s,a_{fs} are bigger than the true value, the estimation errors of b_s are quite large. Comparing with the results of LM algorithm, the results of proposed algorithm has smaller error with parameter $a,b,a_f,b_f,a_s,a_{fs},b_{fs}$, but the estimation of b_s also has a little difference with the true value. Figure 2 shows the pressure volume curves which is obtained by using the two sets of estimated parameter. It is shown that the two sets of estimated parameter are valid estimates of the left ventricular mechanical parameters.

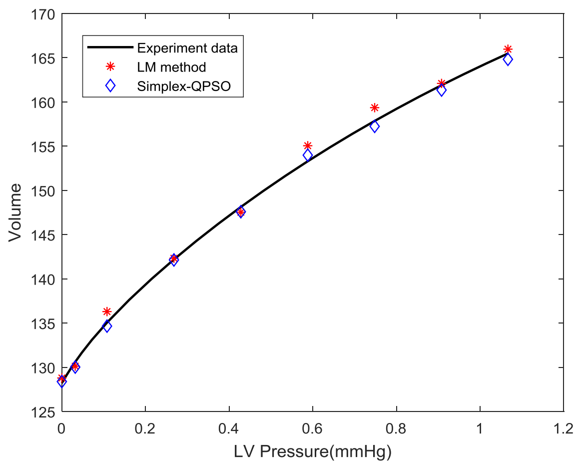

Figure 2. Ventricular pressure volume curve.

The computational efficiency of proposed algorithm is shown in Table 2. Comparing with LM algorithm, both algorithms require a lot of finite element computation, but the proposed algorithm requires less computation time than LM algorithm.

Table 1. Resaults of Parameter Estimation

	a	b	a_f	b_f	a_s	b_s	a_{fs}	b_{fs}
True value[10]	0.236	10.810	20.037	14.154	3.725	5.165	0.411	11.300
LM algorithm	0.238	10.677	18.980	18.459	4.083	3.317	0.433	10.290
HS-QPSO	0.235	10.829	20.015	14.217	3.875	1.900	0.410	11.330

Table 2. Efficiency Comparison of Two Algorithms

	Iterations	Time	Error limit
LM algorithm	773	18h	1e-6

	Iterations	Time	Error limit
HS-QPSO	482	12h	1e-6

5. Conclusions

In this study, a novel framework is established for estimation of myocardium constitutive parameters; this framework contains a finite element analysis model of realistic human left ventricular and a new hybrid simplex and quantum behaviour particle swarm optimization algorithm which is proposed to estimate the constitutive model parameters of myocardium. The realistic geometry of human left ventricular is constructed by using cardiac magnetic resonance image (CMR) and level set method, then mechanical behaviour of human left ventricular during diastolic stage is analysed by finite element method, we consider the left ventricle as an anisotropic nonlinear material, which is described by Holzapfel-Ogden constitutive model. Using this simulation model and the observation data, estimation of mechanical parameters of left ventricular myocardial materials is taken as an inverse problem of finite element analysis. A novel hybrid quantum particle swarm and simplex algorithm is proposed for estimation of mechanical parameters; this approach directly uses the observed data in medical images, and uses the hybrid quantum behaviour particle swam with global convergence. The numerical analysis shows that the method can effectively estimate the constitutive parameters of myocardial materials; this approach provides an effective method for the diagnosis of heart disease.

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