

# Fiber hinge approach for nonlinear analysis of structural members accounting for local buckling under large deformation

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**Abstract.** In order to improve the use of efficiency and economy in steel structures, we always try to make the steel plate with larger width–thickness ratio. Existing analysis methods including plastic hinge method and plastic zone method, but they have their own limitation. In this paper, we proposed an approach based on fiber hinge concept and effective width method which could take local buckling into consideration. Significant parameters of the proposed approach are discussed, including fiber hinge length, effective cross-section width and fiber failure criterion. We verify the accuracy and validity of the proposed approach according to comparison of analysis results and the test data from published literature. By comparing with the plastic zone method, the error of the ultimate bearing capacity is about 4%, at the same time, the calculation speed of this method is 64 times faster than that of the plastic zone method.

## 1. Introduction

In order to improve the use of efficiency and economy in steel structures, we always try to make the steel plate with larger width–thickness ratio. So that the cross section has a larger radius of rotation which can obtain a higher overall stability of the bearing capacity and a better economic effect [1]. Due to large width-to-thickness ratios, the members are prone to local buckling. For the steel structure composed of such members, the premature local buckling will lead to the degeneration of the stiffness, the ultimate bearing capacity of the structure will also be reduced. But local buckling does not mean the ultimate bearing capacity of the steel structures, the steel structure has large deformation capacity instead.

In the plastic zone model, beam-column members are discretized into finite elements, and the cross-section of each finite element is subdivided into many fibers. The plastic zone model can track the spread of plasticity throughout the cross-section and along the member length [2-3]. This model has been accepted as one of the most comprehensive nonlinear analysis models. However, the computing elements of plastic zone method consume too much time and memory, and the connections of structure are very difficult to simulate.

A more simple and efficient way to represent inelasticity in frames is the plastic hinge model. In the plastic hinge model, the beam-column members are modelled appropriately and the further subdivisions are no longer required. The material inelastic behaviours are represented by two plastic hinges located at axial ends of the member [4]. The results are sufficiently accurate for practical designs. The plastic hinge model is simple in formulation as well as implementation, and in particular, it requires the least number of elements to model the member. However, the residual stress and local buckling would not be directly included into the analytical model, and the yield surface is not always available and accurate for every section [5-6].



The plastic zone model is accurate while computationally intensive and time-consuming. The plastic hinge model is simple, but its accuracy is limited. To take advantage of computational efficiency of the plastic hinge model and overcome its disadvantages, a fiber hinge model in which the monitored cross-sections are divided into many fibers was developed [7]. And the inelastic behaviour of material can be considered by tracking the uniaxial stress-strain relationship of each fiber. Fiber hinge model has been proposed in the nonlinear analysis of steel structure [8-10]. However, the fiber hinge model cannot consider the effect of local buckling. A new stress-strain equation accounting for local buckling was proposed based on the energy method [11]. But the method can only take axial compression into consideration.

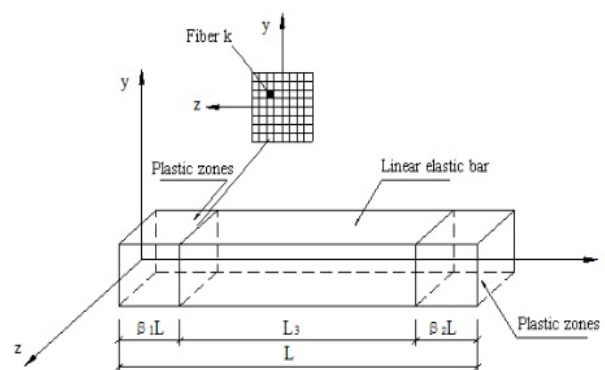
Due to the existing methods have their own limitations, there is an urgent need to propose a method to consider the effect of local buckling which has high accuracy and efficiency.

In order to overcome the above problems, the local buckling fiber hinge model is proposed, which can consider the effect of local buckling under combined bending and axial loading, and provide efficient and accurate analysis method for the whole process of the compression component. This research will lay a solid foundation for the analysis and design of the whole structure to consider the local buckling of the members.

## 2. Development of Local Buckling Fiber Hinge Model

### 2.1. Fiber Hinge Model

The fiber hinge model is based on bar-system structure mechanics and uniaxial constitutive relation. The mechanical response of each fiber is characterized by uniaxial stress-strain relationship of the particular material.



**Figure 1.** A space beam column element with fiber hinges inside

Basic assumptions are as follows:

- All elements are initially straight and prismatic.
- A plane cross-section remains plane after deformation.
- Reductions of torsional and shear stiffnesses are not considered in fiber hinge.
- The panel-zone deformation of the beam-to-column joint is neglected.

Figure 1 represents a space beam column element with fiber hinges inside, the fiber hinge model is developed to track the plastic properties of every fiber's uniaxial stress-strain relationship in the cross-sections, as to track the development of the plastic properties of the structural members along the length of the fiber unit and the height of the section.

There are three aspects requiring special attentions in applying the fiber hinge models, including material constitutive relations, cross section division and fiber hinge unit length. The cross-section divisions and fiber hinge unit length are significant for computational efficiency and accuracy. When using the fiber hinge model to analyse the structure, all plastic deformation of the structure occurs in the length of the fiber hinge unit [12].

## 2.2. Local Buckling Fiber Hinge Model

Fiber hinge model has high computational efficiency, however, it cannot consider the effect of local buckling. The effective width method is an approximate method to calculate the post buckling strength [13]. Under the premise of ensuring the accuracy of calculation, in order to improve the computational efficiency, this paper puts forward the local buckling fiber hinge model based on the fiber hinge method, combined with the effective width method to consider the bearing capacity of the remaining section after the steel plate locally buckled. The cross section of the fiber hinge is divided into the effective section and the failure cross section, and the fiber of effective and the failure cross section are represented by different constitutive relations.

## 2.3. Effective Section and Fiber Failure Criteria

We can calculate the effective cross section [14] of the plate as follows:

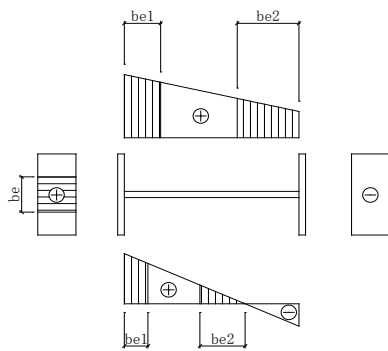
$$\left\{ \begin{array}{ll} \frac{b}{t} \leq 18\alpha\rho & \frac{b_e}{t} = \frac{b_c}{t} \\ 18\alpha\rho < \frac{b}{t} < 38\alpha\rho & b_e = \left( \sqrt{\frac{21.8\alpha\rho}{\frac{b}{t}}} - 0.1 \right) \frac{b_c}{t} \\ \frac{b}{t} \geq 38\alpha\rho & b_e = \frac{25\alpha\rho}{\frac{b}{t}} \cdot \frac{b_c}{t} \end{array} \right. \quad (1)$$

Where,  $b$ ,  $t$  is the width and thickness of the flange and web,  $b_e$  is the effective width,  $\alpha$  and  $\rho$  are the calculating coefficients, the formula are presented in [14].

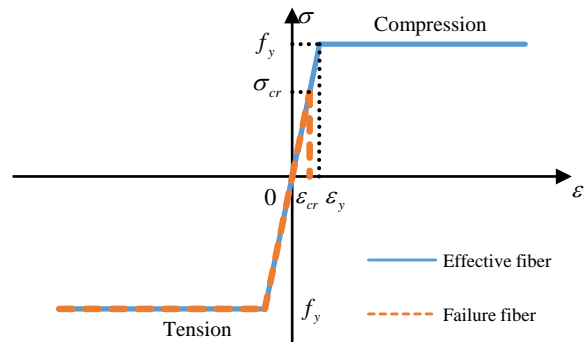
Figure 2 shows the effective cross section of H-shaped section. For flange, the compression plate, the effective width of the compressed flange is distributed in the middle part of the flange, can be calculated according to formula (1), the tension flange is all effective. For web, the effective width can be calculated according to formula (2).

$$\left\{ \begin{array}{ll} \psi \geq 0 & b_{e1} = \frac{2b_e}{5-\psi} \quad b_e = b_e - b_{e1} \\ \psi < 0 & b_{e1} = 0.4b_e \quad b_{e2} = 0.4b_e \end{array} \right. \quad (2)$$

Where,  $\psi$  is stress gradient,  $\psi = \frac{\sigma_{\min}}{\sigma_{\max}}$ .



**Figure 2.** Effective cross section of H-shaped section



**Figure 3.** Constitutive relation of different fiber

After the plate buckled, effective cross section can continue to carry the force. But the failure cross section reached the critical local buckling stresses. In this approach, we defined effective fibers and failure fibers in the effective cross section and failure cross-section separately.

The constitutive relation of the effective fiber is as same as steel fiber, while the failure fiber is shown in Figure 3. Here we defined the failure fiber, the critical local buckling stress of steel plates is shown as formula (3-4),

$$\sigma_{cr} = \frac{k\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2 \quad (3)$$

$$\varepsilon_{cr} = \frac{k\pi^2}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2 \quad (4)$$

#### 2.4. Fiber Hinge Length

In the fiber hinge model, the length of the fiber hinge is set to 10% of the length of the member, which has no theoretical basis [12]. Through the research of the existing literature [15], we concluded that under combined bending and axial loading, local buckling position of slender cantilever columns occurred basically at the bottom of the column, the value is assumed as the height of the cross section.

### 3. Analytical Model Implementation

We implemented the proposed approach in ABAQUS software. The implementation of approach considering local buckling is shown as follows steps:

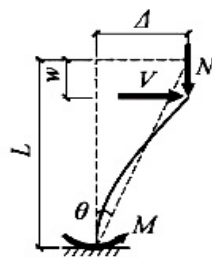
- Step1 Build the basic model in ABAQUS.
- Step2 Calculate the effective cross section as formula (1-2).
- Step3 Calculate the the critical local buckling stress as formula (3-4).
- Step4 Edited the keyword in the inp file, enter the coordinates, area of each fiber.
- Step5 Gave different constitutive relations to different fibers according to Figure 3.

### 4. Verification

The behavior of thin flexible section was studied by Wu X.X. in 2006[15], and the buckling hinge model was proposed to consider the influence of the local buckling of the members, and the experimental results were used to verify the performance of the model.

#### 4.1. Structural Specimen

Section of the cantilever column is H250×150×3.2×4.5, the length of which is 2250mm. At the end of the cantilever, a concentrated force is applied perpendicular to the axial direction of the member, and the concentrated force is kept constant during the whole deformation process, the axial compression ratio is 0.2, and the cantilever end is also applied with a horizontal force and the displacement control is adopted, shown in Figure 4. In buckling hinge model by Wu X. X., it is necessary to make a precise plastic zone model to determine the position of the local buckling and the other parameters, and the process is time-consuming and complex.



**Figure 4.** Schematic diagram of loading

#### 4.2. The Parameters and Calculation of the Proposed Approach

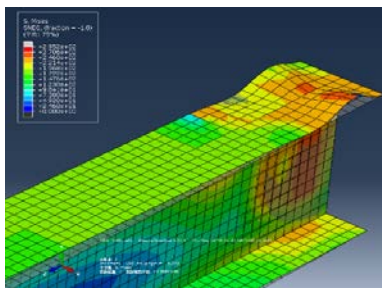
We adopted beam element B31, built a FE model in ABAQUS. For fine calculation, and strictly distinguish the effective fiber and the failure fiber, the section is divided into 100 fibers, the flange is divided into 30 fibers, and the web is divided into 40 fibers.

We calculate the effective section, local buckling stress based on formula (4), the critical local buckling stress of web is  $\sigma_{cr} = 127.35\text{MPa}$ , the flange  $\sigma_{cr} = 183.1\text{MPa}$ . The constitutive relation is shown in Figure 3.

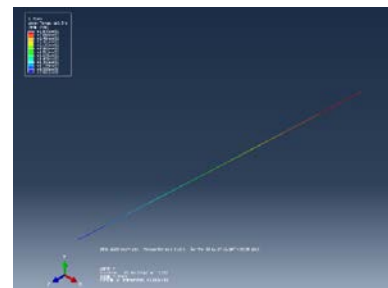
In the proposed approach, the fiber hinge length is essentially equal to the height of the cross section.

#### 4.3. Comparison and Verification

For the above cantilever column, we established the FE model using the plastic zone model and the local buckling fiber hinge model separately, shown in Figure 5-6. Figure 7 shows the  $M-\theta$  curve of proposed approach compared to other approaches using plastic zone model and buckling hinge model. By comparison with that adopting plastic zone model, it is found that using proposed model  $M_u = 50.05\text{kN}\cdot\text{m}$ , plastic zone model  $M_u = 52.31\text{kN}\cdot\text{m}$ , the error is about 4.3%. In the calculation, the proposed approach consumes 3 seconds to complete the calculation, and the approach using plastic zone model takes 2 minutes and 20 seconds, the calculation speed of proposed approach is 64 times faster than that using plastic zone model.

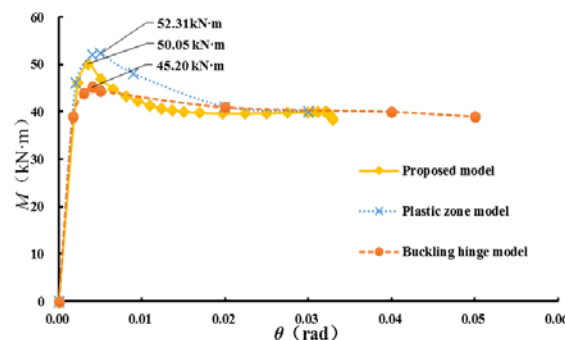


**Figure 5.** Calculation results of plastic zone model



**Figure 6.** Calculation results of proposed model

For the above cantilever column, we established the FE model using the plastic zone model and the local buckling fiber hinge model separately, shown in Figure 5-6. Figure 7 shows the  $M-\theta$  curve of proposed approach compared to other approaches using plastic zone model and buckling hinge model. By comparison with that adopting plastic zone model, it is found that using proposed model  $M_u = 50.05\text{kN}\cdot\text{m}$ , plastic zone model  $M_u = 52.31\text{kN}\cdot\text{m}$ , the error is about 4.3%. In the calculation, the proposed approach consumes 3 seconds to complete the calculation, and the approach using plastic zone model takes 2 minutes and 20 seconds, the calculation speed of proposed approach is 64 times faster than that using plastic zone model.



**Figure 7.** Comparison with plastic zone model and buckling hinge model

## 5. Conclusions

A local buckling fiber hinge model accounting for local buckling effects has been developed for modelling steel members under combined bending and axial loading. We gave the suggestion of the key parameters, and the analytical model implementation. The proposed approach is validated by the data from the existing literature, and the accuracy and efficiency of the calculation results are compared with that of the plastic zone model. We can conclude that:

- The local buckling fiber hinge model can take local buckling into consideration in the whole structure analysis.
- Compared with the plastic zone model, the calculation error is less than 5%.
- Compared with the plastic zone model, the computational efficiency is improved by about 64 times.

Under the premise of ensuring the accuracy of the calculation, the approach proposed in this paper is of high computational efficiency. The proposed approach provides a new idea to consider local buckling in the whole process nonlinear analysis, which is a new way to balance accuracy and efficiency. This study will provide a basis for the influence of the local buckling in the nonlinear analysis of steel frame.

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