

Characteristic analysis of a quasi-zero-stiffness vibration isolator

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Abstract. The frequency range over which a linear passive vibration isolator is effective, is often limited by the stiffness required to support a static load. And this can be improved upon by employing nonlinear negative stiffness elements to get a low dynamic stiffness. In this paper, a quasi-zero-stiffness (QZS) system comprising an air spring with positive stiffness and Euler buckled beams with negative stiffness is studied. Compliant mechanisms are also introduced into the QZS system to achieve quasi-zero-stiffness characteristics under a variable load. Static mechanical characteristics of the system are analyzed by the FEM, and load-displacement curves and stiffness-displacement curves of the system under different working conditions are given. The natural frequency and the response curve of the system are obtained through dynamic analysis. It is concluded that the system has good vibration isolation performance in low frequency range.

1. Introduction

For the traditional linear vibration isolation system, only when the external excitation frequency is more than 1.4 times the inherent frequency of the system, the vibration isolation system can work, which makes the traditional linear vibration isolation system limited in the low frequency vibration range. By reducing the stiffness, the frequency range can be extended to improve its vibration isolation performance, but the reduction of stiffness may lead to the weakness of the load-bearing capacity. Therefore, many researchers try to design a vibration isolation system with high static stiffness and low dynamic stiffness. As a passive nonlinear vibration isolation system, the QZS system can obtain high static stiffness and low dynamic stiffness without using external energy, so it has good low frequency vibration isolation performance on the basis of guaranteeing its bearing capacity. Recently, the research on QZS systems has become a hot topic.

Alabuzhev[1] comprehensively summarizes the design theory of vibration control system with quasi-zero-stiffness characteristics. Brennan[2,3] and Yin Hualin[4] et al. establish simple QZS models by combining vertical springs (positive stiffness elements) and oblique springs (negative stiffness elements), and study static and dynamic characteristics of the systems. The results show that, the effect of the QZS isolation systems is better than linear vibration isolation systems in the low-frequency range.

An Euler column spring has unique mechanical properties of high static stiffness and low dynamic stiffness, which can be used as a low frequency vibration element. Winterfloor[5] et al. design an Euler column spring isolator consisting of two groups of flat springs in the Euler buckling mode. The two flat springs can move in two directions, producing positive and negative stiffness and adjusting the ratio of flexural rigidity of the system, which can produce ultra-low dynamic stiffness. Virgin and Plaut[6,7] et al. study static and dynamic characteristics of a system composed of Euler column springs by experiment. The results show that the system has a small fundamental frequency and a wide range of



vibration isolation frequency, and its effect can be improved by reducing the system's damping. Yu-lin Mei[8] et al. design a kind of Euler spring with varying cross-sections and analyze its static stiffness, dynamic stiffness and load-bearing capacity.

In this paper, a kind of nonlinear vibration isolation system is designed, which has high static stiffness and low dynamic stiffness. The system consists of an air spring, Euler buckled beams and compliant mechanisms. Where, the air spring is a positive stiffness element, and the Euler buckled beams are negative stiffness elements, and the compliant mechanisms are used to make the negative stiffness structure get different prestress under a variable load. Theoretically, the system can achieve quasi-zero-stiffness characteristics in a wide and low frequency range.

2. Design of the Quasi-zero-stiffness System

2.1. Design of the Positive Stiffness Element

As a positive stiffness element, an air spring has nonlinear stiffness characteristics. During the modeling and simulation process of the air spring, in order to get accurate results, it is necessary to consider nonlinearity problems, including the material nonlinearity, the geometric nonlinearity and the change of the internal air pressure of the air spring.

The air spring is made of the composite material vulcanized by cord and rubber, which exhibits complex mechanical anisotropy and nonlinearity. In the paper, the asymptotic homogenization theory is used to calculate equivalent material parameters of the air spring, such as the equivalent elastic modulus and poisson's ratio. In the modeling process, the calculated equivalent material parameters are directly assigned to the corresponding material attributes.

The deformation of the air spring is a geometrically nonlinear problem. The stiffness of the air spring depends not only on its material properties and the initial configuration but also on the stress distribution and the displacement variation under loads. During the finite element analysis, the element type supporting geometric nonlinearity is adopted, and the large deformation effect is activated and a series of nonlinear equilibrium equations are solved iteratively.

The air spring is filled with gas with a certain pressure. With the compression of the air spring, the internal pressure is variable. In order to solve the problem, a multi-step analysis strategy is adopted, that is, in the finite element analysis process, the loading process is dispersed into sufficient steps. By calculating the internal volume in each iterative step, the internal pressure is got by the gas equation

$$PV^n = P_0V_0^n \quad (1)$$

Where P_0, V_0 are initial internal pressure and volume, P, V are internal pressure and volume after loading, n is polytropic exponent.

Based on the above analysis, the air spring model is established, as shown in Fig.1(a). The geometrical parameters of the upper and lower cover plate parts are height 490mm, radius 250mm, and thickness 8mm. Where, the entity unit SOLID185 is adopted, and the material attributes are set according to calculation results by asymptotic homogenization theory.

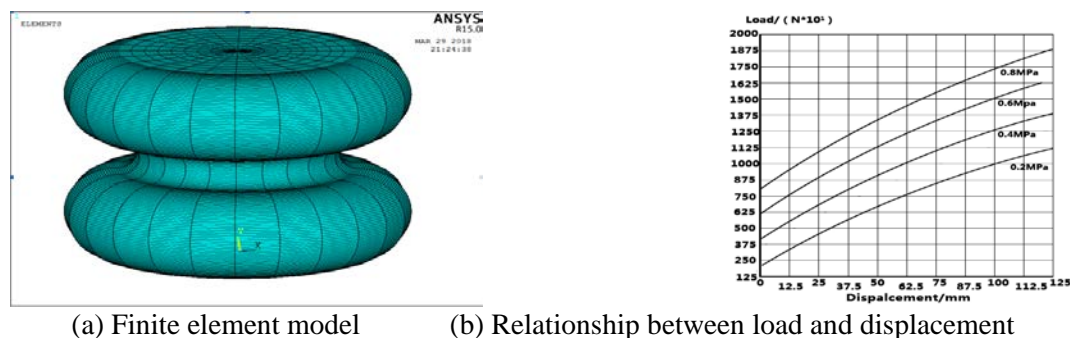


Figure 1. Positive stiffness element: the air spring

During the simulating process, a half of the model is analyzed. The initial air pressure is set as 0.2Mpa, 0.4MPa, 0.6Mpa and 0.8Mpa, respectively. A vertical load is applied to the air spring, assuming the gasbag of the air spring does not leak, then relationship curves between the vertical load and the vertical displacement are obtained, as shown in Fig.1 (b).

2.2. Design of the Negative Stiffness Element

The realization of the negative stiffness is presented in Fig.2. A vertical load F is applied to the beam AC at the midpoint, and two axial loads P are applied at both ends.

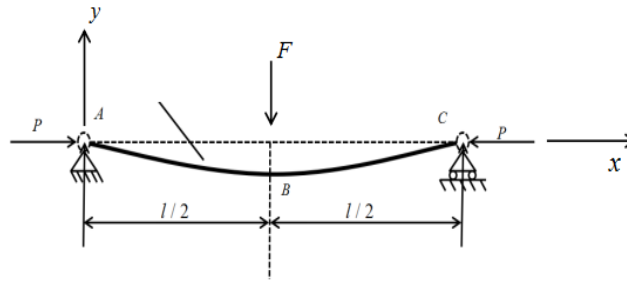


Figure 2. Negative stiffness mechanism

Due to the symmetry of the beam AC, only considering a half of the beam, we have

$$EIy'' = \frac{F}{2}x - Py \quad (2)$$

Setting $k^2 = P/EI$, the solution of Eq.(2) is

$$y = \frac{F}{2EI k^2}x - \frac{F \sin(kx)}{2EI k^3 \cos(kl/2)} \quad (3)$$

Letting $u = kl/2$, $x = l/2$, the maximum displacement at the middle point B is obtained

$$y_{\max} = \frac{Fl^3}{48EI} \frac{3(u - \tan u)}{u^3} \quad (4)$$

K is the beam stiffness at the point B, which can be described as

$$K = \frac{F}{y_{\max}} = \frac{48EI}{l^3} \frac{u^3}{3(u - \tan u)} \quad (5)$$

Obviously, K is variable with the change of u . Setting P_l is the critical pressure of the beam AC, we have $P_l = \pi^2 EI / l^2$. It can be found that the lateral stiffness at point B has been decreasing with the increase of P , and when $u \geq \pi/2$ ($P/P_l \geq 1$), the negative stiffness occurs.

According to the principle of the negative stiffness, a kind of Euler buckled beam is designed, as shown in Fig.3(a), where the length is 460mm, the cross-sectional area is $10 \times 40 \text{mm}^2$ and the critical pressure $P_l = 27643 \text{N}$.

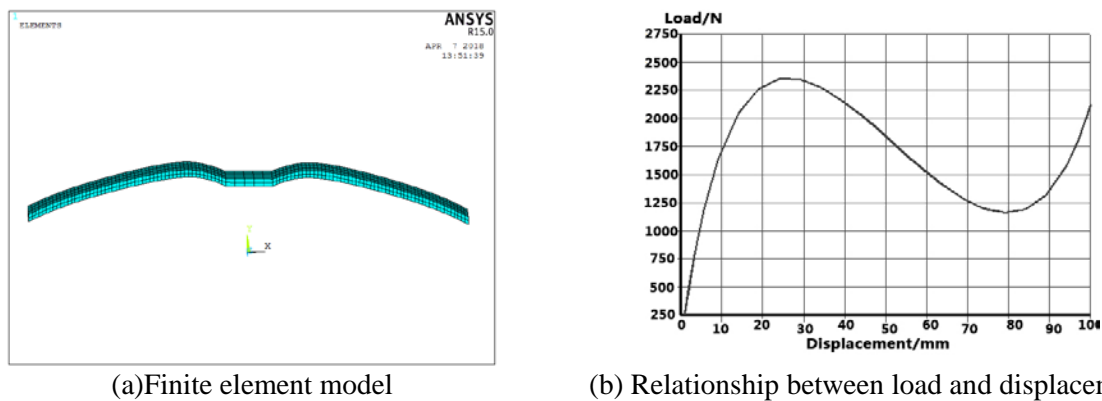


Figure 3. Negative stiffness element: the Euler buckled beam

During the simulating process, the axial loads P are gradually applied to the Euler buckled beam by using the temperature load method, meanwhile, the vertical load F is gradually increased. From the Fig.3(b), P is greater than P_l when the displacement is 25mm, resulting in the negative stiffness.

2.3. Realization of Quasi-zero-stiffness under a Variable Load

For a traditional QZS isolation system, the working load is determined. By combining positive and negative stiffness elements, a quasi-zero-stiffness vibration isolation system can be realized under a certain load condition. However, parameters of the traditional QZS system, such as stiffness and damping, can't be adjusted with the change of environment and excitation, therefore, if the load is slightly changed, the system can't make sure the quasi-zero-stiffness.

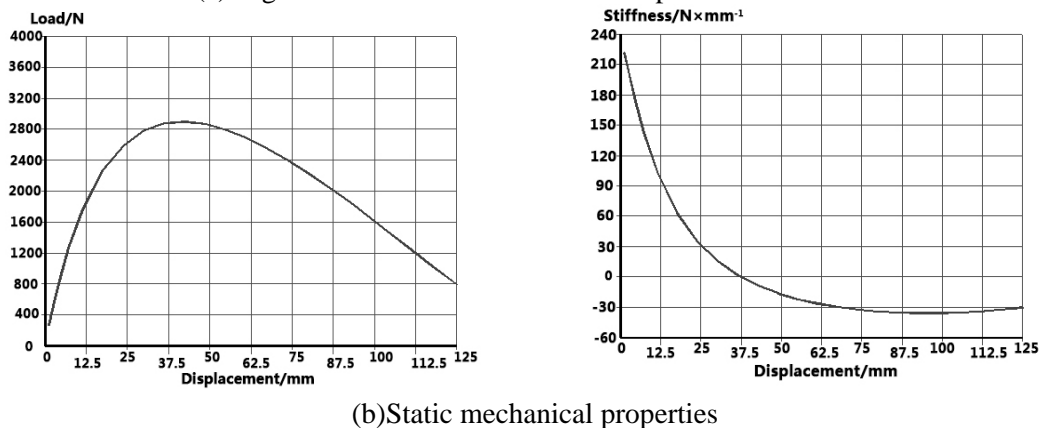
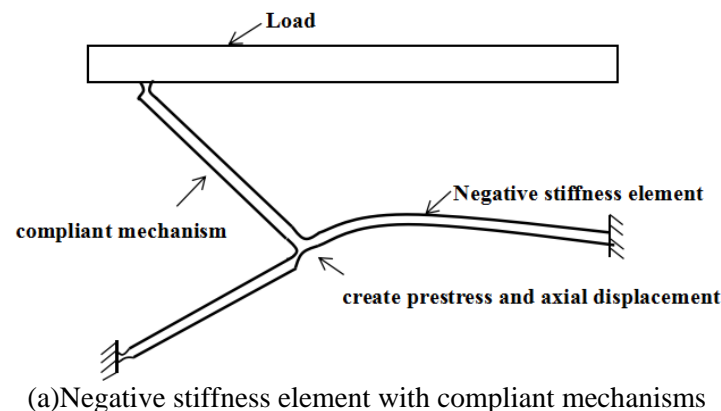


Figure 4. Characteristics of negative stiffness element with compliant mechanisms

The above analysis shows that when the axial load applied to the negative stiffness element is variable, its stiffness will change accordingly. Therefore, the dynamic stiffness of the system can reach zero with this change of the axial load. The compliant mechanism is a new type of transmission and supporting mechanism[9]. In order to design a kind of isolation system with quasi-zero-stiffness characteristics under a variable load, we introduce compliant mechanisms into the combination of positive and negative stiffness elements, and use the compliant mechanisms to make the negative stiffness structure get different prestress under a variable load.

In this section, a compliant mechanism is designed as shown in Fig.4(a). A load can be transferred to a negative stiffness element through the compliant mechanism, in other words, the variable axial load and lateral displacements can be applied to the negative stiffness structure.

During the simulating process, the displacement load is applied to the flexible negative stiffness element. The relationship curves between load, stiffness and displacement are obtained, as shown in Fig.4(b). It can be seen that the absolute value of negative stiffness increases with the displacement load applied. Therefore, the stiffness of the negative stiffness element can be changed with the external load. Consequently, the quasi-zero-stiffness can be realized under the variable load.

3. Analysis of Static Characteristics of the System

3.1. Static Characteristics under a Certain Load

Fig.5 is a QZS isolator without compliant mechanisms, which consists of an air spring and Euler buckled beams.

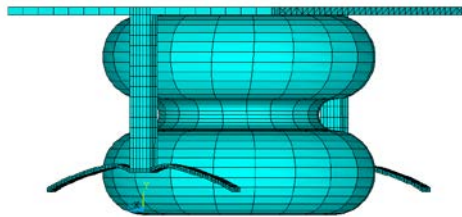


Figure 5. Finite element model of quasi-zero-stiffness system without compliant mechanisms

Here, only mechanical properties in the vertical direction are studied. Considering the isolation system is axisymmetric, during the simulating process, a half of the model is analyzed. The initial air pressure of the air spring is set as 0.2Mpa, 0.4Mpa, 0.6Mpa and 0.8Mpa, respectively. And a vertical load is applied to the system. The simulation data are shown in Table 1, including vertical displacements and vertical reaction forces.

Table 1. Load(L)-displacement(S) relationship

S [mm]	L [Kg]			
	0.2Mpa	0.4Mpa	0.6Mpa	0.8Mpa
0	384	587	795	992
10	598	823	1081	1284
20	735	1000	1248	1452
30	800	1075	1313	1556
40	840	1083	1360	1591
50	846	1088	1364	1600
60	851	1090	1372	1613
70	889	1125	1402	1654
80	960	1187	1451	1723
90	1088	1332	1605	1884
100	1384	1666	1910	2223

According to data in Table 1, taking the initial pressure of 0.6Mpa as an example, Fig.6 is obtained. It can be found that the dynamic stiffness of the vibration isolation system is approaching to zero at $S=50\text{mm}$. In other words, when the load is 1364kg, the dynamic stiffness approaches to zero withholding its static bearing capacity, which satisfies the low frequency isolation condition.

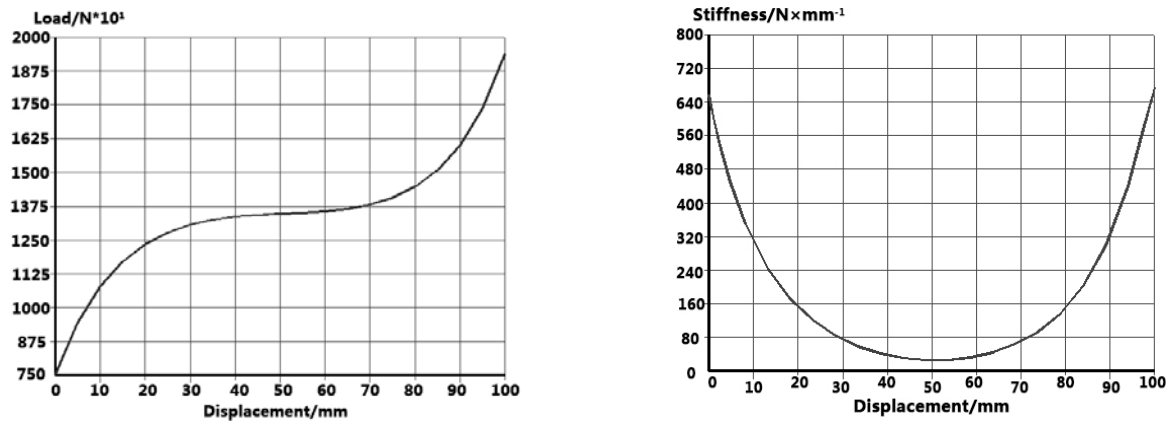


Figure 6. Quasi-zero-stiffness system without compliant mechanisms (0.6Mpa)

3.2. Static Characteristics under a Variable Load

The QZS isolation system analyzed above is only for a certain load condition. When the load changes, compliant mechanisms are introduced to create the corresponding prestress and displacement applying to negative stiffness elements so that the quasi-zero-stiffness can be realized under the variable load. The model is shown in Fig.7.

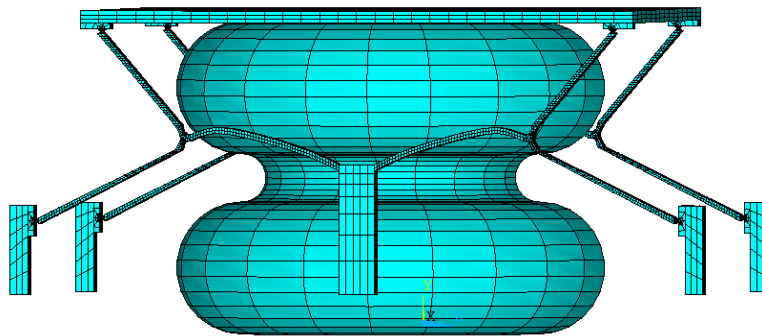


Figure 7. Finite element model of quasi-zero-stiffness system with compliant mechanisms

Taking the initial pressures of 0.4Mpa, 0.6Mpa and 0.8Mpa as examples, a displacement load is applied to the system and simulation results are shown in Fig.8. When the initial pressure is 0.4Mpa, in the range of 1100kg-1400kg, the vibration isolation system has a lower dynamic stiffness; when the initial pressure is 0.6Mpa, in the range of 1300kg-1600kg, the dynamic stiffness is in a lower range; when the initial pressure is 0.8Mpa, in the range of 1600kg-1800kg, the dynamic stiffness is in a lower range. Obviously, initial pressures of the air spring can be changed to widen the loading capacity.

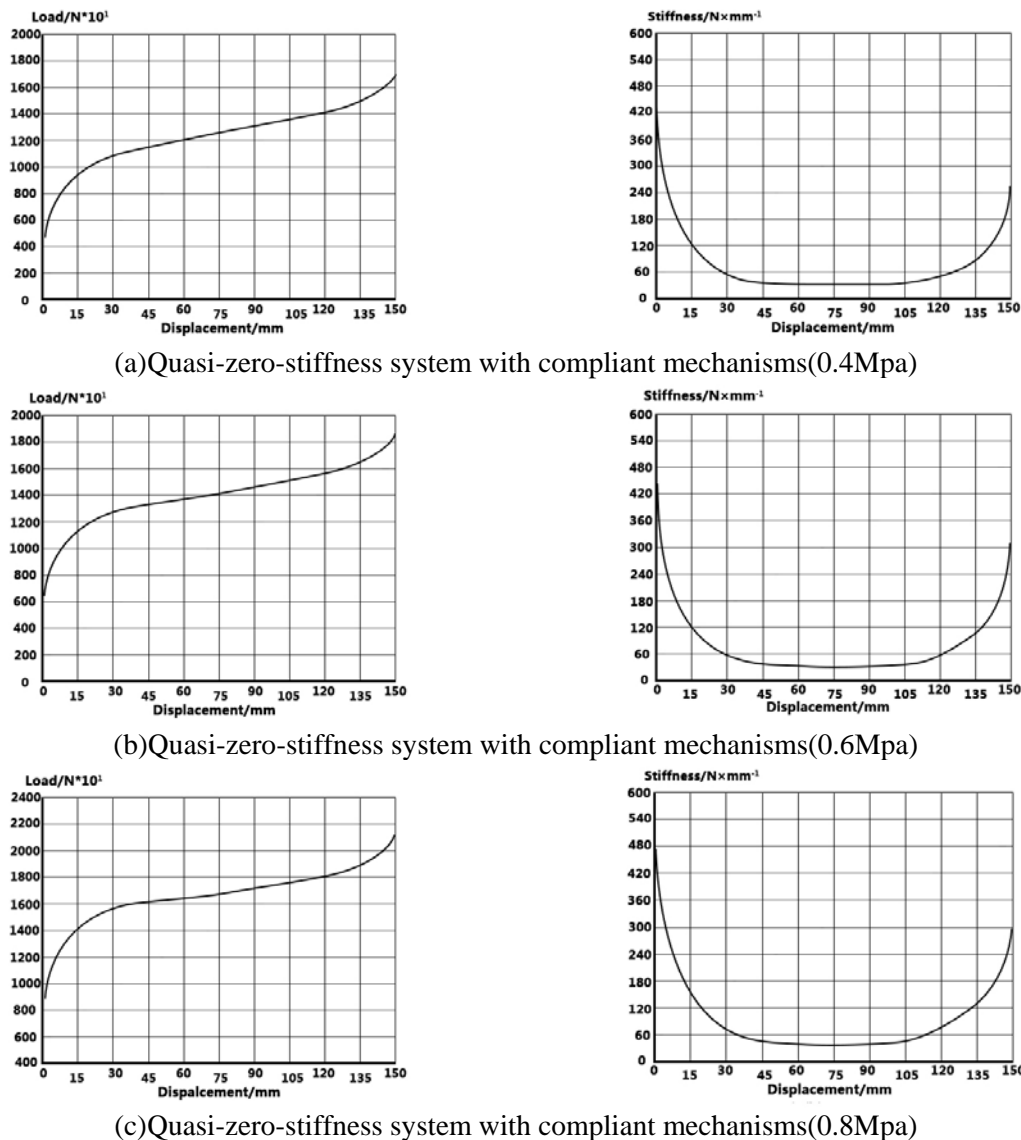


Figure 8. Static mechanical properties under different initial conditions

4. Analysis of Dynamic Characteristics of the System

Static analysis can determine a structure's ability of withstanding stable loads. And structural dynamics can analyse responses of stress, displacement or acceleration under dynamic loads.

4.1. Dynamic Modelling and Solution

A QZS vibration isolation system is a nonlinear system with one-degree-of-freedom, its viscous damping is C . Assuming that the system carries an object with a weight of m , the system keeps in a position of dynamic zero stiffness, and the position is defined as a static equilibrium position. The displacement loading position of 75mm is defined as an initial origin of curves, then displacement loads of 30-120mm are applied for analysis. A cubic polynomial is used to fit load-displacement curves, and the equations can be fitted

$$f = k_3 y^3 + k_1 y = 6.5 \times 10^5 y^3 + 59150 y \quad (6)$$

Where $f/(N)$ is external load applying to the system, $y/(m)$ is the displacement of the system.

According to reference[10], the dynamic modelling of a nonlinear vibration system is established. When a harmonic force of $F = H \cos(\omega t)$ is applied to the system, a slight vibration produces near the equilibrium position. The differential equation of motion for the nonlinear system can be written as

$$m\ddot{y} + c\dot{y} + k_3 y^3 + k_1 y = H \cos(\omega t) \quad (7)$$

By using non-dimensional parameters $\tilde{y} = y/e$, $\gamma = k_3 e^2 / k_1$, $\xi = c / 2m\omega_n$, $\omega_n = \sqrt{k_1/m}$, $\tau = \omega_n t$, $\Omega = \omega / \omega_n$, here $e = H / k_1$, $(k_3 = 0, \omega = 0)$, the non-dimensional equation of motion is got

$$\ddot{\tilde{y}} + 2\xi \dot{\tilde{y}} + \gamma \tilde{y}^3 + \tilde{y} = \tilde{H} \cos(\Omega \tau) \quad (8)$$

The harmonic balance method is used to solve Eq.8, assuming that the solution has the form

$$\tilde{y} = A \cos(\Omega \tau + \theta) = A \cos \varphi \quad (9)$$

which implies

$$\begin{aligned} \dot{\tilde{y}} &= -\Omega A \sin \varphi, \quad \ddot{\tilde{y}} = -\Omega^2 A \cos \varphi \\ \tilde{y}^3 &= A^3 \cos^3 \varphi = \frac{3}{4} A^3 \cos \varphi + \frac{1}{4} A^3 \cos 3\varphi \\ \tilde{H} \cos(\Omega \tau) &= \tilde{H} \cos(\varphi - \theta) = \tilde{H} \cos \varphi \cos \theta - \tilde{H} \sin \varphi \sin \theta \end{aligned} \quad (10)$$

substituting Eq.10 into Eq.8 yields

$$-\Omega^2 A \cos \varphi - 2\xi \Omega A \sin \varphi + \frac{3}{4} \gamma A^3 \cos \varphi + A \cos \varphi = \tilde{H} \cos \varphi \cos \theta + \tilde{H} \sin \varphi \sin \theta \quad (11)$$

Making corresponding coefficients on the both sides of Eq.11 equal, we have

$$\begin{aligned} \frac{3}{4} \gamma A^3 + (1 - \Omega^2) A &= \tilde{H} \cos \theta \\ -2\xi \Omega A &= \tilde{H} \sin \theta \end{aligned} \quad (12)$$

Then a relationship between response amplitudes and excitation frequencies can be got by squaring and adding both equations in Eq.12, which results in

$$\frac{9}{16} A^6 \gamma^2 + \frac{3}{2} (1 - \Omega^2) A^4 \gamma + [(1 - \Omega^2)^2 + 4\xi^2 \Omega^2] A^2 = \tilde{H}^2 \quad (13)$$

According to Eq.13, the response amplitudes A are influenced by the excitation condition and system parameters. Simulation results of amplitude-frequency characteristics are given in Fig.9(a). The horizontal axis is the excitation frequency ω ($\omega = \omega_n \Omega$), and the vertical axis is A , where the corresponding parameters are $k_1 = 59150$, $m = 1400 \text{ Kg}$, $\omega_n = \sqrt{k_1/m} = 6.5 \text{ Hz}$, $\gamma = 0.02$, $\xi = 0.01$, $H = 0.1$.

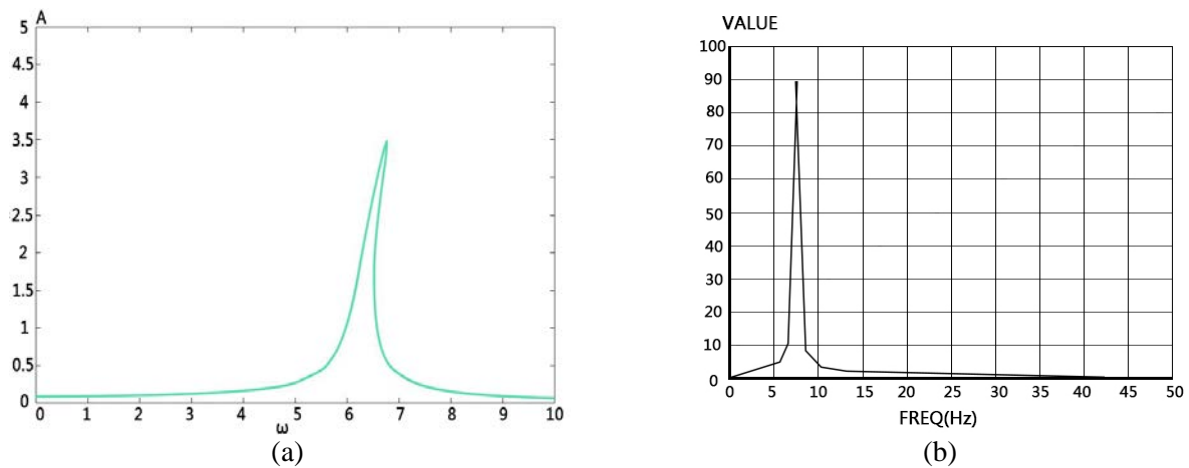


Figure 9. Amplitude-frequency characteristics

4.2. Modal Analysis

In order to determine the vibration properties of the QZS system with compliant mechanisms in Fig.7, natural frequencies and vibration modes of the system are investigated. The whole structure is fully constrained. Assuming that the initial pressure is 0.6MPa and the bearing weight is 1470kg. By using the modals synthesis theory, the first two natural frequencies and mode shapes are extracted and analyzed, as shown in Fig.10.

Simulation results show that the low order modes are excited during the deformation of the air spring element and the Euler buckled beams. The two natural frequencies are 7.00Hz and 9.57Hz, respectively. The actual vibration of the QZS system is the superposition of all modes, and the low order modes have great influence.

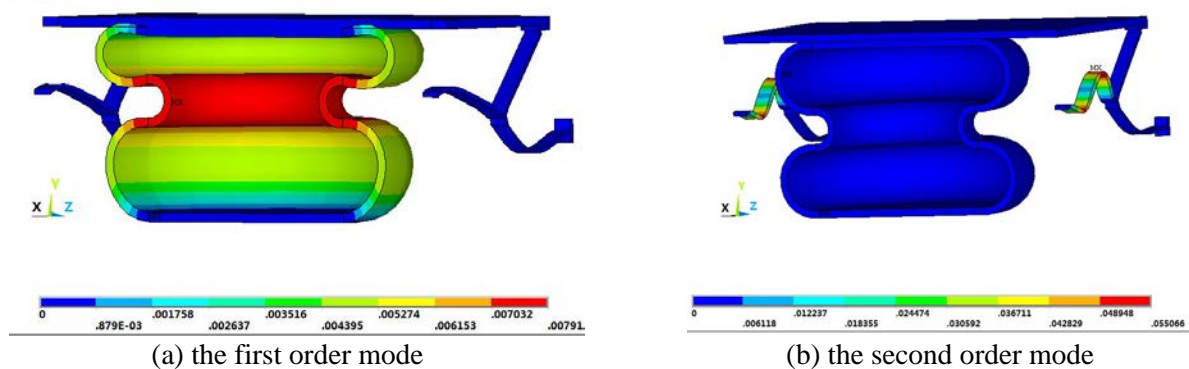


Figure 10. Vibration modes

4.3. Harmonic Response Analysis

In order to further research dynamic responses of the QZS system with compliant mechanisms, the modals superposition method is adopted to analyze a harmonic response of the system.

Assume that there is a mass of 1450kg on the top of the system, and the frequency range of the external vertical sinusoidal excitation is 0-50Hz. A harmonic response curve is shown in Fig.9(b). It can be seen that when the excitation frequency is about 7Hz, a resonance occurs. The results obtained by the harmonic response analysis are consistent with those calculated above by the harmonic balance method, which proves the correctness of the simulation results, and the system can get a quasi-zero-stiffness at a low frequency.

5. Summary

In this paper, a new kind of quasi-zero-stiffness vibration isolator is designed, which consists of positive stiffness elements, negative stiffness elements and compliant mechanisms. System parameters of the vibration isolator, such as stiffness and damping, can be adjusted with changes of environment and excitation. And when external loads are slightly changed, the vibration isolator still has quasi-zero-stiffness characteristics without sacrificing the load-bearing capacity.

By researching static and dynamic characteristics of the new kind of quasi-zero-stiffness vibration isolator, conclusions can be drawn as follows:

(1) By combining positive and negative stiffness elements, a QZS vibration isolator can be realized under a certain working load condition. It has high static stiffness and low dynamic stiffness and can achieve low-frequency vibration isolation.

(2) By introducing compliant mechanisms, a QZS vibration isolator can be realized under a variable working load condition, and the variable working load range can be expanded by parameter optimization designs.

(3) Compared with a traditional QZS vibration isolator, the new kind of quasi-zero-stiffness vibration isolator has better vibration isolation performance in the low frequency range.

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