Influence of aspect ratio on the linear and non-linear steady state forced vibration response of cylindrical open shells

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Abstract. The paper deals with the linear and non-linear steady state periodic response estimation of isotropic cylindrical open shells. The displacement field of first order shear deformation theory has been employed and the analysis is based on finite element method. C^0 continuous, eight-noded serendipity quadrilateral shear flexible element with five nodal degrees of freedom has been used for the analysis. The geometric non-linearity is included in the analysis using von Kármán's assumption for small strains and moderately large deflection. The governing equations of motion have been obtained in the time domain and the nonlinear periodic responses are obtained using shooting technique. The entire non-linear steady state frequency response curve consisting of stable and unstable regimes has been obtained employing shooting technique along with arc length and pseudo-arc length continuation schemes. The present method does not involve any apriori assumption on the participating modes. The comparison of linear and non-linear frequency response curves reveals large differences between the two. Frequency response curve, response history and the phase plane plots have been obtained, for different aspect ratio (L/b), to explore the linear and non-linear forced vibration characteristics of cylindrical open shells. The asymmetric nature of the phase plane plots reveal significant higher harmonic contributions. With the increase in aspect ratio, thin open shells reveal increased peak amplitude in both linear and nonlinear analysis. The presence of secondary peak in the nonlinear frequency response for greater aspect ratio is due to modal interaction between first and higher modes.

1. Introduction

Shells are among the most common structural elements used in many engineering structures, including pressure vessels, pipes, submarine hulls, wings and fuselages of airplanes, exteriors of rockets, missiles as well as integral parts of machines where they are subjected to dynamic loading and hence their vibratory response analysis is important for efficient, reliable and failure-proof designs. Thin cylindrical shells are used, particularly in the aerospace application, as it undergoes buckling/vibration either due to static/dynamic axial load or external pressure and sometimes both. Obtaining an effective design presents a difficult and challenging problem, hence a detailed analysis is required of a shell structure. The analysis of a shell is a challenging task, as it resists the load applied largely by its curvature, that is by just changing the curvature of the shell keeping the thickness and the material same, a totally different load carrying capacity can be obtained for the shell. Hence, parameters such as curvature, thickness and the boundary conditions play a crucial role in determining the behaviour of the shell. The peculiarities of shell structural behaviour, the difficulties of analysis, and the wide use of shell structures have instigated a large research effort in shell analysis. A shell is said to be geometrically

nonlinear if the strain-displacement relations are nonlinear. This paper deals with the linear and geometrically nonlinear forced vibration analysis of isotropic cylindrical open shells.

Structural members such as shells often undergo large amplitude vibrations where the amplitudes are generally of the order of their thickness. The dynamic behaviour obtained based on linear strain-displacement relations generally provide a conservative estimate and may be treated as the first approximation to the actual behaviour. The forced vibration characteristics obtained by considering the geometric non-linearity predicts a closer approximation of the actual dynamic behaviour. The combined effect of geometric nonlinearity and middle surface curvature for shell panels is expected to significantly alter the dynamic behaviour as compared to linear analysis. The need for non-linear steady state forced vibration response arises from the requirement of safe, optimal and efficient design of structural components which are in the form of cylindrical shells.

Most of the research carried out are on free vibration characteristics of cylindrical shells whereas research on forced vibrations analysis of isotropic cylindrical shells are very limited. Alijani & Amabili [1] and Qatu, et al. [2], have carried out extensive literature reviews on the nonlinear vibrations of shells. Liew, et al. [3] presented a review article with bibliography documents, focussed on the developments in the vibration analysis of thin, moderately thick, and thick shallow shells. The nonlinear free flexural vibration behaviour of the isotropic/laminated orthotropic noncircular rings using the finite element approach and Newmark time marching scheme was investigated by Patel et al. [4-5] and reported softening nonlinearity. Kurylov and Amabili [6] analysed the non-linear forced vibration response of simply supported circular cylindrical shells. Alijani and Amabili [7] analysed the nonlinear forced vibrations of laminated circular cylindrical panels. Khan and Patel [8-9] carried out non-linear forced vibration analysis of bimodular plates and cylindrical panels based on modified shooting technique. The effect of boundary conditions on the nonlinear forced vibration response has been analyzed by Amabili [10]. The linear as well as geometrically non-linear steady state periodic response of cylindrical shell panels has been carried out by Khan, et al [11] by employing the displacement field of first order shear deformation theory based on finite element method. C^0 continuous, eight-noded serendipity quadrilateral shear flexible element with five nodal degrees of freedom has been used in the analysis.

From the literature review, it can be noted that most of the study on the non-linear steady state periodic response of cylindrical shells are based on methods requiring apriori assumptions on the participating mode. Further, it is observed that the convergence to a steady state solution is very slow by employing the direct integration approach. In addition, the steady state response cannot be ascertained in the direct time integration approaches. In this paper the nonlinear steady state periodic response of open cylindrical shell is analysed using the first order shear deformation theory. The governing equation of motion is solved using shooting method coupled with New Mark time marching scheme. The unstable portion of the non-linear frequency response curves are obtained using arc-length and pseudo-arc length continuation methods. A detailed parametric study is conducted to study the influence of aspect ratio (L/b) on the linear and nonlinear dynamic behaviour of isotropic cylindrical open shell. The differences between the linear and nonlinear frequency response curves have been analyzed. The phase plane plots and the steady state response history reveal unequal positive/negative half cycle time. The secondary peak in the frequency response of shell with L/b =2 is due to interaction of first and higher modes.

2. Governing Equations and Solution Procedure

The geometry and coordinate system of an isotropic open cylindrical shell are shown in Figure 1. The coordinates x, y and z are along the meridional, circumferential and radial/thickness directions, respectively. The length, width, thickness, included angle and radius of the shell are L, b, h, ϕ and r respectively.
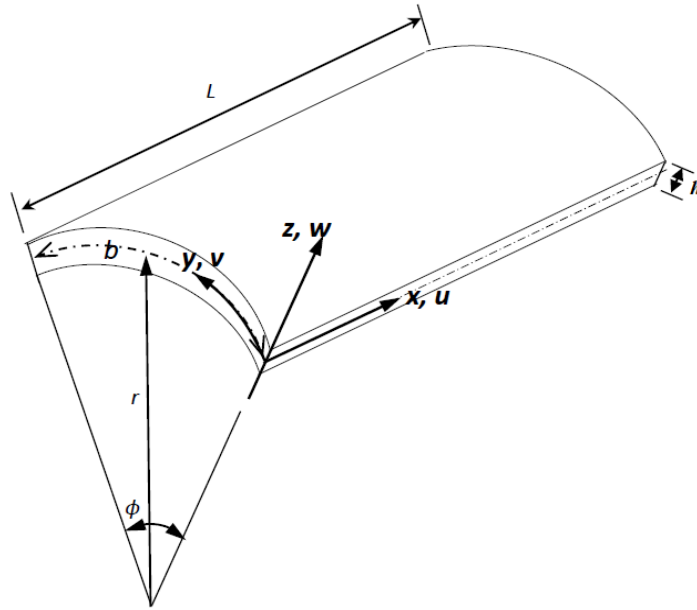
Fig. 1 Geometry and co-ordinate system of open cylindrical shell

The displacement field (u, v, w) at a point(x, y, z) is expressed as a function of middle surface displacements u_0, v_0, w_0 and the independent rotations θ_x and θ_y of the meridional and hoop sections, respectively, using first-order shear deformation theory as:

$$u(x, y, z, t) = u_0(x, y, t) + z\theta_x(x, y, t)$$
$$v(x, y, z, t) = v_0(x, y, t) + z\theta_y(x, y, t) \tag{1}$$
$$w(x, y, z, t) = w_0(x, y, t)$$

Based on Sander's shell theory, strain field in terms of mid-surface deformation variables can be written as:

$$\{\varepsilon\} = \{\varepsilon_{xx} \quad \varepsilon_{yy} \quad \gamma_{xy} \quad \gamma_{xz} \quad \gamma_{yz}\}^T = \begin{Bmatrix} \varepsilon_P^L \\ 0 \end{Bmatrix} + \begin{Bmatrix} z\varepsilon_b \\ \varepsilon_s \end{Bmatrix} + \begin{Bmatrix} \varepsilon_P^{NL} \\ 0 \end{Bmatrix} \tag{2}$$

where, $\varepsilon_P^L, \varepsilon_b, \varepsilon_s$ and ε_P^{NL} representing linear mid-surface membrane, bending, transverse shear and nonlinear mid-surface membrane strain vectors, respectively, are defined as:

$$\varepsilon_p^L = \begin{Bmatrix} u'_{0,x} \\ v'_{0,y} + \dfrac{w_0}{r} \\ u'_{0,y} + v'_{0,x} \end{Bmatrix}, \varepsilon_b = \begin{Bmatrix} \theta'_{x,x} \\ \theta'_{y,y} \\ \theta'_{x,y} + \theta'_{y,x} \end{Bmatrix}, \varepsilon_s = \begin{Bmatrix} \theta_x + w'_{0,x} \\ \theta_y + w'_{0,y} - \dfrac{v_0}{r} \end{Bmatrix}, \varepsilon_P^{NL} = \begin{Bmatrix} \dfrac{1}{2}\left(w'_{0,x}\right)^2 \\ \dfrac{1}{2}\left(w'_{0,y} - \dfrac{v_0}{r}\right) \\ w'_{0,x}\left(w'_{0,y} - \dfrac{v_0}{r}\right) \end{Bmatrix} \tag{3}$$

The membrane stress resultant$\bar{N} = \{N_{xx} \quad N_{yy} \quad N_{xy}\}^T$, moment resultant $\bar{M} = \{M_{xx} \quad M \quad M_{xy}\}^T$ and transverse shear stress resultant $\bar{Q} = \{Q_{xz} \quad Q_{yz}\}^T$ vectors are related to the membrane $\varepsilon_p = \varepsilon_P^L + \varepsilon_P^{NL}$, bending ε_b and transverse shear ε_s strain vectors through the constitutive relation as.

$$\left\{\begin{matrix}\overline{N}\\\overline{M}\end{matrix}\right\} = \begin{bmatrix}A & B\\B & D\end{bmatrix}\left\{\begin{matrix}\varepsilon_p\\\varepsilon_b\end{matrix}\right\}, \quad \overline{Q} = E\varepsilon_s \tag{4}$$

where $B_{ij} = 0$ and the nonzero elements of A_{ij} , D_{ij} ($i, j = 1,2,6$) and E_{ij} ($i, j = 4,5$) for an isotropic shell are

$$A_{11} = A_{22} = \frac{Eh}{1-v^2}, \quad A_{21} = A_{12} = \frac{vEh}{1-v^2}, \quad A_{66} = \frac{Eh}{2(1+v)}$$

$$D_{11} = D_{22} = \frac{Eh^3}{12(1-v^2)}, \quad D_{21} = D_{12} = \frac{vEh^3}{12(1-v^2)}, \quad D_{66} = \frac{Eh^3}{24(1+v)},$$

$$E_{44} = E_{55} = \frac{Eh}{2(1+v)} \tag{5}$$

where [A], [D] and [B] are extensional, bending, and bending-extensional coupling stiffness coefficient matrices, respectively. [E] is the shear stiffness matrix.

The total potential energy functional U consisting of strain energy and potential of the uniformly distributed transverse load is given by:

$$U = \frac{1}{2}\iint\left(d^T K d + \frac{1}{3}d^T K_1 d + \frac{1}{6}d^T K_2 d\right)dxdy - \iint F w_0 dxdy \tag{6}$$

where [K] represents linear stiffness matrix, $[K_1]$ and $[K_2]$ are representing quadratic and cubic non-linear stiffness matrices, respectively.

The kinetic energy of the shell is given by:

$$T = \frac{1}{2}\iint\left(\rho h(\dot{u}_0^2 + \dot{v}_0^2 + \dot{w}_0^2) + \frac{\rho h^3}{12}(\dot{\theta}_x^2 + \dot{\theta}_y^2)\right)dxdy \tag{7}$$

where ρ is the mass density of shell. Dot over the variables denotes the derivative with respect to time.

Using a C^0 continuous, eight-noded serendipity quadrilateral shear flexible shell element with five nodal degrees of freedom $u_{0i}, v_{0i}, w_{0i}, \theta_{xi}, \theta_{yi}$, the field variables are interpolated in terms of their nodal values and shape functions as:

$$(u_0, v_0, w_0, \theta_x, \theta_y) = \sum_{i=1}^{8} N_i^0(u_{0i}, v_{0i}, w_{0i}, \theta_{xi}, \theta_{yi}) \tag{8}$$

Employing Hamilton's principle, using standard finite element assembly procedure and considering dissipative forces, the governing equation of motion can be written as:

$$[M]\{\ddot{\delta}\} + [C]\{\dot{\delta}\} + [K + (1/2)K_1(\delta) + (1/3)K_2(\delta)]\{\delta\} = \{F\} \tag{9}$$

where the damping matrix is defined based on Rayleigh proportional damping model and the damping matrix [C] is taken as
$$[C] = \hat{\alpha}[M] + \beta[K] \tag{10}$$
The governing equations are solved using the procedure described in authors work [8-9] and are not presented here for the sake of brevity. The solution is started at frequency far away from resonance using New-Mark time marching and modified shooting method. At the frequency near bifurcation points where the modified shooting method fails to give converged solution the frequency response curve is continued using arc-legth/pseudo-arc length continuation schemes.

3. Results and Discussion

The influence of aspect ratio on the linear and nonlinear forced vibration response of isotropic cylindrical open shell subjected to uniformly distributed harmonic force with forcing frequency varied in the vicinity of first fundamental frequency has been investigated. The material of the shell is assumed to be Aluminium with Young's Modulous 70 GPa, Density 2778 kg/m^3 and poisons ratio 0.3. Based on mesh convergence a 10x10 mesh size is used to discretize the shell. The boundary condition choosen for the analysis is all edges simply supported.

The nonlinear frequency response curves for both positive and negative half cycle are presented in Fig. 2. It can be inferred from the positive/negative half cycle frequency response curves (Fig. 2 a) that the peak amplitude increase with the increase in aspect ratio (L/b). The nonlinear response amplitude for negative half cycle is significantly greater for L/b=1, 2 as compared to L/b=0.5 indicating that the difference in positive and negative half cycle stress amplitude is greater in the GNL analysis compared to the GL(Fig. 2 b) analysis. The peak amplitude also increases with the increase in aspect ratio for both GNL and GL. The percentage difference between GNL and GL increases with the increase in aspect ratio.

The comparison of peak amplitude in the linear and nonlinear analysis for the positive half cycle reveals that peak amplitude in linear analysis is significantly larger compared to non-linear analysis, with the peak amplitude in the linear analysis being 1.09, 3.02 and 3.77 times the peak amplitude obtained from non-linear analysis for L/b = 0.5, 1 and 2, respectively. The peak amplitude in the linear analysis being 1.06, 2.13 and 2.98 times the peak amplitude obtained from non-linear analysis for L/b = 0.5, 1 and 2, respectively for the negative half cycle. From the positive half cycle GNL response curve it can be observed that the peak amplitude for L/b = 1 and 2 are 2.37 and 4.53 times the peak value for L/b = 0.5 respectively. In the negative half cycle GNL response curve, the peak value for L/b = 1 and 2 are 3.27 and 5.58 times the peak value for L/b = 0.5 respectively.

The steady state response history in the linear analysis (Fig. 3 (b)) reveal equal positive/negative half cycle amplitude with equal time in tension and compression. In contrast the non-linear response history (Fig. 3 (a)) depicts greater negative half cycle time indicating that the panel is in inward motion for greater portion of the periodic cycle. The negative half cycle time is 1.12, 1.29 and 1.10 times the positive half cycle L/b = 0.5, 1 and 2, respectively.

The phase plane plots (Fig. 4 (a) & (b)) corresponding to peak amplitude in the frequency response curves depict asymmetry for the nonlinear case which is due to significant higher harmonic contributions. The secondary peak in the nonlinear response curve for shell with L/b =2, is due to interaction of first and higher modes.

4. Conclusion

The linear as well as nonlinear forced vibration response of isotropic cylindrical open shells have been obtained in time-domain using modified shooting technique and continuation schemes. The complete frequency response curves consisting of stable and unstable regimes have been obtained. It is apt to make a mention here that the direct time integration method fails to predict the unstable regimes. A detailed analysis has been carried out to analyze the influence of aspect ratio on the linear and nonlinear dynamic response of open cylindrical shells. Based on the analysis it is found that with the increase in aspect ratio the peak amplitude increases with the increase in aspect ratio for both linear and nonlinear analysis. It is concluded that the peak amplitude for linear analysis is considerable greater than that for nonlinear analysis and this difference increases with the increase in aspect ratio. It can also be concluded that the negative half cycle amplitude as well as cycle time is greater than the corresponding positive half cycle values with the nonlinear analysis. The nonlinear phase plane plots depict greater higher harmonic contributions. The secondary peak in the frequency response curves of L/b=2 is due to modal interaction between first and third mode.
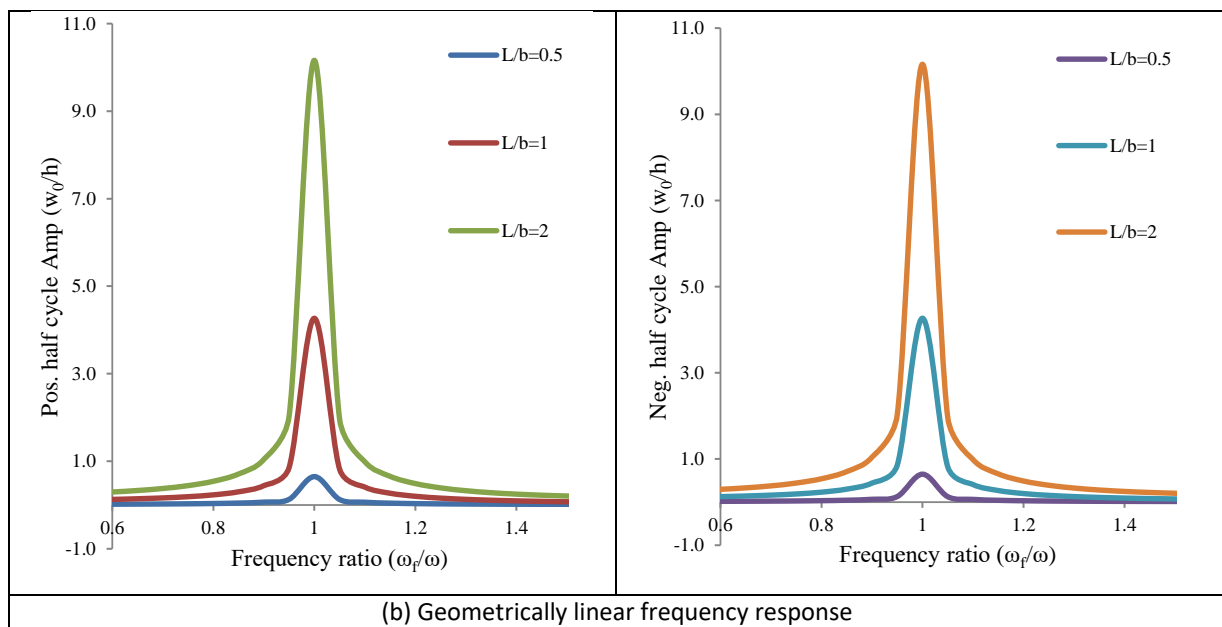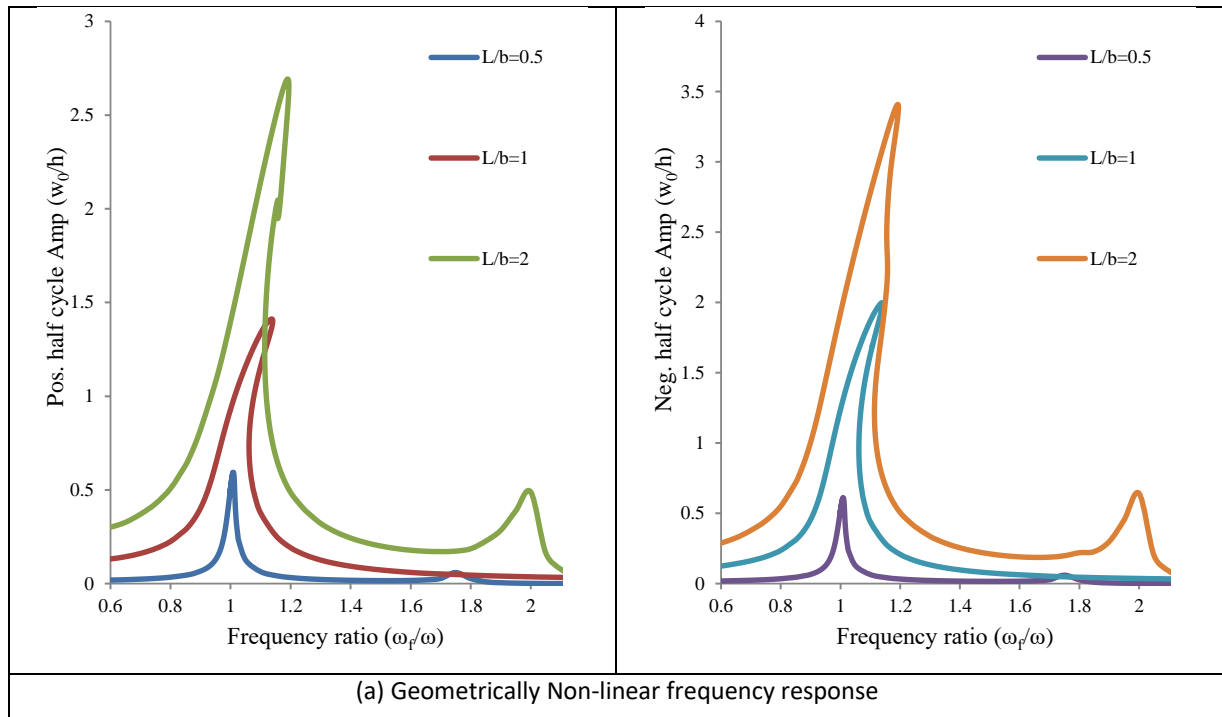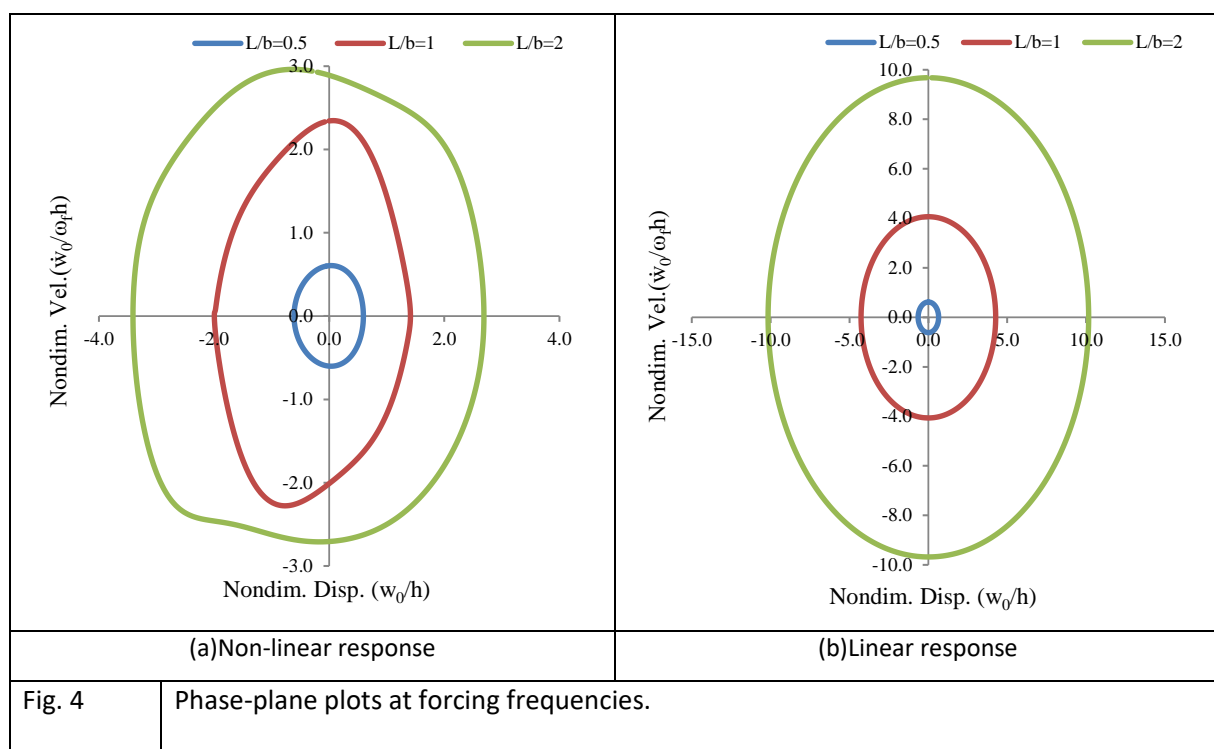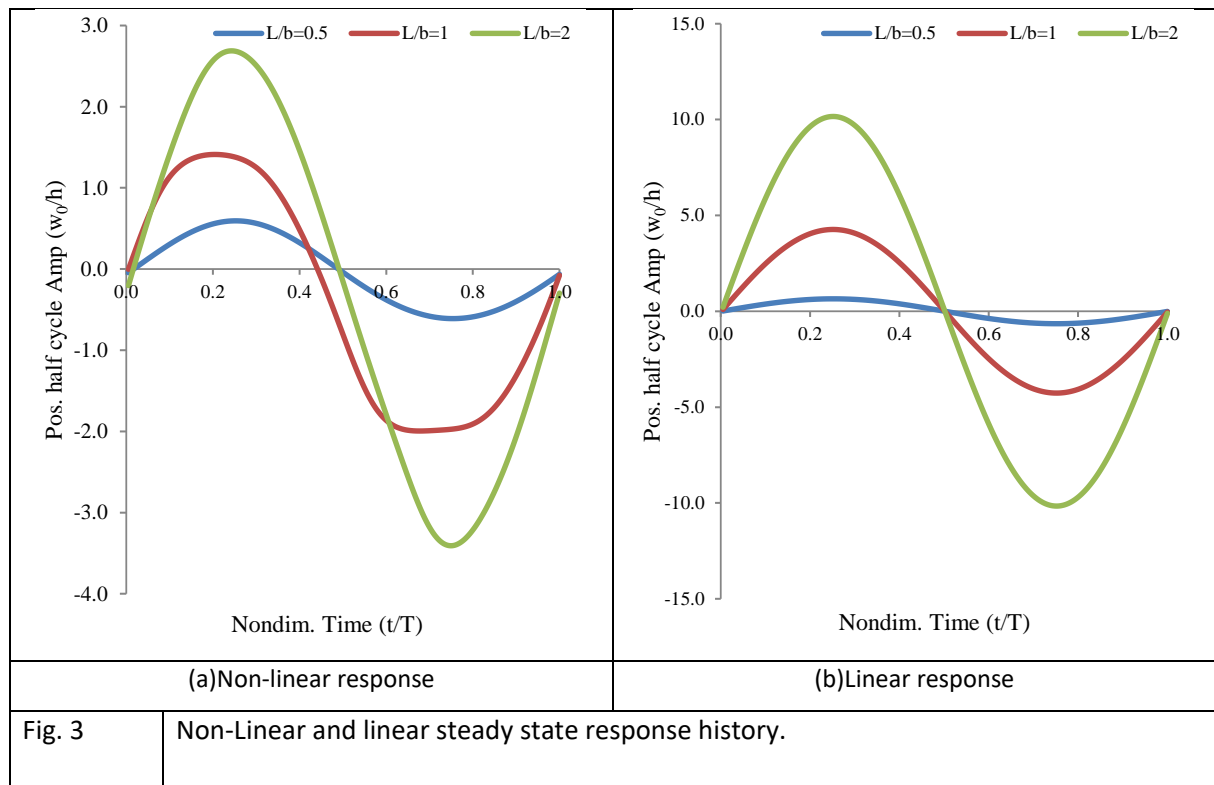
(a) Geometrically Non-linear frequency response

(b) Geometrically linear frequency response

Fig. 2	Steady state forced vibration frequency response curves (a) Non-Linear, (b) linear of cylindrical open shell. (SSSS, b/h=100, b/r=0.1, Load=5 kPa, ξ=0.010)

	(a)Non-linear response	(b)Linear response
Fig. 3	Non-Linear and linear steady state response history.	

	(a)Non-linear response	(b)Linear response
Fig. 4	Phase-plane plots at forcing frequencies.	

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