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Estimation of in-plane thermal conductivity of copper clad board by inverse analysis using artificial neural networks

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Abstract. This study presents a methodology to estimate the in-plane thermal conductivity of copper clad board used in electronic applications. Experiments are performed in vacuum environment and an inverse heat conduction problem (IHCP) is solved employing artificial neural networks to estimate the in-plane thermal conductivity. A comparison of estimates of thermal conductivity as obtained by solving the inverse problem using back propagation artificial neural networks trained using two algorithms namely Levenberg-Marquardt and Scaled Conjugate Gradient are presented.

1. Introduction

In electronic applications copper clad boards are used as substrate material for mounting electronic components. The copper clad boards are etched to make printed circuit boards. These boards are in-plane isotropic in nature. The objective of the present study is to estimate the in-plane thermal conductivity $(k_x = k_y = k)$ of copper clad boards by solving an inverse heat conduction problem (IHCP) using steady state temperature measurements. The inverse analysis is performed using backpropagation artificial neural networks (ANNs) employing both Levenberg-Marquardt and Scaled Conjugate Gradient algorithms.

There are different techniques proposed by researchers in the literature which are implemented to solve a variety of inverse problems. Beck [1] proposed a nonlinear parameter estimation technique to estimate the thermal transport properties such as thermal conductivity and volumetric heat capacity of solids simultaneously under transient conditions. Garnier et al. [2] introduced a novel technique to carry out surface temperature measurements on the test specimen to estimate thermal properties of composites. The advantages of parameter estimation techniques in terms of more information and faster results in comparison with other steady state techniques such as guarded hot plate or transient methods such as line source method, flash method etc. were brought out. Sawaf and Ozisik [3] performed inverse analysis for the simultaneous estimation of the principal thermal conductivities of orthotropic materials using conjugate gradient method and Levenberg-Marquardt method. Mejias et al. [4] carried out a comparative study on non-linear parameter estimation techniques like conjugate gradient method and Levenberg-Marquardt method for estimating thermal conductivities of an orthotropic solid by solving an inverse problem. Gobbé et al. [5] conducted an experimental study for the measurement of thermal conductivity of multilayer orthotropic media characterized by isotropic behaviour along planes parallel to layers.

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Artificial neural networks have now been used extensively for solving inverse heat transfer problems. Krejsa et al. [6] presented the advantage and limitations of various back propagation algorithms that are implemented to obtain the successful solution of inverse heat conduction problems using ANNs. Cortés et al. [7] harnessed the capability of ANN for solving the inverse problem to retrieve the heat source generation term of the hot plate in a guarded hot plate apparatus. Chanda et al. [8] presented a novel methodology for the simultaneous estimation of the thermal conductivities along principal directions of an anisotropic honeycomb composite by solving an inverse heat conduction problem using ANN trained with Levenberg-Marquardt algorithm.

The literature review shows the immense capabilities of inverse methodology for estimating the thermal conductivities. Also ANNs have been successfully employed to solve inverse heat transfer problems. These facts motivate to undertake the present study. Hence, the present study involves numerical simulation of direct problem, temperature measurements using of steady state experiments and estimation of in-plane thermal conductivity using artificial neural networks trained using Levenberg-Marquardt and scaled conjugate gradient algorithms.

2. Direct problem

The direct problem is formulated to simulate the exact experimental conditions numerically and to obtain the required temperature distribution. The copper clad test board is modeled as a homogenous and in-plane isotropic board with $k_x = k_y = k$, where the in-plane thermal conductivity need to be determined and out-plane thermal conductivity k_z is already known. Commercial software Ansys Fluent is employed for solving the direct problem. The governing equation for the direct problem is given by equation (1).

$$k_x \frac{\partial^2 T}{\partial x^2} + k_y \frac{\partial^2 T}{\partial y^2} + k_z \frac{\partial^2 T}{\partial z^2} = 0 \tag{1}$$

where k_x, k_y and k_z are the thermal conductivities in x, y and z directions. The direct problem geometry consists of a square board of dimensions $180 \times 180 \times 1.60$ mm with a foil heater fixed at the centre of the top surface of the board. The whole assembly is mounted on a copper frame which serves as a heat sink. The direct problem geometry is depicted in figure 1.

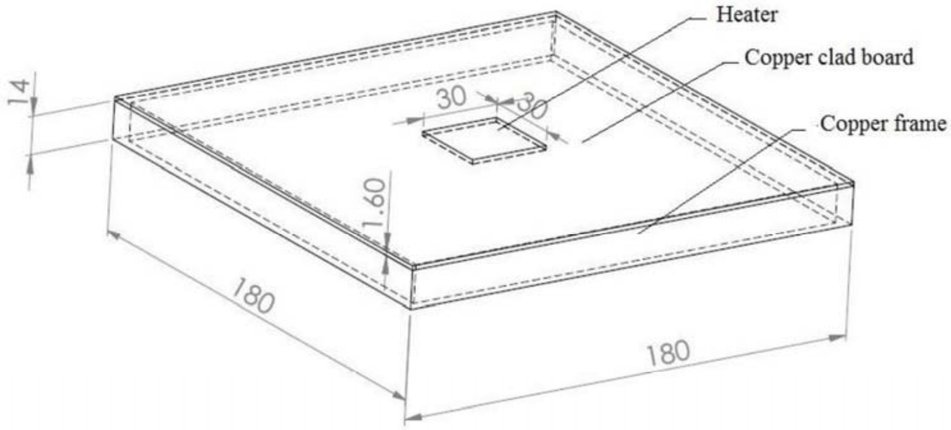

Figure 1. Geometry for direct problem simulation

2.1. Meshing of direct problem geometry

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The direct problem geometry is discretised into quadrilateral cells. Regular auto meshing with proximity option is used to mesh the geometry. A grid independence study is conducted to obtain the optimum number of cells to save the computational time while not affecting the accuracy of the solution. The results of the grid independence study are shown in figure 2. A mesh with 0.24 million nodes yielded grid independent results, hence it was used for the present study.

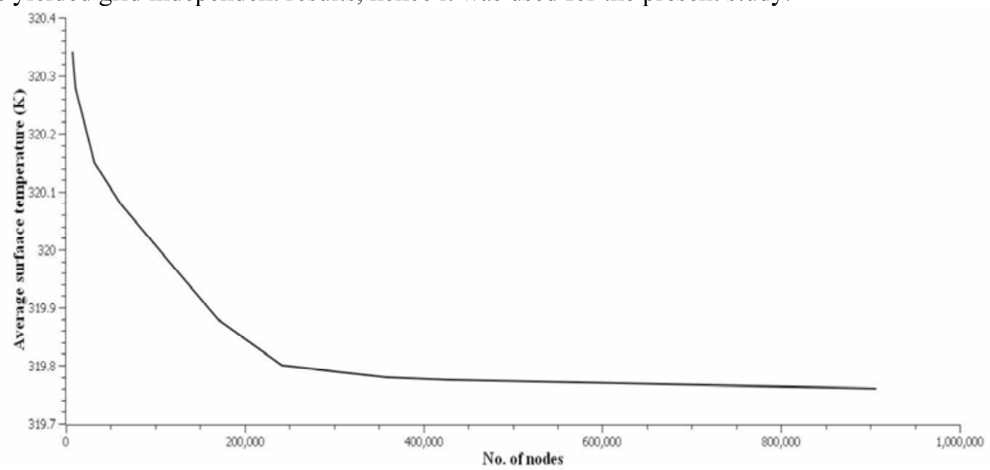

Figure 2. Grid independence study

2.2. Boundary conditions

The following boundary conditions are imposed on the direct problem,

- A heat flux of 1111 W/m^2 is applied to heater domain.
- Isothermal condition (298 K) is imposed on copper frame.
- All exposed surfaces of copper clad board except the surface in contact with the copper frame are given adiabatic condition.

2.3. Direct problem solution

The governing heat conduction equation is solved using Ansys Fluent. The convergence criterion is said to be reached when the temperature residuals fall below 10^{-9}.

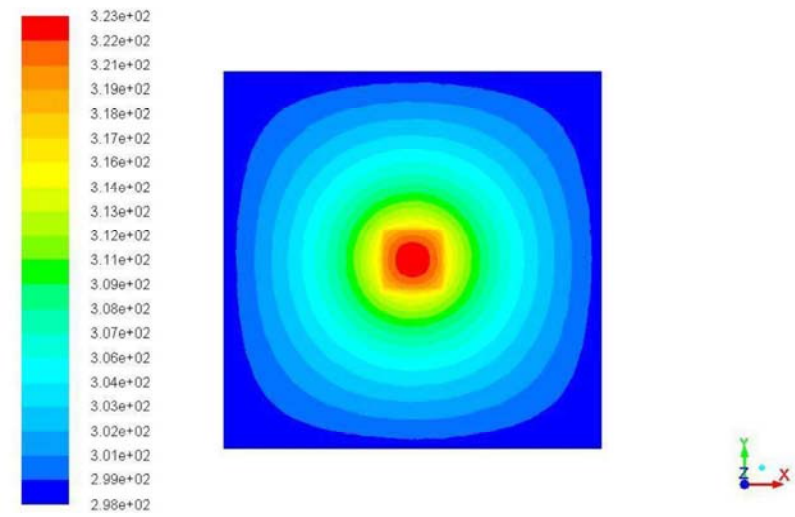

Figure 3. Temperature contour for the simulation case

The results for a sample simulation case with $k_x = k_y = k$ = 10W/mK and k_z = 0.34W/mK is shown in figure 3. As seen in figure 3, the temperature contour assumes a circular profile implying equal heat transfer in the in-plane direction which corroborates well with the laws of heat transfer. The temperature is maximum near the heater and appreciable variations of temperatures are observed along both X and Y directions to a certain distance after which appreciable temperature gradients do not exist as depicted in figure 4. These temperature gradients hold the key to the successful estimation of thermal conductivity.

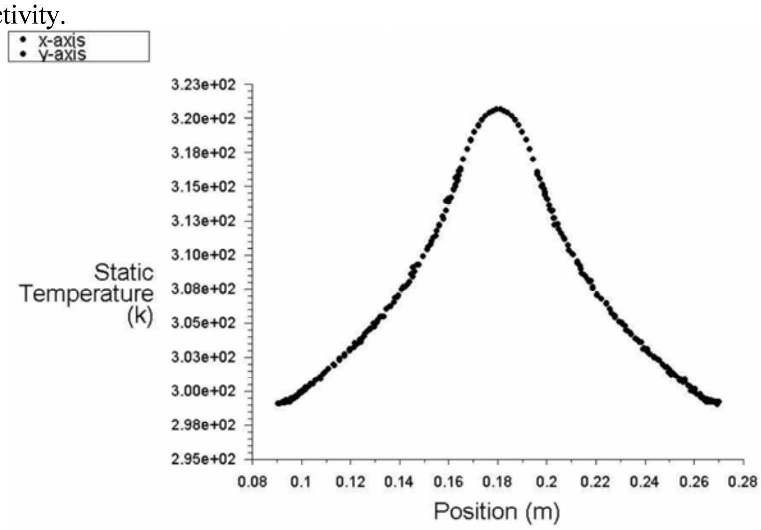

Figure 4. Temperature profile along X and Y axis of test sample

3. Experimental setup and methodology

The photograph of the entire experimental setup is depicted in figure 5. It consists of a vacuum chamber that develops a vacuum level of 10^{-5} mbar with the help of vacuum pumps.

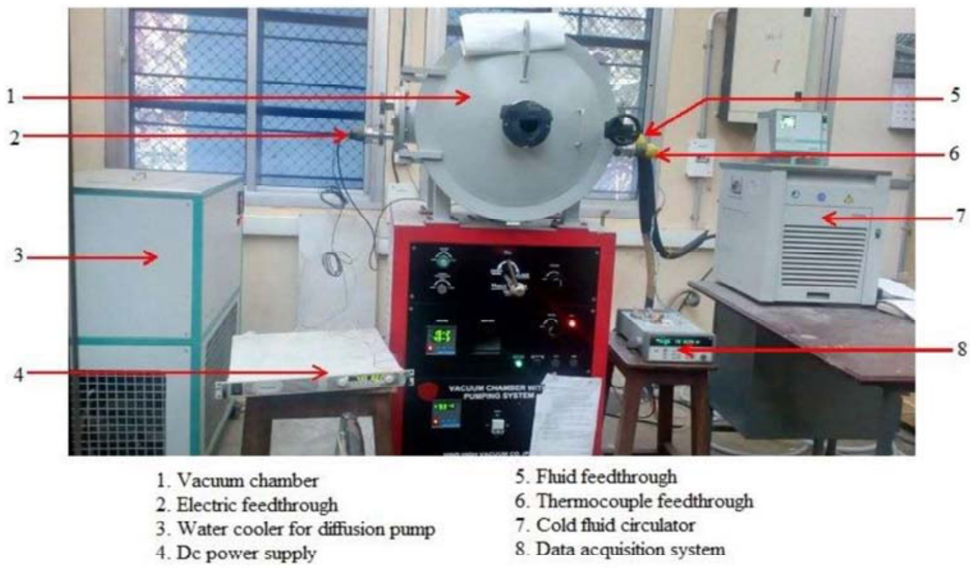

Figure 5. Photograph of the experimental setup

The vacuum is maintained to eliminate convection losses. A square copper frame along with an aluminium cold plate serves as heat sink. A constant temperature water bath is used to circulate cold fluid to aluminium plate to maintain it at a constant low temperature.

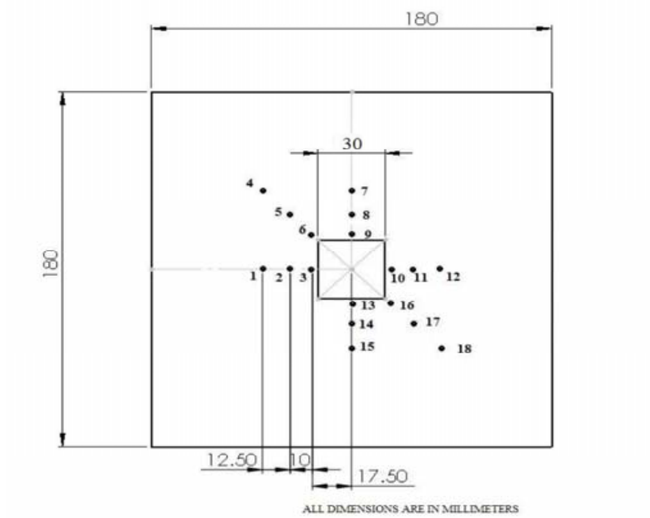

Figure 6. Locations of thermocouples in the test sample

T type thermocouples are fixed at the locations on the top surface of the test as shown in figure 6. A foil type heater is placed at the middle of the board (test sample). A low emissivity sheet is used to cover all the exposed surfaces of the board to minimize radiation losses along with a multilayer insulation cover. This entire test assembly is placed in the vacuum chamber. A vacuum level of 10^{-5} mbar is maintained in the chamber. The foil heater is energized to 1W and the sink is kept at 25 ^{o}C. The steady state is deemed to be achieved when the temperature sensor readings do not change by ±0.1^{o} C for a time period of one hour. These steady state temperatures are used for the estimation of thermal conductivity of copper clad board by solving the inverse problem using ANN. Details of the experimental set up are provided in study by Chanda et al. [8]. More details are omitted for the sake of brevity.

An uncertainty analysis is carried out to determine the uncertainty in both measured and derived quantities. For the direct measurements like temperature, voltage, current and vacuum pressure the least count of the instruments are taken as uncertainty. Uncertainty in the derived quantities i.e. power is evaluated using equation (2) as given by Venkateshan [9].

$$\sigma_P = \sqrt{\left(\frac{\partial P}{\partial V}\sigma_V\right)^2 + \left(\frac{\partial P}{\partial I}\sigma_I\right)^2} \tag{2}$$

where σ is the uncertainty in measurement, P is the power, V is the voltage and I is the current. The results of the uncertainty analysis are given in table 1.

Table 1. Uncertainties in measured and derived quantities

Sl. no.	Quantity measured	Uncertainty
1	Temperature	±0.5 K
2	Voltage	±0.001V
3	Current	±0.0001A
4	Power	0.102%
5	Vacuum pressure	10^{-6}mbar

4. Inverse methodology and artificial neural network

The inverse methodology involves the solution of inverse heat conduction problem to obtain the in plane thermal conductivity using ANN where the experimental temperatures are fed as inputs to the ANN.

Artificial neural networks are computational structures used to simulate nonlinear and complex relationships between input and output data. By mimicking the human brain ANNs can identify the correlated patterns between inputs and outputs. Therefore ANNs trained for sample set of inputs and outputs can be used to predict the outputs when presented with input data set in the similar range.

In the present study artificial neural network based on feed forward back propagation principle is implemented. Temperature data at nine pre assigned locations are used as inputs and in-plane thermal conductivity as output of the neural network. A neuron independence study is undertaken to determine the number of neurons required in the hidden layer of the artificial neural network to accurately simulate the problem under consideration. Using direct problem simulations, 300 different values of $k_{xx} = k_{yy} = k$ lying in the range from 0.5 to 15 W/mK is generated. Out of these, 210 samples (70%) are used for training and 90 samples (30%) for testing the neural network. The number of hidden neurons is varied from 3 to 12. Using performance metrics such as mean relative error (MRE) and correlation coefficient (R^2) as given by equation (3) and equation (4), the optimum number of hidden neurons is determined.

$$MRE = \frac{1}{m}\sum_{i=1}^{m}\left|\frac{k_{actual} - k_{retrieved}}{k_{actual}}\right| \tag{3}$$

$$R^2 = 1 - \left[\frac{\sum_{i=1}^{m}(k_{actual} - k_{retrieved})^2}{\sum_{i=1}^{m}(k_{actual} - \bar{k}_{actual})^2}\right] \tag{4}$$

Results of the neuron independence study are given in table 2.

Table 2. Neuron independent study results

Sl. no.	No. of hidden neurons	MRE	R^2
1	3	0.5786	0.9458
2	5	0.5728	0.9332
3	7	0.6098	0.9371
4	9	0.8461	0.9387
5	**10**	**0.4985**	**0.9568**
6	12	0.9134	0.9012

A network with 10 neurons in the hidden layer is found to have lowest MRE and highest R^2 values of 0.4985 and 0.9568 respectively and is used for the estimation of in-plane thermal conductivity.

5. Results of the estimation of thermal conductivity of copper clad board

As stated earlier, the test specimen is an in-plane isotropic copper clad board of dimensions 180×180×1.6 mm. Steady state experiments are performed on the test sample instrumented with T-type thermocouples placed at pre-assigned locations. These steady state temperatures are used as an input to the inverse model. The training range of ANN and the search range used in this problem for performing parameter estimation has been limited to $2 \leq k \leq 15$ W/mK. The artificial neural network used is trained with a data set comprising of 210 samples obtained by simulating the direct problem in the range as stated earlier, with temperatures as inputs and thermal conductivities as outputs.

Artificial neural network used in this study is trained using the feed forward backpropagation technique. The algorithms employed to train the ANN in the current study are Levenberg - Marquardt algorithm and scaled conjugate gradient algorithm with back propagation of errors. Levenberg - Marquardt algorithm is one of the robust algorithms implemented by many researchers because of its efficacy in retrieving the parameters of interest from inverse problems. Scaled conjugate gradient algorithm utilises less computational resources. The tan sigmoid transfer function is applied to all the neurons in the hidden layer and pure-linear to the neurons in the output layer. A single hidden layer network with 10 neurons is used in the network. The neural networks are trained for 1000 epochs. The results of the estimated values of thermal conductivity as obtained from ANNs trained using two different algorithms are listed in table 3.

Table 3. Retrieved values of thermal conductivity

Sl. no.	Algorithm	Thermal conductivity(W/mK)
1	Levenberg - Marquardt	10.08
2	Scaled conjugate gradient	9.60

Figure 7 shows the parity plot with error bars depicting the comparison of measured temperatures with those obtained from direct problem simulation using the retrieved value of thermal conductivity using Levenberg - Marquardt algorithm and scaled conjugate gradient algorithm. The error bar corresponding to maximum measurement error in temperature i.e. 0.5 K

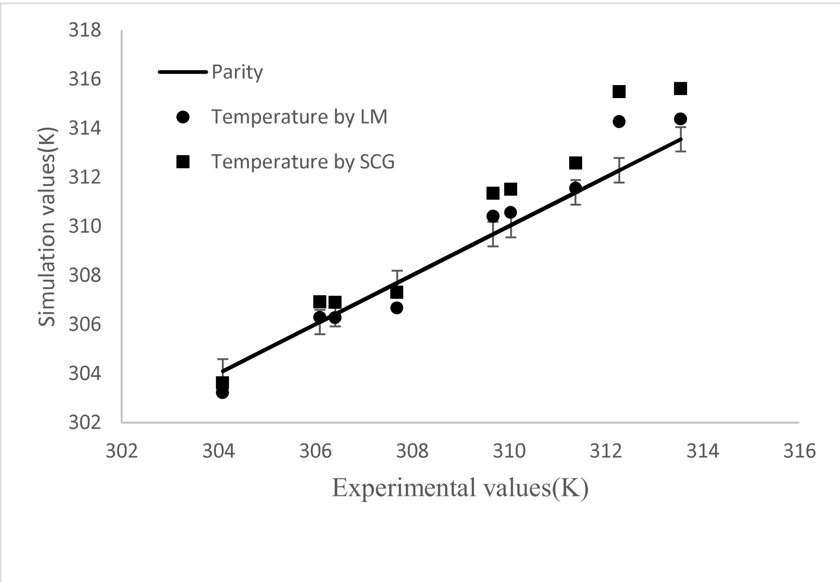

Figure 7. Parity plot of measured and simulated temperatures of copper clad board

As evident from figure 7, the temperature values obtained from simulation and measurement agrees well with each other. The Levenberg - Marquardt algorithm gives a slightly better approximation than scaled conjugate gradient algorithm. The bias free plot ensures that the governing physics has been captured well in the model. The thermal conductivity of the copper clad board estimated by using this technique agrees well with that reported by Azar and Graebner [10]. Therefore the present study using inverse methodology employing ANN is able to retrieve the in-plane thermal conductivity of copper clad board successfully.

6. Conclusion

The present study focused on estimating the in-plane thermal conductivity of copper clad board by employing synergistic combination of direct problem solution, experiments and inverse methodology employing artificial neural networks using Levenberg - Marquardt algorithm and scaled conjugate gradient algorithm. Two ANN models trained using different algorithms namely Levenberg - Marquardt and Scale Conjugate Gradient with 210 sample data obtained from direct problem simulations were used to retrieve thermal conductivity from experimental temperature data. Both the ANNs were found to predict the thermal conductivity values with good accuracy. It was also seen that, ANN trained using Levenberg Marquardt algorithm out performed that trained using Scale Conjugate Gradient algorithm slightly in terms of predicted estimates.

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