

Influence of Time-varying Mesh Stiffness Excitation on Vibration of Planetary Automatic Transmission

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Abstract. Based on the energy method, the time-varying mesh stiffness models of the internal and external meshing of the planetary gears are established. The translation-rotation coupled dynamics differential equation of the planetary transmission system is established through the concentrated mass method. Analyze the influence of time-varying mesh stiffness excitation on the vibration of planetary gear transmission. The results show that the vibration response of the planetary gear sets is mainly related to the connection form under the excitation of the time-varying mesh stiffness. The time-varying mesh stiffness has the greatest influence on the vibration of the planet gear. The vibration of the planet gear is obviously more serious than the other components of the same planetary. When multiple planetary gear sets are connected, the vibration of the output component is smaller than the other components. Moreover the vibration of the input planetary set is generally greater than the rear planetary set except the planet gear.

1. Introduction

During the operation process of the gear transmission system, there are not only external excitations caused by driving torque and load changes, but also internal excitations. Normally, the contact ratio of the meshing gear pair is not an integer. In the meshing process, the single and double tooth meshing occurs alternately, and the periodical change with time will change the overall mesh stiffness of the gear tooth [1, 2]. Gear mesh stiffness is the main reason of the vibration in gears [3]. Therefore, in order to study the influence of gear internal excitation on the dynamic response of the gear system, it is necessary to research the influence of gear mesh stiffness variation on the system vibration. This is the foundation for studying on the gear fault diagnosis, system dynamic characteristics analysis and life prediction. The influence of time-varying mesh stiffness on the tooth crack and tooth wear has been researched by scholars [4, 5]. Planetary gears are increasingly used in automatic transmissions because of their advantages such as small size, light weight, and large transmission ratio. Planetary gear meshing is different from that of fixed-axle gear train. Carrier drives the planet gears to orbit so that the planet gears mesh with the sun gear and the ring gear simultaneously [6]. Now domestic and foreign scholars have begun to analyze the influence of mesh stiffness on planetary gear transmission. Kahraman [7] and Zhu [8] have analyzed the load-sharing characteristics of planetary gear set considering the mesh stiffness.



In view of the above situation, this paper analyzes the influence of time-varying mesh stiffness excitation on vibration of planetary automatic transmission. Based on the energy method, this paper establishes the internal and external mesh stiffness models of planetary gear sets, and establishes the translation-rotation coupling dynamics differential equation of the planetary transmission system through the concentrated mass method. The research of vibration response on mesh stiffness excitation will help improve the fault diagnosis and optimization design of the gear transmission system.

2. Mesh stiffness modeling

2.1. External mesh stiffness modeling

The spur gear tooth two-dimensional variable section cantilever beam model is shown in Figure 1. In this figure, F is the meshing force perpendicular to the tooth face; h represents half the tooth thickness at the position where the meshing force acts; α_1 is the angle between the meshing force and the tooth thickness direction; d represents the effective length, that is, the distance between the meshing force acting position and the fixed part of the root circle; dx and $2hx$ respectively represent the width and length of the micro section of the position x from the meshing force.

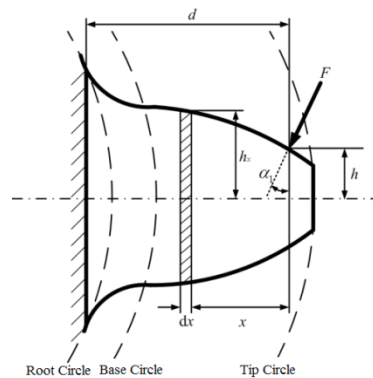


Figure 1. Non-uniform cross-section cantilever beam model for external gear.

Based on the principle of potential energy, through Hertz contact theory, elastic mechanics and the beam deformation theory in material mechanics, the expressions of bending, shear, compression and Hertz contact stiffness are calculated. In addition to the effect of gear teeth deformation on the mesh stiffness, the deformation of the gear wheel body also greatly influences the gear mesh stiffness. The equivalent stiffness on the mesh line caused by the wheel body deformation is given.

$$\left\{ \begin{array}{l} \frac{1}{K_b} = \int_0^d \frac{(x \cos \alpha_1 - h \sin \alpha_1)^2}{EI_x} dx \\ \frac{1}{K_s} = \int_0^d \frac{1.2 \cos^2 \alpha_1}{GA_x} dx \\ \frac{1}{K_a} = \int_0^d \frac{\sin^2 \alpha_1}{EA_x} dx \\ \frac{1}{K_h} = \frac{4(1-\nu^2)}{\pi EW} \\ \frac{1}{K_f} = \frac{\cos^2 \alpha_m}{WE} \left\{ L^* \left(\frac{u_f}{S_f} \right)^2 + M^* \left(\frac{u_f}{S_f} \right) + P^* (1 + Q^* \tan^2 \alpha_m) \right\} \end{array} \right. \quad (1)$$

Where u_f , S_f and other parameters can be gained by the article[9]. Therefore, by synthesizing the equivalent stiffness of meshing lines corresponding to the above-mentioned tooth bending deformation,

shear deformation, axial compression deformation, Hertzian contact deformation, and wheel body deformation, the mesh stiffness of the external meshing gear can be given.

$$K_e = 1 / \left(\frac{1}{K_{b1}} + \frac{1}{K_{s1}} + \frac{1}{K_{a1}} + \frac{1}{K_{f1}} + \frac{1}{K_{b2}} + \frac{1}{K_{s2}} + \frac{1}{K_{a2}} + \frac{1}{K_{f2}} + \frac{1}{K_h} \right) \quad (2)$$

In the formula, the footnotes 1 and 2 respectively represent the pinion gear and the wheel gear in the gear meshing pair.

2.2. Internal mesh stiffness modeling

Similar to the calculation of the external spur gear tooth stiffness, the internal gear teeth are regarded as variable cross-section cantilever beams. The potential energy principle is used to derive the internal gear tooth stiffness calculation formula. Therefore, the single tooth mesh stiffness of the internal gear pair can be given. In the formula, the indexes 1 and 2 respectively represent the external gear and the internal gear in the gear meshing pair. Here, it is assumed that the internal gear wheel body is rigid, and then the influence of the flexible deformation of the internal gear ring is ignored.

$$\left\{ \begin{array}{l} \frac{1}{K_b} = \int_0^d \frac{(x \cos \alpha - h \sin \alpha)^2}{EI_x} dx \\ \frac{1}{K_s} = \int_0^d \frac{1.2 \cos^2 \alpha}{GA_x} dx \\ \frac{1}{K_a} = \int_0^d \frac{\sin^2 \alpha}{EA_x} dx \\ \frac{1}{K_h} = \frac{4(1-\nu^2)}{\pi EW} \\ \frac{1}{K_f} = \frac{\cos^2 \alpha_m}{WE} \left\{ L \left(\frac{u_f}{S_f} \right)^2 + M^* \left(\frac{u_f}{S_f} \right) + P^* (1 + Q^* \tan^2 \alpha_m) \right\} \\ K_I = 1 / \left(\frac{1}{K_{b1}} + \frac{1}{K_{s1}} + \frac{1}{K_{a1}} + \frac{1}{K_{f1}} + \frac{1}{K_{b2}} + \frac{1}{K_{s2}} + \frac{1}{K_{a2}} + \frac{1}{K_{f2}} + \frac{1}{K_h} \right) \end{array} \right. \quad (3)$$

3. Planetary gear set dynamic modeling

The mass-concentration method is applied to establish the “translation-rotation” coupling dynamical differential equations of planetary gear by the Lagrange theorem. The dynamic model is shown as an example in Fig. 2, which is a planetary gear with internal and external meshing of four planet gears.

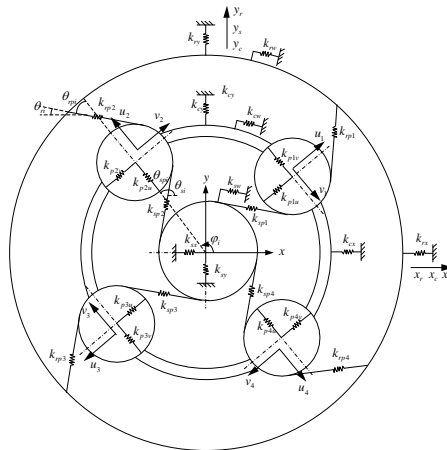


Figure 2. Four-planet-gear planetary set dynamic model.

The “translation-rotation” coupled dynamical differential equations of the four-planet-gear planetary set is given.

$$\begin{cases}
 m_s \ddot{y}_s + k_{sy} y_s = N_{sp1} \sin \theta_{s1} - N_{sp2} \sin \theta_{s2} - N_{sp3} \sin \theta_{s3} + N_{sp4} \sin \theta_{s4} \\
 m_s \ddot{x}_s + k_{sx} x_s = -N_{sp1} \cos \theta_{s1} - N_{sp2} \cos \theta_{s2} + N_{sp3} \cos \theta_{s3} + N_{sp4} \cos \theta_{s4} \\
 I_s \ddot{\theta}_s = T_s - N_{sp1} R_{sb} - N_{sp2} R_{sb} - N_{sp3} R_{sb} - N_{sp4} R_{sb} - N_{sw} R_{sb} \\
 m_{p1} \ddot{u}_{p1} + k_{p1u} u_{p1} = N_{sp1} \cos \theta_{sp1} - N_{rp1} \cos \theta_{rp1} \\
 m_{p1} \ddot{v}_{p1} + k_{p1v} v_{p1} = N_{sp1} \sin \theta_{sp1} + N_{rp1} \sin \theta_{rp1} \\
 I_{p1} \ddot{\theta}_{p1} = -T_{p1} + N_{sp1} R_{p1b} - N_{rp1} R_{p1b} \\
 m_{p2} \ddot{u}_{p2} + k_{p2u} u_{p2} = N_{sp2} \cos \theta_{sp2} - N_{rp2} \cos \theta_{rp2} \\
 m_{p2} \ddot{v}_{p2} + k_{p2v} v_{p2} = N_{sp2} \sin \theta_{sp2} + N_{rp2} \sin \theta_{rp2} \\
 I_{p2} \ddot{\theta}_{p2} = -T_{p2} + N_{sp2} R_{p2b} - N_{rp2} R_{p2b} \\
 m_{p3} \ddot{u}_{p3} + k_{p3u} u_{p3} = N_{sp3} \cos \theta_{sp3} - N_{rp3} \cos \theta_{rp3} \\
 m_{p3} \ddot{v}_{p3} + k_{p3v} v_{p3} = N_{sp3} \sin \theta_{sp3} + N_{rp3} \sin \theta_{rp3} \\
 I_{p3} \ddot{\theta}_{p3} = -T_{p3} + N_{sp3} R_{p3b} - N_{rp3} R_{p3b} \\
 m_{p4} \ddot{u}_{p4} + k_{p4u} u_{p4} = N_{sp4} \cos \theta_{sp4} - N_{rp4} \cos \theta_{rp4} \\
 m_{p4} \ddot{v}_{p4} + k_{p4v} v_{p4} = N_{sp4} \sin \theta_{sp4} + N_{rp4} \sin \theta_{rp4} \\
 I_{p4} \ddot{\theta}_{p4} = -T_{p4} + N_{sp4} R_{p4b} - N_{rp4} R_{p4b} \\
 m_c \ddot{y}_c + k_{cy} y_c = N_{cp1u} \sin \varphi_1 + N_{cp2u} \sin \varphi_2 + N_{cp3u} \sin \varphi_3 + N_{cp4u} \sin \varphi_4 \\
 m_c \ddot{x}_c + k_{cx} x_c = N_{cp1u} \cos \varphi_1 + N_{cp2u} \cos \varphi_2 + N_{cp3u} \cos \varphi_3 + N_{cp4u} \cos \varphi_4 \\
 I_c \ddot{\theta}_c = -T_c - N_{cw} R_{cb} + N_{cp1v} R_{cb} + N_{cp2v} R_{cb} + N_{cp3v} R_{cb} + N_{cp4v} R_{cb} \\
 m_r \ddot{y}_r + k_{ry} y_r = N_{rp1} \sin \theta_{r1} + N_{rp2} \sin \theta_{r2} - N_{rp3} \sin \theta_{r3} + N_{rp4} \sin \theta_{r4} \\
 m_r \ddot{x}_r + k_{rx} x_r = N_{rp1} \cos \theta_{r1} - N_{rp2} \cos \theta_{r2} - N_{rp3} \cos \theta_{r3} + N_{rp4} \cos \theta_{r4} \\
 I_r \ddot{\theta}_r = T_r + N_{rw} R_{rb} + N_{rp1} R_{rb} + N_{rp2} R_{rb} + N_{rp3} R_{rb} + N_{rp4} R_{rb}
 \end{cases} \quad (4)$$

In the formula, $i=1, 2, 3, 4$.

Where m_s, m_c, m_r and m_{pi} are the masses of the sun gear, planet carrier, ring gear, and planet gear i respectively. I_s, I_c, I_r and I_{pi} are the rotary inertias of the sun gear, planet carrier, ring gear, and planet gear i respectively. $k_{sy}, k_{sx}, k_{cy}, k_{cx}, k_{ry}$ and k_{rx} are the support stiffness of the sun gear, planet carrier and ring gear in the y and x directions respectively. The value is $1 \times 10^9 N/m$.

k_{piu} and k_{piv} are the support stiffness of the planet gear i in the u_i and v_i directions, respectively. The value is $1 \times 10^9 N/m$;

$x_s, y_s, \theta_s, x_c, y_c, \theta_c, x_r, y_r$ and θ_r are the translational and rotational displacements of the sun gear, planet carrier and ring gear in the x and y directions, respectively;

u_{pi}, v_{pi} and θ_{pi} are the translational displacements in the u_i and v_i direction and rotational displacement of the planet gear i , respectively;

R_{sb} , R_{cb} , R_{rb} and R_{pi} are the base circle radius of the sun gear, planet carrier, ring gear, and planet gear i . φ_i is the angle between the planet gear i and the horizontal line in the counterclockwise direction.

N_{spi} , N_{rpi} , N_{cpiu} and N_{cpiv} are respectively the dynamic meshing forces of the sun gear and the planet gear i , the dynamic meshing force between the planet gear i and the gear ring, and the planet carrier forces in the u and v directions from the planet gear i . These forces are given.

$$\begin{cases} N_{spi} = k_{spi} \delta_{spi} \\ N_{rpi} = k_{rpi} \delta_{rpi} \\ N_{cpiu} = k_{cpiu} u_{pi} \\ N_{cpiv} = k_{cpiv} v_{pi} \end{cases} \quad i = (1, 2, 3, 4) \quad (5)$$

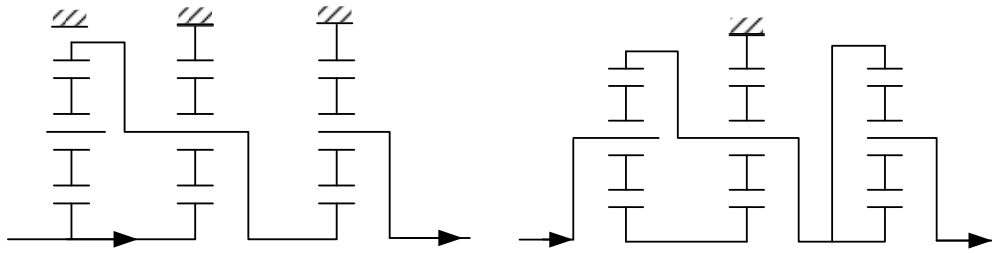
In the formula: k_{spi} and k_{rpi} are the time-variable mesh stiffness of the sun gear and the planet gear i , the planet gear i and the ring gear; δ_{spi} and δ_{rpi} are the gear mesh deformations.

$$\begin{cases} \delta_{sp1} = -(y_s - y_{p1}) \sin \theta_{s1} + (x_s - x_{p1}) \cos \theta_{s1} + R_{sb} \theta_s - R_{p1b} \theta_{p1} \\ \delta_{sp2} = (y_s - y_{p2}) \sin \theta_{s2} + (x_s - x_{p2}) \cos \theta_{s2} + R_{sb} \theta_s - R_{p2b} \theta_{p2} \\ \delta_{sp3} = (y_s - y_{p3}) \sin \theta_{s3} - (x_s - x_{p3}) \cos \theta_{s3} + R_{sb} \theta_s - R_{p3b} \theta_{p3} \\ \delta_{sp4} = -(y_s - y_{p4}) \sin \theta_{s4} - (x_s - x_{p4}) \cos \theta_{s4} + R_{sb} \theta_s - R_{p4b} \theta_{p4} \\ \delta_{rp1} = (y_{p1} - y_r) \sin \theta_{r1} + (x_{p1} - x_r) \cos \theta_{r1} + R_{p1b} \theta_{p1} - R_{rb} \theta_r \\ \delta_{rp2} = (y_{p2} - y_r) \sin \theta_{r2} - (x_{p2} - x_r) \cos \theta_{r2} + R_{p2b} \theta_{p2} - R_{rb} \theta_r \\ \delta_{rp3} = -(y_{p3} - y_r) \sin \theta_{r3} - (x_{p3} - x_r) \cos \theta_{r3} + R_{p3b} \theta_{p3} - R_{rb} \theta_r \\ \delta_{rp4} = (y_{p4} - y_r) \sin \theta_{r4} + (x_{p4} - x_r) \cos \theta_{r4} + R_{p4b} \theta_{p4} - R_{rb} \theta_r \\ y_{pi} = u_{pi} \sin \varphi_i - v_{pi} \cos \varphi_i \\ x_{pi} = u_{pi} \cos \varphi_i + v_{pi} \sin \varphi_i \end{cases} \quad (6)$$

The method of dynamic modeling of the planetary set with six planet gears is the same as that of four planet gears. According to the change of the number of planet gear, the dynamic equations of the planetary set are gained.

4. Analysis of planetary gears vibration

Taking the planetary transmission as the research object, the connection structure of the planetary sets at first gear is shown in Figure 3. The first planetary set does not work, and ring gears of the second and third planetary set are braked. The power input from the sun gear of the second planetary set and output from the carrier. After that the power goes to the third set's sun gear and output from the third carrier. In the case of the fourth gear, the connection structure is shown in Figure 4. The first set and second set work for the power flow. The third set does not work because the sun gear, ring gear and carrier rotate together. The parameters of planetary gears are shown in Table 1. The masses and rotary inertias of set are shown in Table 2 and Table 3.

**Figure 3.** The power flow of the first gear.**Figure 4.** The power flow of the fourth gear**Table 1.** The parameters of planetary gears

Planetary Set Parameters	First	Second	Third
The tooth of sun gear	31	37	54
The tooth of ring gear	73	75	90
The tooth of planet gear	21	19	18
Module of gear	4.0	4.0	4.0
Gear pressure angle	25°	25°	25°

Table 2. The masses and rotary inertias of the first set

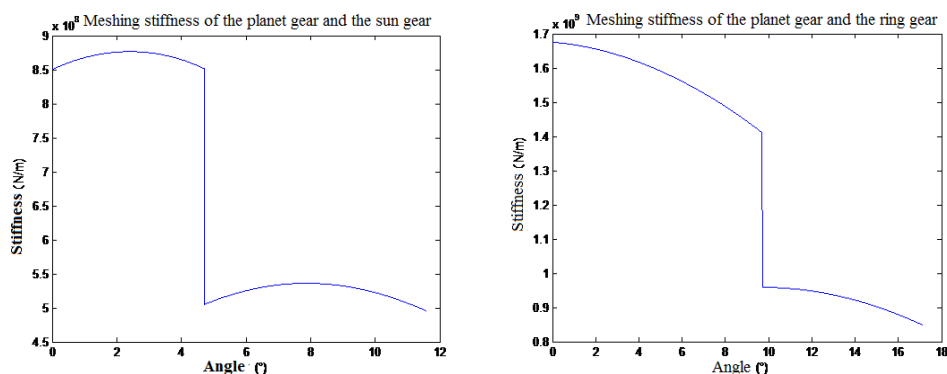
	First			
	sun	planet	ring	carrier
Mass/kg	11.58	0.6321	28.97	20.63
Rotary Inertia/kg*m ²	0.0626	0.001173	0.3811	0.5758

Table 3. The masses and rotary inertias of the second and third set

	Second				Third			
	Sun	Planet	Ring	Carrier	Sun	Planet	Ring	Carrier
Mass/kg	11.58	0.7870	7.28	28.97	28.97	0.6175	32.99	51.09
Rotary Inertia/kg*m ²	0.0626	0.000707	0.259	0.3811	0.3811	0.000588	0.2738	0.356

4.1. Time-varying Mesh stiffness

According to the gear mesh stiffness modeling method in this paper, the mesh stiffness of the planetary gear is calculated. The results of the first planetary set are shown in Figure 5. The change rules of the mesh stiffness of the planetary external gear are different from the internal meshing.

**Figure 5.** The time-varying mesh stiffness of the first planetary set

4.2. Dynamic response of mesh stiffness excitation

The calculated mesh stiffness is brought into the dynamics differential equations of the gear system, and the dynamic responses of the planetary gear train are calculated. The dynamic response results of the second set's planet gear are shown in Fig. 6, which are the accelerations in x and y directions. The dynamic response results of the third carrier are shown in Fig. 7, which are the accelerations in x and y directions. The results of the vibration responses of the planetary gear train at the first gear are shown in Table 4.

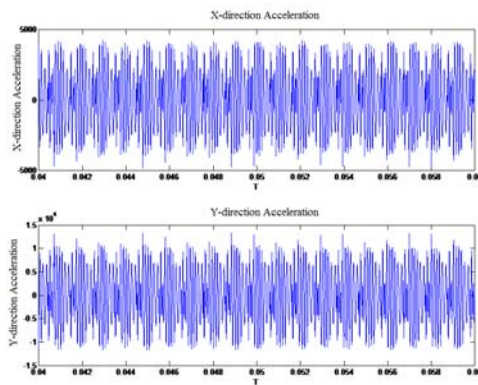


Figure 6. Dynamic response of the planet gear

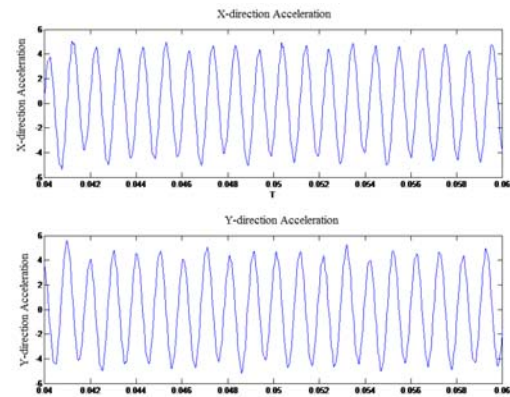


Figure 7. Dynamic response of the third carrier

Table 4. The vibration response of the planetary gear train at the first gear

Acceleration Amplitude	Second Set				Third Set			
	Sun	Planet	Ring	Carrier	Sun	Planet	Ring	Carrier
X direction	804.73	4968.42	1080.14	75.21	75.21	17982.68	7.8	5.3
Y direction	810.54	13628.21	1055.76	79.98	79.98	6124.07	6.1	5.8

The results show that the planet gear vibration is much greater than the vibration of other components in the same planetary set. The vibration responses of gear and carrier are almost the same in the x and y directions except the planet gear. The reason is that the gear and carrier are symmetrical about the x and y axes. However the symmetrical axes of the planet gears are different from other components. We can also conclude that the input vibration amplitude of the first planetary set components is larger than that of the latter planetary set except the planet gear under the time-varying mesh stiffness excitation. For example, under the first gear condition, the vibration amplitudes of the second ring gear and the carrier are significantly larger than the third ring gear and the planet carrier.

The results of the vibration response of the planetary gear train at the fourth gear are shown in Table 6.

Table 5. The vibration response of the planetary gear train at the fourth gear

Acceleration Amplitude	First Set				Second Set			
	Sun	Planet	Ring	Carrier	Sun	Planet	Ring	Carrier
X direction	712.99	8288.69	53.75	57.29	712.99	3638.84	503.77	53.75
Y direction	738.67	5754.14	52.78	57.36	738.67	3767.95	491.58	52.78

By comparing the calculation results between the first gear and the fourth gear, we can conclude that when the connection form of the planetary sets is different, the vibration response of the same planetary set component is also significantly different. The vibration response of planetary set components is mainly related to the connection form. The vibration of the output member is obviously

smaller than that of other planetary set components. For example, under the first gear condition, the vibration of the third carrier is the smallest; under the fourth-speed operation condition, the vibration of the second planetary carrier is the smallest.

5. Conclusion

This paper establishes the time-varying mesh stiffness model of planetary gears based on the energy method, and applies the concentrated mass method to establish the translation-rotation coupling dynamics model of the planetary gear transmission system. The vibration response of the planetary gear trains with time-varying mesh stiffness is analyzed and the following conclusions are obtained.

(1) The change rules of the mesh stiffness of the planetary external gears are different from the internal meshing.

(2) Under the excitation of time-varying mesh stiffness, the vibration response of planetary set components is mainly related to the connection form. The time-varying mesh stiffness has the greatest influence on the vibration of the planet gears, and the planet gear vibration is significantly much greater than the vibration of other components in the same planetary set. The vibration responses of gear and carrier are almost the same in the x and y directions except the planet gear.

(3) When multiple planetary sets are connected, the vibration of the output member is obviously smaller than that of other planetary set components. The vibration at the input planetary set is generally greater than that at the rear planetary set except the planet gear.

Acknowledgments

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References

- [1] Li Runfang, Wang Jianjun. Gear system dynamics-vibration, shock, noise [M]. Beijing: Science Press, 1997: 11-12.
- [2] Yue Xiyao, Ding Wenshi, Ding Kang, et al. Numerical calculation of planetary gear train involute modified gear time-varying meshing stiffness [J]. Journal of Chongqing University of Technology (Natural Science), 2016, 30 (6): 38- 44.
- [3] Chang Lehao, He Zhaoxia, Liu Wei et al. Express method for determining the transmission error and mesh stiffness of helical gears [J]. Journal of Vibration and Shock, 2017, 36(6): 157-162.
- [4] Wan Zhiguo, Yan Yanyang, Cao Hongrui et al. Time-varying mesh stiffness algorithm correction and tooth crack dynamic modeling [J]. Journal of Mechanical Engineering, 2013, 49 (11): 153-160.
- [5] Feng Song, Mao Junhong, Xie Youbai. Analysis and Calculation of Gear Mesh Stiffness with Tooth Wear [J]. Journal of Mechanical Engineering, 2015, 51 (15): 27-32.
- [6] Rao Zhengang. Planetary Gear Design [M]. Beijing: Chemical Industry Press, 2014.
- [7] Kahraman A. Free torsional vibration characteristics of compound planetary gear sets [J]. Mechanism and Machine Theory, 2001, 36(8): 953 – 971.
- [8] Zhu Weilin, Wu Shijing et al. Influence of position errors on the load-sharing characteristics of compound planetary gear sets considering the variable stiffness coefficient [J]. Journal of vibration and shock, 2016, 35(12): 77-85.
- [9] Wu S, Zuo M J, Parey A. Simulation of spur gear dynamics and estimation of fault growth [J]. Journal of Sound and Vibration, 2008, 317 (3/4/5): 608-624.