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Variable Gain Iterative Learning Control with Forgetting Factor

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Abstract. In order to improve the convergence speed of iterative learning control and reduce the fluctuation of the system error, a class of linear steady-state systems is considered. The convergence of the algorithm and error fluctuations are studied by introducing the variable-gain idea into the D-type iterative learning control algorithm with variable forgetting factor. According to the related properties of the λ norm theory, the convergence of the improved iterative learning algorithm is proved. Compared with iterative learning control with forgetting factor and iterative learning control with variable gain, MATLAB simulation analysis is performed. The simulation results show that the algorithm is effective. The improved iterative learning law not only makes the iterative error smoother, but also improves the convergence speed.

1. Introduction

Iterative Learning Control (ILC) [1] is one of the intelligent control algorithms proposed by the 1980s that is extremely suitable for some repetitive motion characteristics. The ILC algorithm was proposed to solve the optimization problem of the system control input [2]. Under the effect of the ILC learning law, the system output trajectory follows the target trajectory [3] within a limited time interval, and even tracks are fully tracked. Due to its simple structure and excellent tracking performance, it has attracted the attention of a large number of research scholars and has achieved a lot of research results. Some scholars have introduced forgetting factor [4] in the ILC algorithm to reduce the adverse effects of the system's pre-control items on the system tracking process through the oblivion factor forgetting effect, so that the output error fluctuation of the system under the effect of the ILC gradually decreases, making the iteration the process is more stable. The reference [5] proposed the forgetting factor ILC algorithm with feed forward feedback, which has good robustness to the nonlinear system and can ensure that the algorithm error converges smoothly. The reference [6] introduced the forgetting factor in high-order ILC to improve the robustness of the system. However, the algorithm itself is too complex, the convergence proves too cumbersome, and the simulation results are not ideal. The reference [7] uses an ILC algorithm with variable learning gains to adjust the gain of the ILC in batches. Simulation results show that the convergence speed can be improved. The disadvantage is that the batch-to-batch process is cumbersome. The reference [8] proposes to accelerate the ILC algorithm through variable gain. Although the fixed gain ILC has a faster convergence speed, the robustness of the error in the convergence process is slightly oscillating. The reference [9] introduces a forgetting factor into iterative learning for a class of P-type ILC, and enhances the robust performance of the system through the effect of forgetting factors. However, the convergence speed of the algorithm is not fast enough.

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For a class of linear time-invariant systems, a D-type ILC algorithm is studied in this paper. The idea of variable gain is introduced into the variable forgetting factor iterative learning. Under the effect of the learning law, the control is given according to the related properties of the λ norm theory. The mathematical derivation of the algorithm's convergence proves that it not only makes the iterative error curve smoother, but also the system's convergence speed is effectively improved. Finally, through Matlab simulation results, it can be clearly seen that the comparison with the general ILC algorithm under the control of the learning law used in this paper shows that the variable-gain D-type ILC algorithm with forgetting factor not only greatly speeds up the convergence speed of the control system, but also The iterative output error of the system is attenuated more smoothly, making the ILC's tracking performance more optimal.

2. System Description and Learning Law

Consider a linear steady-state system with repeated properties that has the following form

$$\begin{cases}\dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t)\end{cases} \tag{1}$$

In equation (1), x(t) $\in R^n$, u(t) $\in R^m$and y(t) $\in R^p$ are the state matrix of the system, the control input matrix and the output matrix, respectively, $A \in R^{n\times n}$,$B \in R^{n\times m}$ and $\boldsymbol{C} \in \boldsymbol{R}^{p\times n}$ are all real numbers, and t $\in [0, \mathrm{T}]$ is a time variable.

Suppose 1: The desired trajectory $y_d(t)$ is reachable at any time in the time interval [0, T], ie. There is an expected output $y_d(t)$ corresponding to the control input $u_d(t)$ at any time, such that

$$\begin{cases}\dot{x}(t) = Ax_d(t) + Bu_d(t) \\ y(t) = Cx_d(t)\end{cases} \tag{2}$$

Suppose 2: For system (1) it meets:$x_k(0) = x_d(0)$, k=0,1,2,3…

The purpose of the ILC is to find the optimal input. And the paper the variable gain ILC with forgetting factor learning law:

$$u_{k+1}(t) = \big(1 - r(k)\big)u_k(t) + r(k)u_0(t) + \mathrm{\beta}L\dot{e}_k(\mathrm{t}) \tag{3}$$

In equation (3), k is the number of iterations set by the system, r(k) is a variable forgetting factor, a function that takes the number of iterations as a variable, and $r(k) \in [0,1)$; L is the learning gain matrix of the ILC learning law; β(t) is the exponential variable gain; the systematic error is $e_k(t) = y_d(t) - y_k(t)$.

3. Convergence analysis

Theorem: If the ILC system described in equation (1) satisfies Hypothesis 1 and Hypothesis 2, the following conditions must be satisfied:

$$\left\|\big(1 - r(k)\big)\mathrm{I} - \beta(t)LCB\right\| < 1 \text{ And} \lim_{k\to\infty} r(k) = 0.$$

When$k \to \infty$, the iterative output $y_k(t)$ of system (1) uniformly converges to the desired output trajectory $y_d(t)$ within the time interval [0, T].That is $\lim_{k\to\infty} y_k(t) \to y_d(t)\ (t \in [0, T])$

Prove: According to equation (3), the K+1 iteration control input error of system (1) can be described as

$$\Delta u_{k+1}(t) = \big(1 - r(k)\big)\Delta u_k(t) + r(k)\Delta u_0(t) + \mathrm{\beta}L\dot{e}_k(t) \tag{4}$$

Where u$\Delta u_k(t) = u_d(t) - u_k(t)$. The Kth system iteration error is expressed as

$$e_k(t) = y_d(t) - y_k(t) = Cx_d(t) - Cx_k(t) \tag{5}$$

Bring equation (2) into equation (5)

$$e_k(\mathrm{t}) = Ce^{At}\left(x_d(0) - x_k(0)\right) \ = \int_0^t Ce^{A(t-\tau)}B(u_d(\tau) - u_k(\tau))d\tau \tag{6}$$

Equation (6) can be obtained according to suppose 2

$$e_k(\mathrm{t}) = \int_0^t Ce^{A(t-\tau)}B\Delta u_k(\tau)d\tau \tag{7}$$

Derivatives for both sides of equation (7)

$$\dot{e}_k(\mathrm{t}) = CB\Delta u_k(t) + \int_0^t Ce^{A(t-\tau)}B\Delta u_k(\tau)d\tau \tag{8}$$

Substituting equation (8) into equation (4)

$$\Delta u_{k+1}(t) = \left(1 - \mathrm{r(k)}\right)\mathrm{I} - \upbeta(\mathrm{t})LCB)\Delta u_k(t) + r(k)\Delta u_0(t)\int_0^t \upbeta(\mathrm{t})LCAe^{A(t-\tau)}B\Delta u_k(\tau)d\tau \tag{9}$$

In equation (9):$M = \left(1 - r(k)\right)I - \beta(t)LCB)$ and$N = \beta(t)LCA$.

We can get: $\Delta u_{k+1}(t) = M\Delta u_k(t) + r(k)\Delta u_0(t) + \int_0^t Ne^{A(t-\tau)}B\Delta u_k(\tau)d\tau$ (10)

Apply the norm to both ends of equation (10)

$$\left||\Delta u_{k+1}(t)|\right| \le \left||M|\right|\left||\Delta u_k(t)|\right| + r(k)||\Delta u_0(t)|| + \int_0^t \left||Ne^{A(t-\tau)}B|\right| \left||\Delta u_k(\tau)|\right|d\tau \tag{11}$$

Equation (11) is multiplied at both ends by $e^{-\lambda t}$ ($\mathrm{t} \in [0, \mathrm{T}]$)

$$\mathrm{e}^{-\lambda \mathrm{t}}\left||\Delta u_{k+1}(t)|\right| \le \mathrm{e}^{-\lambda \mathrm{t}}\left||M|\right|\left||\Delta u_k(t)|\right| + r(k)\mathrm{e}^{-\lambda \mathrm{t}}||\Delta u_0(t)|| + a_1\int_0^t e^{-\lambda(t-\tau)}\mathrm{e}^{-\lambda \mathrm{t}}||\Delta u_k(\tau)||d\tau \tag{12}$$

In the equation (12), $\mathrm{a}_1 = \sup_{t\in[0,T]}||Ne^{At}B||$.

$$\left||\Delta u_{k+1}(t)|\right|_\lambda \le \left\{\left||M|\right| + a_1\frac{1-\mathrm{e}^{-\lambda \mathrm{T}}}{\lambda}\right\}\left||\Delta u_k(t)|\right|_\lambda + \mathrm{r(k)}\left|\left|\Delta u_0(t)\right|\right|_\lambda \tag{13}$$

$$\left||\Delta u_{k+1}(t)|\right|_\lambda \le \uprho\left||\Delta u_k(t)|\right|_\lambda + r(k)\varepsilon \tag{14}$$

In the equation (14): $\uprho = \left||M|\right| + a_1\frac{1-\mathrm{e}^{-\lambda \mathrm{T}}}{\lambda}$, $\upvarepsilon = \left||\Delta u_0(t)|\right|_\lambda$.

When λ chooses a sufficiently large value, the conditional expression (1) satisfying the system convergence can be expressed as ρ<1.

$$\left||\Delta u_k(t)|\right|_\lambda \le \frac{\varepsilon}{1-\rho}\mathrm{r(k)} \text{ And} \lim_{k\to\infty}\left||\Delta u_k(t)|\right|_\lambda \le \frac{\varepsilon}{1-\rho}\lim_{k\to\infty} r(k) = 0.$$

According to the system, we can see $\lim_{k\to\infty}\left||\Delta u_k(t)|\right|_\lambda \ge 0$, then

$$\lim_{k\to\infty}\left||\Delta u_k(t)|\right|_\lambda = 0 \tag{15}$$

It is known from equation (7)

$$\left\|e_k(t)\right\|_\lambda \leq cb\frac{1-e^{(a-\lambda)T}}{\lambda-a}\Delta u_k(t) \tag{16}$$

In equation (16), $> a$, $a = ||A||$, $b_1 = ||B||$, $c = ||C||$.

According to equation (14) and equation (15) available $\lim_{k\to\infty}\left\|e_k(t)\right\|_\lambda = 0$.

Then according to $\lim_{k\to\infty}\sup_{t\in[0,T]}||e_k|| \leq e^{\lambda T}\left\|e_k(t)\right\|_\lambda$, it can be proved that $\lim_{k\to\infty}\sup_{t\in[0,T]}\left\|e_k(t)\right\| = 0$. Proof completed.

4. Simulation Analysis

To illustrate the effectiveness of the algorithm used in this paper, according to the following linear stationary system

$$\begin{cases}\dot{x}(t) = Ax(t) + Bu(t)\\ y(t) = Cx(t)\end{cases}$$

$$x(t) = \begin{bmatrix}x_1(t)\\x_2(t)\end{bmatrix}, u(t) = \begin{bmatrix}u_1(t)\\u_2(t)\end{bmatrix}, y(t) = \begin{bmatrix}y_1(t)\\y_2(t)\end{bmatrix}, A = \begin{bmatrix}-1 & 1\\-1 & 0\end{bmatrix}, B = \begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}, C = \begin{bmatrix}2 & 0\\0 & 1\end{bmatrix}.$$

In the $t \in [0, T]$ interval, the desired trajectory is $y_d(t) = \begin{bmatrix}y_{1d}(t)\\y_{2d}(t)\end{bmatrix} = \begin{bmatrix}1.5t\\4\sin t\end{bmatrix}$, It is required to fully track the system output in the iterative time domain.

The ILC algorithm is used to learning law (3), where the initial control input is$u_1(k) = 0,\ u_2(k) = 0$. The variable forgetting factor takes$r(k) = \frac{1}{k^3}$, the variable gain coefficient takes $\beta(t) = e^{0.3t}$, and the gain takes $\boldsymbol{L} = \begin{bmatrix}0.5 & 0\\-0.5 & 1\end{bmatrix}$.

Convergence condition $\left\|\left(1 - r(k)\right)I - \beta(t)LCB\right\| < 1$.

Both $\beta(t)$ and $r(k)$ are monotonous and easy to know to satisfy the condition; $\lim_{k\to\infty} r(k) = 0$ the same reason also satisfies the condition.

In order to illustrate the effectiveness of the ILC learning law used in this paper, the MATLAB simulation was compared with the ILC learning law with the forgetting factor and the variable gain ILC learning law. The simulation diagram is shown in Fig.1-Fig.6.

Fig.1 shows the process effect trace of the output trajectory tracking the desired trajectory with the variable gain D-type ILC algorithm with forgetting factor. It can be seen from the figure that the systematic output trajectory and expected output trajectory of the variable-gain ILC with forgetting factor have large deviations from the beginning of tracking, and completely coincide at the end of the iteration, which is a process in which the deviation gradually decreases. Fig.2 shows that under the action of the ILC learning law in this chapter, it is completed iteratively. The system trajectory completely tracks the desired trajectory and can achieve complete tracking. Fig.3 shows the trajectory tracking error curve of the conventional PID type ILC learning law. Fig.4 and Fig.5 show the tracking error curves of the ILC learning law with the forgotten factor and the gain-based ILC learning law, respectively. Fig.6 shows the trajectory tracking error curve for the gain-gain D-type ILC learning law with forgetting factor. From the comparison of Fig.3, Fig.4 and Fig.5, it can be seen that the error variability of the conventional PID-type ILC learning law in the iterative tracking process is relatively large, and the convergence speed is slightly slower. And in the simulation running process, the running time of the PID type ILC learning law is generally longer than the other two ILC learning rules, which means that the other two ILC learning laws are more time efficient. Fig.6 and Fig.3, Fig.4, Fig.5 can be seen, this chapter of the ILC learning law in the tracking process error converges to zero faster, and the error decay to zero faster, Only 5 times are needed to converge the error to zero and the error converges smoothly. In other words, the ILC learning law in this chapter not only has a faster convergence speed, but also has a smoother convergence process.

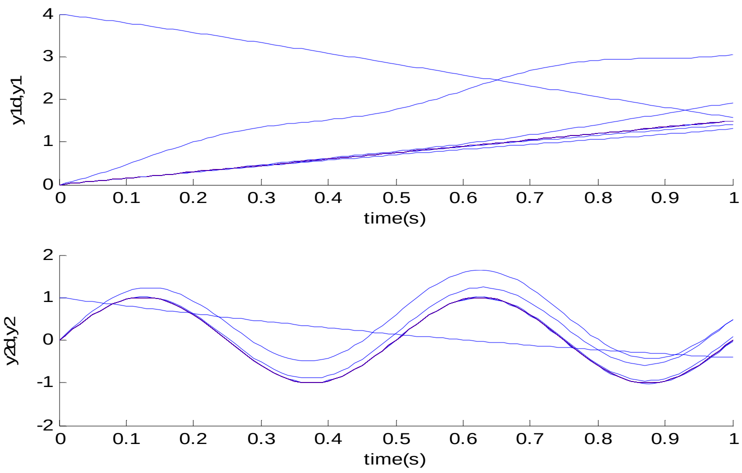

Figure 1. Variable gain ILC with forgetting factor trajectory tracking process

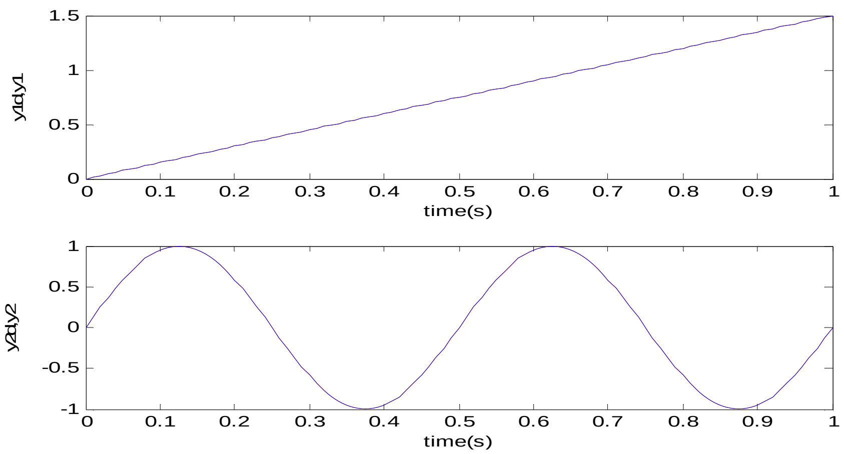

Figure 2. Expected trajectory trace after 10 iterations of gain-gain ILC with forgetting factor

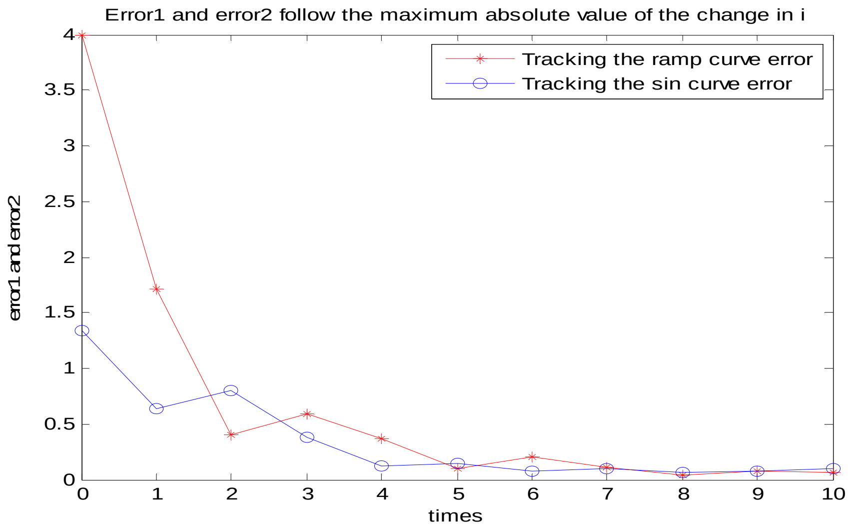

Figure 3. Error curve of conventional PID-ILC

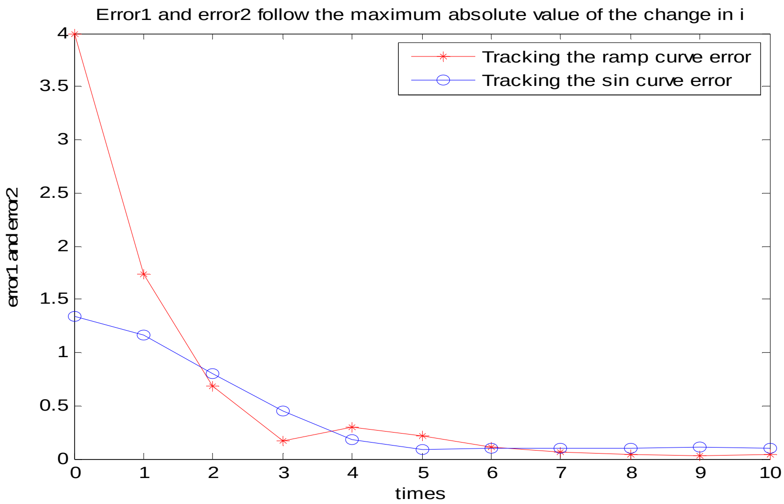

Figure 4. Error curve of ILC with forgetting factor

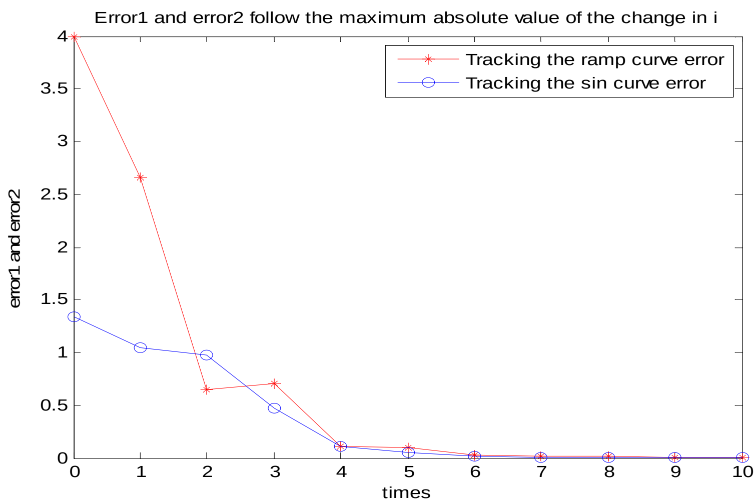

Figure 5. Error curve of Variable gain ILC

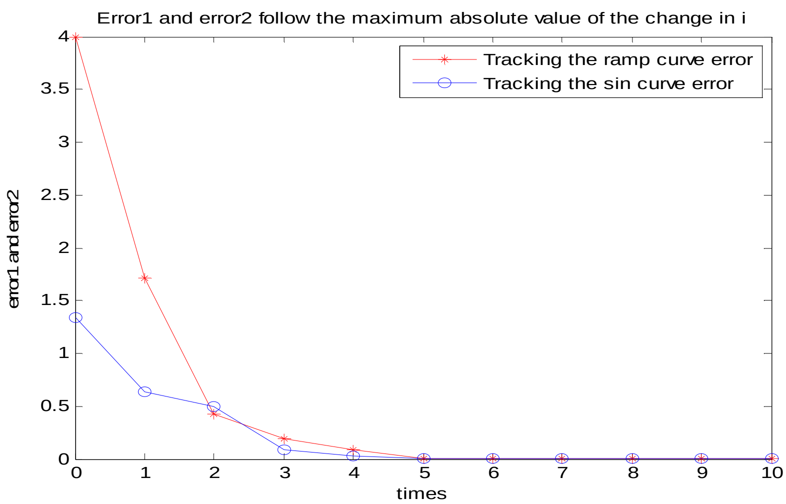

Figure 6. Error curve of Variable gain ILC with forgetting factor

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5. Conclusion

Forgetting factor and variable gain have the effect of improving the error fluctuation and improving the convergence speed for the ILC algorithm, respectively, but it is generally difficult to combine both advantages. This paper discusses a class of linear time-invariant systems with repetitive motion characteristics using a control strategy with variable gain D-type ILC algorithm with forgetting factor. The convergence of the algorithm proves that the rigorous proof is given by using the relevant properties of the λ norm. This proof method simplifies the system to achieve complete tracking of the system within the control time zone under the learning law control and the error converges to zero. Comparing the error absolute value curves of the ILC algorithm with the forgetting factor and the variable gain ILC algorithm by simulation, it can be known that the variable-gain ILC algorithm with forgetting factor cannot only speed up the convergence speed of the system, but also make the convergence error curve of the algorithm more stable, ensuring that the system can completely track the desired system.

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