

# The Comparison of Negative Linear Compressibility of Hexagonal Honeycomb under Different Layouts

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**Abstract.** Hexagonal honeycomb as a structure with a nice negative linear compressibility (NLC) is analyzed here from the aspect of its two layouts to be full of space, these two layouts are compared in terms of minimal negative value of linear compressibility and the angle range of NLC to obtain a better way of layout.

## 1. Introduction

Negative linear compressibility is a rare elastic property which makes materials or structures expand along one direction when compressed hydrostatically unlike conventional one shrinking in all directions. Ever since 1922, when Bridgman first discovered NLC in trigonal crystal of tellurium, it has been received much attention, many subsequent researches [1-4] further show that NLC really exists in natural-occurring materials and artificial structures and it doesn't violate the laws of thermodynamics.

In the past few years, many man-made structures have sprung up in our eyes including honeycomb [1.2], 3D wine rack model [3], hexagonal dodecahedron [4], what is worth mentioning is that honeycomb as a simple and basic form with NLC is selected here to explore it further.

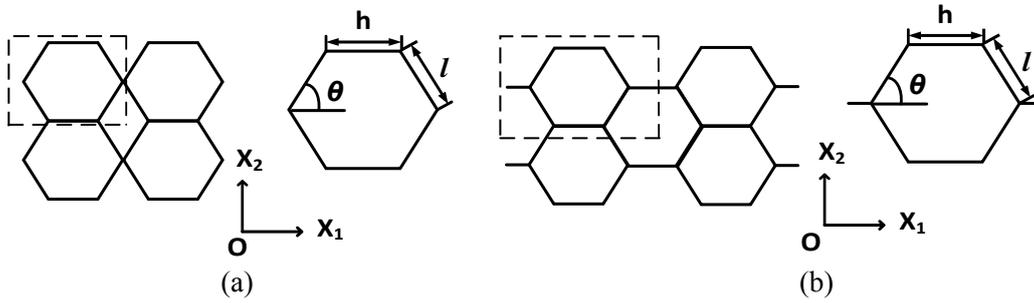
In the earlier work, I. G. Masters [1] and K. E. Evans analyzed in detail elastic deformation of the honeycomb from tensile moduli, shear moduli and Poisson's ratio under different conditions of flexure, stretching and hinging mechanism. Since then, Joseph N et al. [2] calculated the on-axis linear compressibility based on previous inferential basis of elastic deformation indicating that hexagonal honeycomb exhibits negative linear compressibility (NLC) by setting specified range of geometric parameters. However, it should be noted that the layout of honeycomb is a non-ignorable factor to affect value of NLC of honeycomb. In this paper, two arrangements of honeycomb are listed to discuss the differences of coefficients of compressibility and comprehensive assessment to two structures.

## 2. Analytical model

For honeycomb configuration studied by aforesaid researches is shown in figure 1(a). The difference is that a new derivation method is presented here to apply to the following two kinds of arrangement to get the expression of linear compressibility of honeycomb based on hinges at their vertices, we bring in rotational stiffness constant  $k_h$  to describe the relationship between the moment ( $M$ ) and the change of angle ( $d\theta$ ) as follows.

$$M = K_h \cdot d\theta \quad (1)$$





**Figure 1.** (a) The hexagonal honeycomb in array-based layout and its unit cell (b) the hexagonal honeycomb in homogeneous layout and its unit cell

As you can see in figure 1. (a), projected lengths of honeycomb unit cell along two vertical directions OX1 and OX2 are as follows:

$$X_1 = h + 2l \cos \theta \quad (2)$$

$$X_2 = 2l \sin \theta \quad (3)$$

In view of practicality and validity of the model, it should be pointed out that  $h > 0, l > 0, 0^\circ < \theta < 90^\circ$ , furthermore, we assume that the thickness of unit cell perpendicular to the direction of paper is  $b$ . Using the derivation method from [4] we can work out the on-axis Poisson's ratio  $\nu_{ij}$  ( $i=1, 2; j=1, 2$ ) and Young's moduli  $E_i$  ( $i=1, 2; j=1, 2$ ) by adding force respectively in OX1 and OX2 direction.

$$E_1 = \frac{2K_h \sin \theta (h + l \cos \theta)}{bl^3 \sin^3 \theta} \quad (4)$$

$$E_2 = \frac{2K_h \sin \theta}{bl \cos^2 \theta (h + l \cos \theta)} \quad (5)$$

$$\nu_{12} = \frac{\cos \theta (h + l \cos \theta)}{l \sin^2 \theta} \quad (6)$$

$$\nu_{21} = \frac{l \sin^2 \theta}{\cos \theta (h + l \cos \theta)} \quad (7)$$

Then the on-axis linear compressibility  $\beta_{11}$  and  $\beta_{22}$  in OX1 and OX2 direction can be obtained based on above Poisson's ratio  $\nu_{ij}$  and Young's moduli  $E_i$ :

$$\beta_{11} = \frac{1}{E_1} - \frac{\nu_{21}}{E_2} = \frac{bl^3 \sin \theta (\sin^2 \theta - \frac{h}{l} \cos \theta - \cos^2 \theta)}{2K_h (h + l \cos \theta)} \quad (8)$$

$$\beta_{22} = \frac{1}{E_2} - \frac{\nu_{12}}{E_1} = \frac{bl^2 \cos \theta (2 \cos^2 \theta + \frac{h}{l} \cos \theta - 2 \sin^2 \theta)}{4K_h \sin \theta} \quad (9)$$

The same method is also used to calculate the array-based arrangement mode as shown in Figure 1(b), The projected length of the unit in the OX1 and OX2 are given by:

$$X_1 = 2(h + l \cos \theta) \quad (10)$$

$$X_2 = 2l \sin \theta \quad (11)$$

The parameters should also meet the condition of  $h > 0, l > 0, 0^\circ < \theta < 90^\circ$ , Using the same method above from [4] we can obtain the on-axis Poisson's ratio  $\nu_{ij}$  and Young's moduli  $E_i$  by adding force respectively in OX1 and OX2 direction.

$$E_1 = \frac{K_h (h + 2l \cos \theta)}{bl^3 \sin^3 \theta} \quad (12)$$

$$E_2 = \frac{4K_h \sin \theta}{bl \cos^2 \theta (h + 2l \cos \theta)} \quad (13)$$

$$\nu_{12} = \frac{\cos \theta (h + 2l \cos \theta)}{2l \sin^2 \theta} \quad (14)$$

$$\nu_{21} = \frac{2l \sin^2 \theta}{\cos \theta (h + 2l \cos \theta)} \quad (15)$$

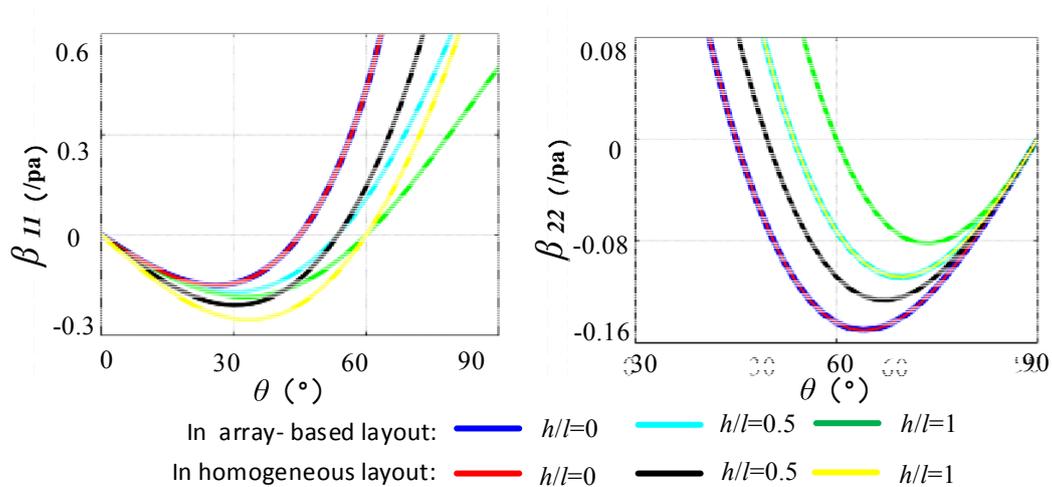
Likewise, we can get the expression for on-axis linear compressibility  $\beta_{11}$  and  $\beta_{22}$  in OX1 and OX2 direction based on above Poisson's ratio  $\nu_{ij}$  and Young's moduli  $E_i$ .

$$\beta_{11} = \frac{1}{E_1} - \frac{\nu_{21}}{E_2} = \frac{bl^3 \sin \theta (\sin^2 \theta - \frac{h}{l} \cos \theta - \cos^2 \theta)}{K_h (h + 2l \cos \theta)} \quad (16)$$

$$\beta_{22} = \frac{1}{E_2} - \frac{\nu_{12}}{E_1} = \frac{bl^2 \cos \theta (\cos^2 \theta + \frac{h}{l} \cos \theta - \sin^2 \theta)}{2K_h \sin \theta} \quad (17)$$

### 3. Discussion

We can see from the equations above that linear compressibility can be viewed as the function of  $\theta$  and  $h/l$  in the condition of assuming that  $b, l$  and  $K_h$  are constant, so the linear compressibility of honeycomb in two different arrangement is expressed by plots of  $\theta$  and different  $h/l$  as shown in Fig. 2.



**Figure 2.** Linear compressibility of honeycomb under two different layouts across various angles of  $\theta$  with  $k_h = 1 \text{ KJ/rad}^2$ ,  $l = 1 \text{ mm}$ ,  $b = 1 \text{ mm}$ .

As seen in figure. 2, the trend of the line in OX1 and OX2 direction in a uniform arrangement is almost same indicating that the value of linear compressibility is from zero to negative to positive with the angle increasing and the slope is gradually smaller as the ratio between  $h$  and  $l$  increasing along OX1 orientation, however, the trend is just reverse in OX2 orientation, in addition, the lines of  $\beta_{11}$  in two layouts when  $h/l$  is zero are coincident because the formula is not a bit different and at this point, both forms can be regarded as wine rack model which is also the reason for superposition for two lines of  $\beta_{22}$  in the condition of  $h/l=0$ . It should be noted that the line of  $\beta_{22}$  in array-based arrangement under situation of  $h/l=0.5$  coincides with the line of  $\beta_{22}$  in the way of homogeneity on situation of  $h/l=1$  which can be explained from the formula of  $\beta_{22}$  that if the value of  $h/l$  in homogeneous way is twice as many as one in array-based way, the formulas of  $\beta_{22}$  in two ways are completely equal, i.e. two layouts will have the same effect in  $\beta_{22}$  as long as satisfying this double relationship.

**Table 1.** The minimum value and null point of linear compressibility

	$\beta_{11} = 0$	$\beta_{22} = 0$	$\beta_{11min}$	$\beta_{22min}$
Array-based layout	$\theta=0, 60^\circ$	$\theta=60^\circ, 90^\circ$	-0.1845	-0.0821
Homogeneous layout	$\theta=0, 60^\circ$	$\theta=54^\circ, 90^\circ$	-0.2534	-0.1084

What is the effect of two ways on linear compressibility? Here we pick the condition of  $h/l = 1$  (the situation is same when the ratio between  $h$  and  $l$  is constant) to discuss how two layouts affect minimal negative value of linear compressibility and the angle range of NLC. One can calculate the specific value in the light of expression of linear compressibility as shown in table. 1. In OX1 orientation, the angle ranges of NLC are both  $0 < \theta < 60^\circ$  while minimum value that can be achieved in homogeneous layout is smaller than array-based layout showing a 37% rise in homogeneous layout. As for OX2 orientation, the way of homogeneity is superior to array-based layout whether in angle ranges or minimum value of NLC and this property is improved by 32%. Through the above analysis, it suggests that homogeneous layout is better than array-based layout of honeycomb in NLC.

#### 4. Conclusion

Based on the above, two layouts for honeycomb are shown here through calculating relevant data indicating that homogeneous layout is superior to array-based layout not only in minimal negative value of linear compressibility but the angle range of NLC, but beyond that, variation trends of linear

compressibility for two ways are uniform. Given the importance of this paper, it is prospected that this work can provide a reference for selecting honeycomb as a required ideal material.

### **Acknowledgments**

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### **References**

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