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Modelling and simulation of the load in the epicyclic rotary pump with trochoidal gear profiles

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Abstract. This paper presents an analysis of the load distribution in the trochoid pump with the planetary movement of the impeller. There are a lot of theoretical and numerical analyses conducted for the gerotor of the pump with fixed positions of the gear axis. Design of planetary gerotor pump has a different load distribution than gerotor with fixed axes. A planetary gerotor consists of a fixed outer gerotor gear with internal gearing and inner gear with an external gearing. The centre axis of the inner gear has a circular motion around the central axis of the fixed gear, because of the eccentricity on drive shaft. The special shape of the gerotor teeth forms chambers with varying volume that allows the flow and pressure of the fluid. During the pump working process the forces of different direction and intensity are present in each of the working chambers. It is necessary to determine the pressure in each of the chambers in order to obtain the load model of the gerotor. Load model will enable the calculation of load distribution in the zone of the maximum contact stresses. This calculation should help in dimensioning of the pump and the achievement of better performance characteristics.

1. Introduction

The principle of trochoidal gears meshing is equivalent to the principle of the classical planetary mechanism with parallel axes, with most of the design solutions applying epicyclic motion, which enable gain of high transmission ratios and compact design [1]. For pumps with internal trochoidal gearing, the profile of one gear is defined by trochoid (curve from the cycloid family), while the meshed profile is an appropriate inner or outer envelope. Because of the specific geometry, the entire profile can be applied for meshing. Equidistant modification provides a profile with better functional characteristics. These advantages are used in the design of the gerotor pumps.

Gerotor pumps have wide application in various hydraulic systems, including vehicles [2] and flow meters [3], because they have numerous advantages such as: compact design, small pulsations (therefore lower noise), precise and stable operation at high speeds. Although they have simple design, they are complex from the kinematics aspect and require additional study, particularly a gerotor pump with planetary movement of the gears. In the last few decades, a large number of papers have been published covering all the main aspects in the research field of the gerotor pumps. The most important investigations of the hydraulic pump parameters are given in [2], [4], [5], the model of computer dynamics of the fluid could be found in [6], and profile of the gears in [7]. Research of the gap calculation between the teeth profiles is presented in [8]. Significant publications that relate the stress in the teeth contact of the trochoidal meshing are [9-11]. The problems of friction in cycloidal gearing

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were investigated in [12], [13]. The precise calculation of the cycloid rotary pump flow and the flow rate irregularity is described in detail in [14].

Requirements in terms of pumps performance are increasingly stringent in modern industrial applications. In addition, the environmental protection standards prescribe the development of hydraulic systems without any noise and without leakage of the fluid. Taking into account the quality of the gerotor pumps and the challenges they face in modern applications, as well as the requirements of the standards, the authors have developed a new concept of the gerotor pump whose prototype has been presented in the paper [15]. New topics related to gerotor pumps and trochoid gearing are, generally, numerical approaches supported by experimental investigation [16], [17].

This paper investigates the forces and moments that act on the gear pair of gerotor pumps with planetary movement of the inner gear. In that case, the inner gear is mounted on the drive shaft, while the outer gear is fixed. The main objective of this analysis is to set up a force model, which would further enable the definition of conditions for reducing contact forces. The reduction of contact forces would reduce the wear. The problem of determining the contact forces is complex since the load is transmitted simultaneously at several points of contact. In addition, the pressure forces of the fluid that act on the tooth side of the gears are considered, which depend on a large number of influencing parameters. For these reasons, a simple physical model and an appropriate analytical method were applied.

2. Load analysis of the trochoid pumps with planetary movement of working elements

For the analytical determination of the fluid pressure force, Hall's method is presented, which is described in the reference [18]. It is necessary to emphasize that the basic aim of the research in this chapter is to define the load distribution, so certain approximations can be accepted in the force calculation.

During the pump working process, the forces of different intensity and direction are acting in each of the working chambers, as a result of the pressure of the fluid. Modelling of the gear pair loads requires the intensity of the pressure in each chamber at an arbitrary point. For the force calculation of the fluid pressure in the pump operating chambers, a model shown schematically in Figure 1 is being considered. In addition, the following assumptions are introduced:

- the working fluid is incompressible,
- there is no friction between fluids and surfaces of working elements,
- there is a change in pressure in each chamber as a result of changing the current volume of the chamber,
- each opening of the valve is modelled as a damper with the same discharge coefficient C_p.

Compressive forces in each of the chambers, as well as their resultant, will be defined via their projections onto the axis of a stationary coordinate system of the envelope. The position of the inner gear at the arbitrary time is determined by the position of the reference line with the use of angle ψ.

When the gears are rotating, the intensity and direction of the force change is related to the change in pressure in the working chambers. Starting from the above assumptions, a general pattern for determining the flow in the chamber K_i can be written [18]:

$$Q_i = \frac{dV_i}{dt} = C_p A_0 \left[\frac{2\Delta p_i}{\rho_f} \right]^{\frac{1}{2}} \tag{1}$$

where: Δp_i is the pressure drop due to the flow of liquid, A_0 is the cross-sectional area of the opening of the distribution valve and ρ_f is the fluid density.

During the suction phase, the volume of the chamber increases and the current geometric flow is positive. Therefore, the following expression for determining the geometric flow for the chamber K_i in the suction phase can be written:

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$$Q_i = C_P A_0 \left[\frac{2(p_{us} - p_i)}{\rho_f} \right]^{\frac{1}{2}} \tag{2}$$

During the discharge phase, the volume of the chamber decreases and the current geometric flow is negative. Therefore, the following expression can be written for determining the geometric flow for the chamber K_i in the discharge phase:

$$Q_i = -C_p A_0 \left[\frac{2(p_i - p_{pot})}{\rho_f} \right]^{\frac{1}{2}} \tag{3}$$

Based on equation (2), the expression for determining the pressure for the chamber K_i in the suction phase can be written:

$$p_i = p_{us} - \frac{\rho_f [dV/dt]^2}{2C_p^2 A_0^2} \tag{4}$$

i.e., according to equation (3), in the discharge phase:

$$p_i = p_{pot} + \frac{\rho_f [dV/dt]^2}{2C_p^2 A_0^2} \tag{5}$$

The pressure force in the chamber is a continuous force that can be represented by an equivalent concentrated pressure force, $\mathbf{F}_{pi}$, which attack line coincides with the symmetry line of the path $\mathbf{P}_i\mathbf{P}_{i+1}$ connecting the two adjacent points of contact for the observed chamber, as shown in Figure 1. The equivalent pressure force in a chamber can be expressed in vector form as:

$$\mathbf{F}_{pi}^{(a)} = p_i b \mathbf{k}_a \times (\mathbf{P}_i\mathbf{P}_{i+1})^{(a)} \tag{6}$$

Vector $\mathbf{P}_i\mathbf{P}_{i+1}$ that connects the two consecutive points of contact can be expressed as the difference of the position vector of those points in the coordinate system of the envelope $O_a x_a y_a z_a$:

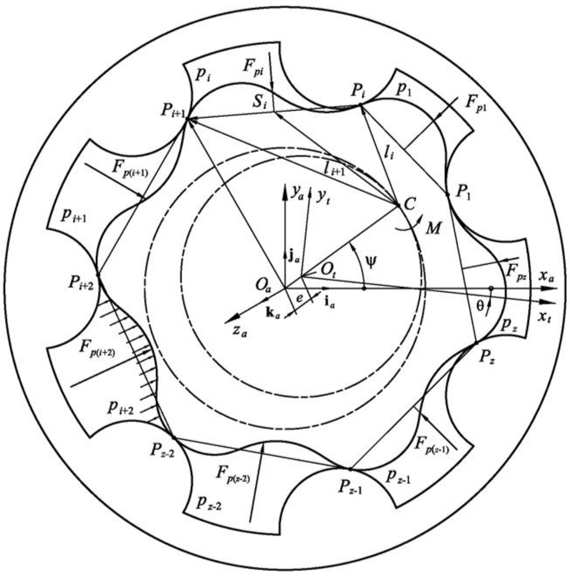

Figure 1. The pressure forces of the fluid acting on the inner gear in an arbitrary position.

$$(\mathbf{P}_i\mathbf{P}_{i+1})^a = \mathbf{r}_{P_{i+1}}^{(a)} - \mathbf{r}_{P_i}^{(a)} = \begin{bmatrix} x_{P_{i+1}}^{(a)} - x_{P_i}^{(a)} \\ y_{P_{i+1}}^{(a)} - y_{P_i}^{(a)} \\ 0 \end{bmatrix} \tag{7}$$

When equation (7) is included in equation (6), the following is obtained:

$$\mathbf{F}_{pi}^{(a)} = p_i b \begin{vmatrix} \mathbf{i}_a & \mathbf{j}_a & \mathbf{k}_a \\ 0 & 0 & 1 \\ x_{P_{i+1}}^{(a)} - x_{P_i}^{(a)} & y_{P_{i+1}}^{(a)} - y_{P_i}^{(a)} & 0 \end{vmatrix} = p_i b\left[-\left(y_{P_{i+1}}^{(a)} - y_{P_i}^{(a)}\right)\mathbf{i}_a + \left(x_{P_{i+1}}^{(a)} - x_{P_i}^{(a)}\right)\mathbf{j}_a\right] \tag{8}$$

The vector position of contact point P_i in the coordinate system of the envelope, can be written in the form of the following matrix expression [19]:

$$\mathbf{r}_{P_i}^{(a)} = \begin{bmatrix} x_{P_i}^{(a)} \\ y_{P_i}^{(a)} \\ 1 \end{bmatrix} = \begin{bmatrix} e\left\{z\lambda\cos\frac{\pi(2i-1)}{z} - c\cos\left[\frac{\pi(2i-1)}{z} + \delta_i\right]\right\} \\ e\left\{z\lambda\sin\frac{\pi(2i-1)}{z} - c\sin\left[\frac{\pi(2i-1)}{z} + \delta_i\right]\right\} \\ 1 \end{bmatrix} \tag{9}$$

where:

$$\delta_i = \arctan\frac{\sin\left[\frac{\pi(2i-1)}{z} - \psi\right]}{\lambda - \cos\left[\frac{\pi(2i-1)}{z} - \psi\right]} \tag{10}$$

Angle τ_i between the symmetry line of the teeth of the external gear and the coordinate axis x_a can be expressed as [19]:

$$\tau_i = \frac{\pi(2i-1)}{z} \tag{11}$$

while, for the adjacent profile, this angle can be calculated using the following expression:

$$\tau_{i+1} = \frac{\pi(2i+1)}{z} \tag{12}$$

Starting from equation (9), and taking into account equations (10), (11) and (12) and applying the corresponding transformations, a final equation is obtained for the pressure force of the fluid in the chamber K_i:

$$\mathbf{F}_{pi}^{(a)} = \begin{bmatrix} -p_i be\left\{2z\lambda\sin\frac{\pi}{z}\cos\frac{2\pi i}{z} + c\left.\frac{\sin\psi - \lambda\sin\tau}{\left[1+\lambda^2 - 2\lambda\cos(\tau-\psi)\right]^{\frac{1}{2}}}\right|_{\tau_i}^{\tau_{i+1}}\right\} \\ -p_i be\left\{2z\lambda\sin\frac{\pi}{z}\sin\frac{2\pi i}{z} - c\left.\frac{\cos\psi - \lambda\cos\tau}{\left[1+\lambda^2 - 2\lambda\cos(\tau-\psi)\right]^{\frac{1}{2}}}\right|_{\tau_i}^{\tau_{i+1}}\right\} \\ 0 \end{bmatrix} \tag{13}$$

The resultant of all pressure forces acting on the inner gear is obtained as their vector sum:

$$\mathbf{F}_p = \sum_{i=p}^{q} \mathbf{F}_{pi} \tag{14}$$

with its acting point coinciding with the pitch point C.

The torque $\mathbf{M}$ acts on the inner gear and it is equal to the sum of the equivalent pressure forces in the chambers:

$$\mathbf{M}^{(a)} = M\mathbf{k}_a = \sum_{i=1}^{z} \mathbf{M}_{pi}^{(a)} \tag{15}$$

According to the adopted convention, force moments are calculated in relation to the instantaneous pitch point C. Therefore, the moment of force equivalent to the pressure in the chamber K_i can be expressed as the following vector product:

$$\mathbf{M}_{pi}^{(a)} = \mathbf{CS}_i^{(a)} \times \mathbf{F}_{pi}^{(a)} \tag{16}$$

where S_i is the middle vector point $\mathbf{P}_i\mathbf{P}_{i+1}$ (Figure 1). Based on the geometric relationships in Figure 1, the following vector relationships can be written:

$$\mathbf{CS}_i = \mathbf{CP}_i + \mathbf{P}_i\mathbf{S}_i \tag{17}$$

$$\mathbf{P}_i\mathbf{S}_i = \frac{1}{2}\mathbf{P}_i\mathbf{P}_{i+1} \tag{18}$$

$$\mathbf{P}_i\mathbf{P}_{i+1} = \mathbf{CP}_{i+1} - \mathbf{CP}_i \tag{19}$$

and the following is gained:

$$\mathbf{CS}_i = \frac{1}{2}\left(\mathbf{CP}_{i+1} + \mathbf{CP}_i\right) \tag{20}$$

Applying the rules for vector summation, the following is obtained:

$$\mathbf{CS}_i^{(a)} = \frac{1}{2}\left[\left(x_{\mathbf{CP}_{i+1}}^{(a)} + x_{\mathbf{CP}_i}^{(a)}\right)\mathbf{i}_a + \left(y_{\mathbf{CP}_{i+1}}^{(a)} + y_{\mathbf{CP}_i}^{(a)}\right)\mathbf{j}_a\right] \tag{21}$$

If equation (19) is replaced by the equation (6), the equivalent pressure force can be expressed as:

$$\mathbf{F}_{pi}^{(a)} = p_i b\left[-\left(y_{\mathbf{CP}_{i+1}}^{(a)} - y_{\mathbf{CP}_i}^{(a)}\right)\mathbf{i}_a + \left(x_{\mathbf{CP}_{i+1}}^{(a)} - x_{\mathbf{CP}_i}^{(a)}\right)\mathbf{j}_a\right] \tag{22}$$

Substituting equations (21) and (22) in equation (16), the moment of force equivalent to the pressure in the chamber K_i can be expressed in the form of the following relation:

$$\begin{aligned}\mathbf{M}_{pi}^{(a)} &= \begin{vmatrix} \mathbf{i}_a & \mathbf{j}_a & \mathbf{k}_a \\ \frac{1}{2}\left[x_{\mathbf{CP}_{i+1}}^{(a)} + x_{\mathbf{CP}_i}^{(a)}\right] & \frac{1}{2}\left[y_{\mathbf{CP}_{i+1}}^{(a)} + y_{\mathbf{CP}_i}^{(a)}\right] & 0 \\ -p_i b\left[y_{\mathbf{CP}_{i+1}}^{(a)} - y_{\mathbf{CP}_i}^{(a)}\right] & p_i b\left[x_{\mathbf{CP}_{i+1}}^{(a)} - x_{\mathbf{CP}_i}^{(a)}\right] & 0 \end{vmatrix} = \\ &= \frac{p_i b}{2}\left\{\left[x_{\mathbf{CP}_{i+1}}^{(a)}\right]^2 - \left[x_{\mathbf{CP}_i}^{(a)}\right]^2 + \left[y_{\mathbf{CP}_{i+1}}^{(a)}\right]^2 - \left[y_{\mathbf{CP}_i}^{(a)}\right]^2\right\}\mathbf{k}_a\end{aligned} \tag{23}$$

which can be presented in a simpler form as:

$$\mathbf{M}_{pi}^{(a)} = \frac{p_i b}{2}\left(l_{i+1}^2 - l_i^2\right)\mathbf{k}_a \tag{24}$$

Based on the equation (24) and using the trigonometric transformation, the final expression for determining the intensity of the moment of pressure force in the chamber K_i can be written:

$$M_p = p_i b e^2 z^2 \left\{ 2\lambda \sin\frac{\pi}{z}\sin\left(\frac{2\pi i}{z} - \psi\right) - \frac{c}{z}\left[1+\lambda^2 - 2\lambda\cos(\tau-\psi)\right]^{\frac{1}{2}} \Big|_{\tau_i}^{\tau_{i+1}} \right\} \tag{25}$$

Starting from the law of conservation of energy in the system, a connection can be established between the drive torque, $\mathbf{M}_1$, and the resulting torque of equivalent pressure forces relative to the pitch point, **M**. For the considered configuration of the pump, it has the following form:

$$\mathbf{M}_1 = -\mathbf{M} \tag{26}$$

Based on the analysis, it can be concluded that, in the pumps with planetary motion of working elements, the overall momentum of fluid pressure is balanced by drive torque. Unlike the pump model with fixed axes shafts, where equations of equilibrium contain support reaction force [19], in the observed model with planetary motions, fluid pressure force is balanced by the resultant of the contact forces. From this condition and the equations of balance of forces acting on the gears, it is possible to determine the distribution and size of the contact force.

3. Results

In this part of the paper, the application of the developed mathematical model for the trochoidal gearing loads calculation for the particular case of gear pair will be presented. Parameters required for calculation are as follows: z = 6, e = 3.56 mm, b = 16.46 mm, λ=1.375, r_c = 9.79 mm (c = 2.75), C_p=0.63, A_0=178 mm^2, Δp = 0.6 MPa, ρ_f = 900 kg/m^3, n_1= 1500 rpm and ω_1=50π s^{-1}. The results were obtained by testing the computer programs based on mathematical model for the adopted parameters of the gear pump, and graphically presented in the following figures. Figure 2 shows the fluid pressure force diagram, and Figure 3 shows the resulting torque of fluid pressure force, depending on the drive angle ψ. Figure 4 shows the current pressure forces in the individual chambers of the pump, on the basis of which their size and the course of their changes can be perceived. The results are presented as a function of drive angle ψ, for a period corresponding to the phase difference between the two adjacent chambers, i.e. $\psi_s = 2\pi / z$, as well as for the period of a revolution of the drive shaft (Figure 5).

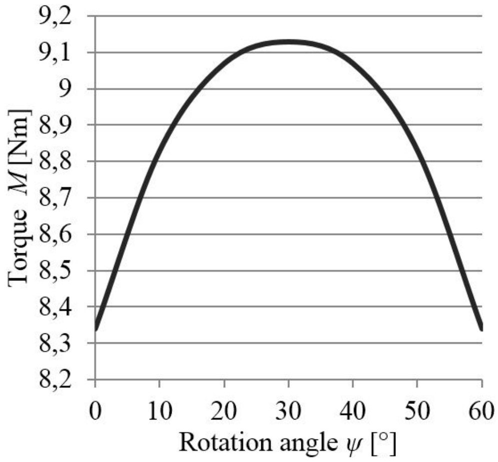

Figure 2. The resulting force of the fluid pressure force.

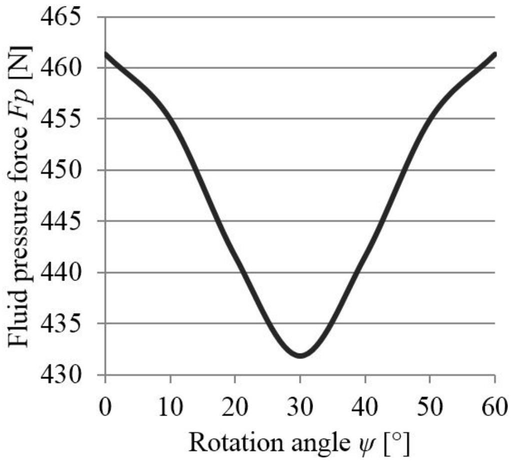

Figure 3. The resulting torque of fluid pressure force.

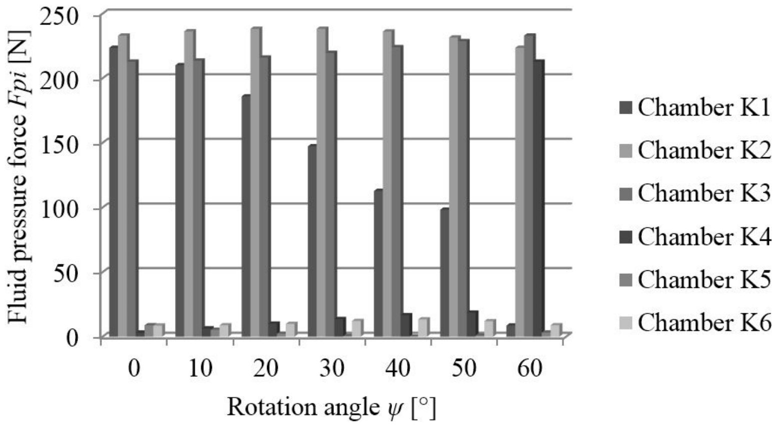

Figure 4. Change in the force of fluid pressure in the pump chamber for one phase $\psi=2\pi/z$.

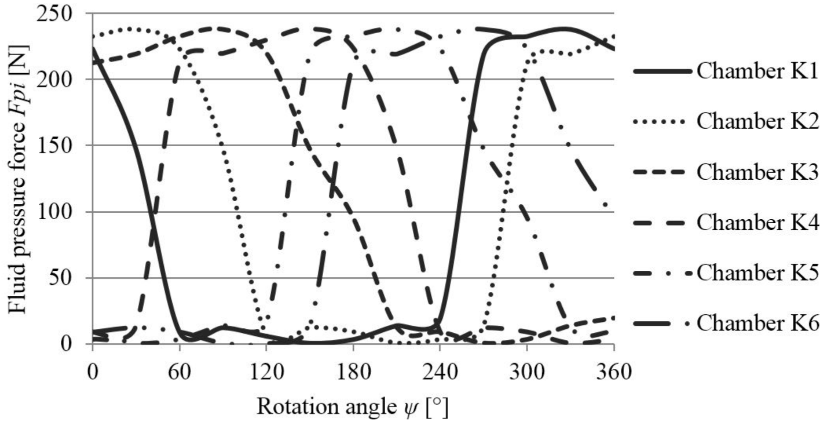

Figure 5. The fluid pressure forces in the pump chambers for the full rotation of the drive shaft.

4. Conclusions

One of the basic problems in the design of internal trochoidal gearing is the impossibility of wear compensation, which occurs as a result of large contact stresses in certain phases of meshing. Related to this assertion, one of the objectives of the paper was to define the initial conditions that would further mitigate the reduction of maximum contact stresses by changing the geometrical parameters of the teeth profiles. This had required a detailed analysis of the forces and moments that act on the toothed pair of the trochoid rotary pumps. The obtained results of this research can be used to analyse the load distribution at the simultaneously meshed tooth pairs and its impact on the strength of the teeth side. Such an analysis would be carried out by the finite elements method (FEM) in some of the PLM software. In carrying out such analyses, the gearing parameters, given in Chapter 3 can be varied, in order to obtain the optimum geometry of the gearing, which would reduce the contact stresses. These possibilities will be the subject of future research.

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