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Relieving the teeth of hobs for cutting the Wildhaber-Novikov gears

A I Sandler [1] and S A Lagutin[2]

[1]SELECTION Ltd, Dubninskaya St., 12-1-269, 127540 Moscow, Russia
[2]EZTM JSC, Krasnaya St. 19, 144005, Electrostal, Moscow reg., Russia

E-mail: sandli@aha.ru

Abstract. Screw and relieved surfaces with a profile of substantial and variable curvature are often used in machine parts and cutting tools. Among them, a special place is occupied by worm hobs for cutting the teeth of Wildhaber-Novikov gears with the profile of the generating worm, variable not only in radius, but also in sign of curvature. A technique for forming the tooth flanks of these hobs by a disk grinding wheel is developed, which allows the grinding and ground profiles in the axial section of the generating worm to be brought together as close as possible. The method for calculating the settings-up and profile coordinates of the grinding wheel is given for its performance in real production conditions.

1. Introduction

The peculiarity of the formation of screw and relieved surfaces by disk grinding wheels is the inevitable deviation of the profile of the ground surface from the profile of the wheel, which was called the organic error. For the straight basic rack this organic error is minimized as much as possible, or at the attained degree of minimization an obtained profile is taken as the basic rack [1]. For screw and relieved surfaces with a profile of substantial and variable curvature, this organic error must be eliminated to the maximum degree.

A well-known example of a solution to this problem is the method for forming a profile of a unruled worm of the ZT2 type, proposed in the book by F.L. Litvin [2] (in the monograph [3] such worms are considered as Flender worms ZF-II). In this example, the axial profile of the grinding wheel was set in the form of a circle arc. And then, from the condition that this arc is the contact line of the wheel and the ground surface, the angle of installation of the grinding wheel axis and the axial profile of the worm thread were sought. Obviously, to obtain a worm hob with relieved surfaces of teeth close in profile to the found worm threads, a solution of the inverse problem is required.

In [4 - 6], the authors proposed the solution of a similar problem with respect to the convex thread profile of the worm and the hob for cutting the wheel teeth to gears with liquid friction. We specify a substantially curvilinear axial profile of the working worm (or the hob generating worm) and also take the profile of the grinding wheel in the axial section of the worm. The setting angle of the grinding wheel axis is determined from the condition that this axis intersects two normal lines to the thread axial profile and it is in a plane parallel to the axis of the worm. The proposed method ensures minimization of the organic error in the profiling of the worm thread, and taking into account a similar organic error in tool profiling, the necessary identity of the profile of the hob generating worm.

In the present paper, we outline the general principles of using an analogous profiling method for the formation of relieved surfaces with significant and variable curvature when the axis of the grinding wheel is located in a plane not parallel to the axis of the hob and specify the conditions of profiling with reference to the relieving of the worm hobs for cutting Wildhaber-Novikov gears.

First-time, helical gears with an initial point tangency of the circular-helical surfaces of the teeth were proposed by the eminent American Inventor E. Wildhaber in one of his 279 patents in 1926 [7]. But at that time this invention went unnoticed, and such gears received rapid development only after M L Novikov had formulated the general principle of their formation and showed that such gears will have the increased load capacity, primarily in contact endurance [8].

Initially, M L Novikov proposed a version of helical overcentrode gears, in which tooth profiles, convex on the pinion and concave on the wheel, were delineated with the circle arcs in the face section of gears. Later, V N Kudryavtsev showed that the same effect can be achieved in the gears synthesized on the basis of two incongruent basic racks with the circle arc profiles and cut by two worm hobs: one for the pinion with convex teeth, the other for the wheel with concave ones [9]. An interesting version of the overcentrode W-N gears, insensitive to the errors of assembly, was proposed in the work [10].

However, at present, basic racks are standardized and widely used in production with a full tooth profile, which provide in gears two lines of action and allow for cutting the pinion and the wheel by one worm hob [11]. In particular, according Russian Standard GOST 15023-76, the basic rack consists of a convex addendum, a concave dedendum and a short straight section between them. Geometry of Novikov gears with double line of action is calculated acc. Russian Standard GOST 17744-72.

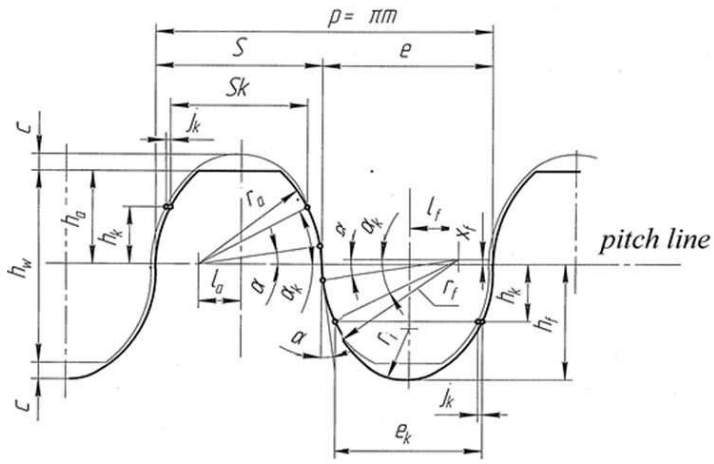

Figure 1. Basic rack for Novikov gears with double line of action acc. GOST 15023-76.

Worm hobs for spur and helical gears have a design profile (a generating rack) in the normal section of the generating worm, which is tangent to the screw front surface. For single-thread hobs this section and front surface practically coincides in the profiling zone. When hobbing the teeth on the gear hobbing machine, the plane of the generating rack is installed perpendicular to the direction of the teeth of the cut wheel, while the hob axis is set taking into account the lead angle of the hob front surface. That is, when machining spur gears, the angle of inclination of the hob axis to the horizontal plane corresponds to the lead angle of the front surface of the hob teeth, when machining helical gears, the angle of installation of the hob axis is increased by the helix angle of the wheel teeth.

When the teeth of the worm hob are relieved, its axis is installed in the centers of the grinding-relieving machine, that is, in the horizontal plane. And for a reliable reproduction of the profile of the generating rack in the normal section of the hob, with the adopted profiling method it is necessary to recalculate the parameters of the profile of the generating rack into the axial section of the tooth surface to be relieved.

First of all, it is necessary to determine the axial profile of the helical surface, on which the cutting edges of the hob teeth are located. To do this the prescribed angle α of the normal profile of the straight section should be replaced by an angle α_1 of the axial profile according to the formula:

$$\tan \alpha_1 = \tan \alpha / \cos \gamma_1 \qquad (1)$$

where γ_1 is the lead angle of the helical surface of the cutting edges on the hob pitch cylinder.

The curvature radii ρ_a and ρ_f of the two parts of active profile in the normal section are replaced by the corresponding radii r_{a0} and r_{f0} in the axial section. For single-thread hobs, these radii with a sufficient approximation are determined by the Meunier theorem from the expressions:

$$r_{a0,\, f0} = \rho_{a,\, f} / \cos \gamma_1 \qquad (2)$$

The arc radius ρ_i on the head of the hob tooth, which is processed fillet of the wheel tooth, in the normal section is replaced with the corresponding radius r_0 in axial section of the hob. According to Euler's formula, these radii are related by the expression:

$$r_0 = \rho_i / \cos^2 \gamma_1 \qquad (2a)$$

The thread thickness b in the axial section should be determined on the minimum ground radius r_{min} of the hob, proceeding from the thickness b_n of the normal section at the end point of machining:

$$b = b_n / \cos \gamma_{(r\ min)} \qquad (3)$$

where $\gamma_{(r\ min)} = \text{atan}\ (p/r_{min})$ is the lead angle on the hob dedendum cylinder, p is the helical parameter.

The parameters of the axial section of the helical surface of the cutting edges of the hob teeth are also parameters of the axial section of the relieved surfaces of the teeth.

2. Parameters of radial-axial relieving

In the general case of radial-axial relieving of the teeth of the worm hob (Figure 2) the direction of relieving motion comprises with perpendicular to the hob axis an angle φ_c, which ensures the obtaining of a given rear angle $\lambda_б$ near the cutting edges of the hob teeth [1, 4, 6]. The wheel axis $O_и$-$O_и$ is crossed with the axis O_1-O_1 of the ground product at an angle of $\beta_и$ to the horizontal plane, and the vertical plane of the wheel axis is not parallel to the axis of the ground hob, but makes with it the angle φ_0. That is, the wheel axis $O_и$-$O_и$ is turned (in the projection to the horizontal plane) at an angle φ_0 to the hob axis and is simultaneously inclined to this plane by the angle $\beta_и$.

Such a method of relieving allows for providing on the flanks the necessary rear angles near the cutting edges of the teeth, to exclude large differences in the curvature of the grinding surface, to improve the conditions for grinding and to increase the service life of the grinding wheel.

The hob parameters are the radius r_F of its pitch cylinder, the axial module m, the threads number z_0, the screw parameter $p = 0{,}5mz_0$ of the generating surface, the number of teeth $z_ф$ in the end section. For hobs with a curvilinear profile of a generating worm the current profile angle α_0 of the axial section is essentially variable (Figure 1) and, respectively, the lateral rear angle λ_b of the relieved surface is also not constant.

During the relieving, the hob rotates around its axis O_1-O_1 with the angular velocity ω_1, the grinding wheel moves along the hob axis at a speed $p\omega_1$ and performs reciprocating motion with a speed $k\omega_1$, where $k = Kz_ф/2\pi$ is the relieving parameter; K is the recession of the Archimedean spiral of the cam of the relieving mechanism on the angular pitch of the hob teeth. The relieving carriage slides are turned relative to the perpendicular to the hob axis by an angle φ_c which allows increasing the rear angle $\lambda_б$ on the lateral surface of the tooth.

The current radius $r_и$ of the grinding wheel is determined at the point of its contact with the tooth of the hob on its current cylinder. For the initial position of the wheel, it is advisable to take the tangency of its maximum radius $r_{им}$ with the minimum radius of the work piece surface $r_{i\ min}$.

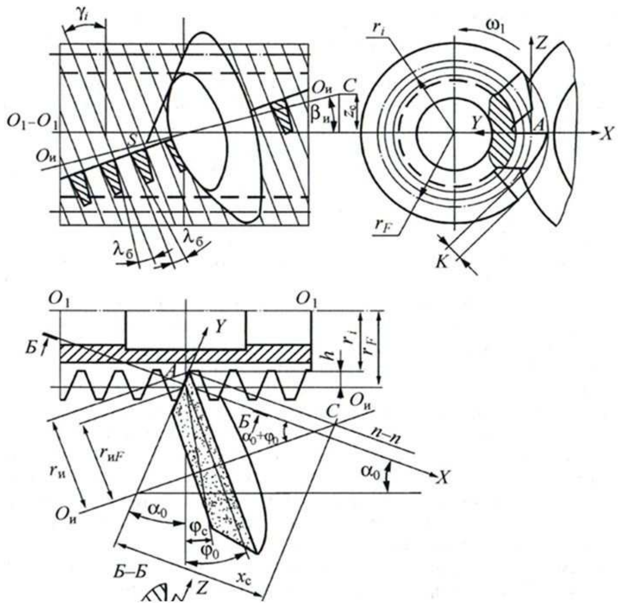

Figure 2. Scheme of radial-axial relieving.

With radial-axial relieving, the lead angle γ_0 of the relieved surface at each of the two selected points ($i_{1,2}$) is determined from the expression

$$\tan \gamma_{0i} = [p \pm k \sin (\alpha_i + \varphi_c)/\cos \alpha_i]/x_{0i} \quad (4)$$

In special cases:

- with a pure radial relieving: $\varphi_c = 0$, $\tan \gamma_{0i} = (p \pm k \tan \alpha_i)/x_{0i}$; (4.1)
- with a pure axial relieving: $\varphi_c = 90°$, $\tan \gamma_{0i} = (p \pm k)/x_{0i}$, (4.2)

where the radius x_{0i} of the location of the selected point on the ground surface is determined from the drawing of the product.

3. Selection of initial points for design normals

The method for selection two points A_1 and A_2 of the axial profile of the ground surface with the parameters $A_1(x_{01}, z_{01}, \alpha_{01})$ and $A_2(x_{02}, z_{02}, \alpha_{02})$,) proceeds from, first, the fact that the profile section between them as much as possible fully covered the working part of the tooth to be ground, and secondly, these points should be boundary for sections of different curvature radii.

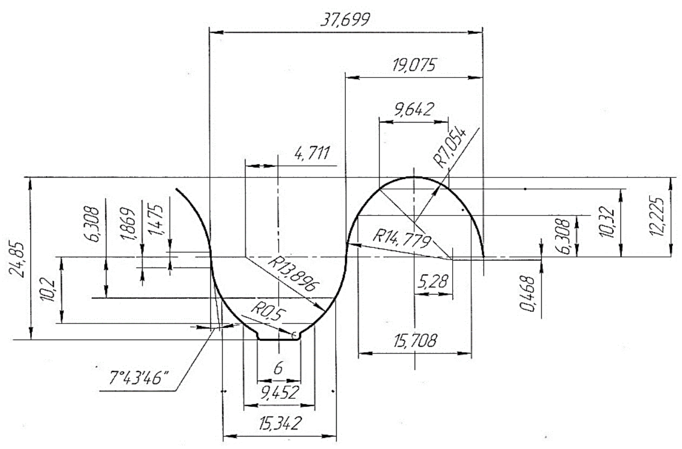

Figure 3. An example of the basic rack of the hob for gears with the module m = 12 mm.

For clarity, let us consider Figure 3, which shows an example of the basic rack for gears with the module m = 12 mm. For the full-profile Novikov gears, the pitch line of basic rack divides the tooth height in half. The active parts of the hob tooth profile are limited by the dimension 10.2, 10.32 from the midline. At the same points, there is a joint of the radius sections of the profile. The tooth thickness in the normal section at the minimum machining diameter is b_n = 31.7 mm.

The tooth of the hob in the normal section is formed according to the dimensions of the space of the basic rack. Similarly, the space between the hob teeth forms the tooth of the wheel to be cut. The center of curvature radius $\square_f$, contouring the tooth addendum, is shifted to the hob axis on value x_f relative to the pitch line. The calculated coordinates of the axial profile of the helical surface of the cutting edges are related to the coordinates of the generating rack profile by the dependencies:

- for point A_1 on the tooth dedendum: $x_{01} = x_{n1}$; $z_{01} = z_{n1}/\cos\gamma_{x01}$; $\tan\alpha_{01} = \tan\alpha_{n1}/\cos\gamma_{x01}$,

where $z_{n1} = 0{,}5\pi\, m_n + l_a - (r_F - x_{n1})\cot\alpha_{n1}$; $\gamma_{x01} = \mathrm{atan}\,(p/x_{01})$,

- for point A_2 on the tooth addendum: $x_{02} = x_{n2}$; $z_{02} = z_{n2}/\cos\gamma_{x02}$; $\tan\alpha_{02} = \tan\alpha_{n2}/\cos\gamma_{x02}$,

where: $z_{n2} = (x_{n2} - r_F + x_f)\cot\alpha_{n2} - l_f$; $\gamma_{x02} = \mathrm{atan}\,(p/x_{02})$. (5)

4. Determination of the angle β of inclination of the grinding wheel axis

In the accepted *XYZ* coordinate system (Figure 4), the *Z* axis is directed along the hob axis, the *X* axis is aligned with the axis of symmetry of the turn, *XZ* is the plane of the hob's axial section, the *Y* axis is perpendicular to this section. The plane of the location of the grinding wheel axis is parallel to the *Y* axis, intersects the *X* axis at a distance *a* to the *Z* axis and makes an angle $\varphi_0 \neq 0$ with this axis.

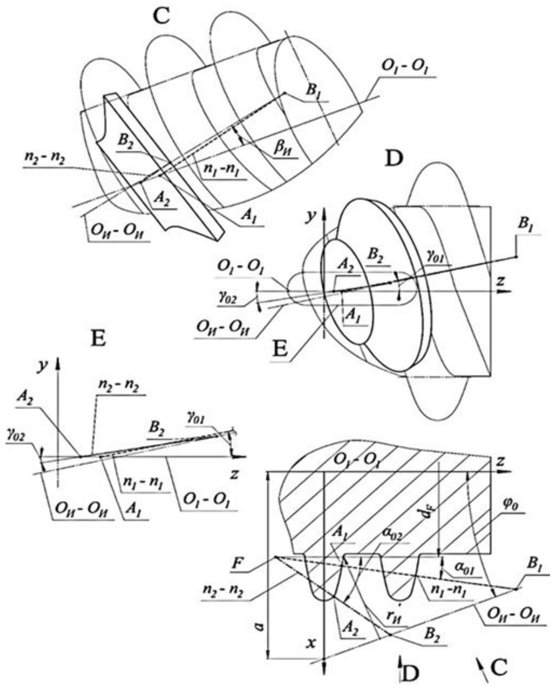

Figure 4. Calculation scheme for determining the setting angle $\beta_и$ of the grinding wheel axis for radial-axial relieving of surfaces with a profile of variable curvature.

The value of *a* is given by expression:

$$a = r_{i\,min} + 0{,}5b\tan\varphi_0 + r_{им}/\cos\varphi_0 \quad (6)$$

where *b* is the thickness of the ground worm turn at the minimum radius $r_{i\,min}$.

For the chosen *a*, the equation of the plane containing the axis of the grinding wheel is written as:

$$x(z) = a - z \tan \varphi_0 \tag{7}$$

The coordinates z_i of the intersection of the normals with the plane (7) are:

$$z_{i(1,2)} = \frac{a - x_{0i} + z_{0i} \tan\alpha_i}{\tan\alpha_i + \tan\varphi_0} \tag{8}$$

The other two coordinates of each of the points of intersection of the normals with the plane of the axis of the grinding wheel are determined by z_i:

$$x_{i(1.2)} = a - z_i \tan \varphi_0; \quad y_{i(1.2)} = (z_i - z_{oi}) \tan \gamma_{0i} \tag{9}$$

The value of the angle $\beta_и$ of the inclination of the grinding wheel axis is determined in the tangent function by the formula:

$$\tan\beta_и = \frac{y_1 - y_2}{\sqrt{(x_1 - x_2)^2 + (z_1 - z_2)^2}} \tag{10}$$

For a special case of relieving with an angle $\varphi_0 = 0$, the plane of the wheel axis is parallel to the axis of the product, and is separated from it at a distance

$$a = r_{i\ min} + r_{им} = x_{0i} + (z - z_{0i}) \tan \alpha_i. \tag{11}$$

The coordinates z_i, y_i of the points of intersection of the normals with the plane containing the grinding wheel axis take the form:

$$z_{i(1,2)} = z_{0i} + (a - x_{0i}) \cot \alpha_i; \qquad y_{i(1,2)} = (z_i - z_{0i}) \tan \gamma_{0i} = (a - x_{0i}) \tan \gamma_{0i} \cot \alpha_i, \tag{12}$$

and the angle of inclination of the wheel axis relative to the axis of the ground product in these particular cases is determined by the expression:

$$\tan\beta_и = \frac{y_1 - y_2}{z_1 - z_2} = \frac{(a - x_{01})\tan\gamma_{01}\cot\alpha_1 - (a - x_{02})\tan\gamma_{02}\cot\alpha_2}{(a - x_{01})\cot\alpha_1 - (a - x_{02})\text{con}\alpha_2 + z_{01} - z_{02}} \tag{13}$$

5. The choice of angles φ_c and φ_0 for relieving the tooth flanks

In contrast to the relieving of the hob teeth with a straight profile, the axial component of the relieving motion for the curvilinear profiles is variable along the profile. For the hobs with a large difference in the profile angles, it is not possible to apply a purely radial relieving (φ_c=0).

This is due, first, to the insufficiency of the rear angles at small profile angles $\alpha_{0\ min}$, in particular in the area adjacent to the pitch cylinder, and secondly, by a large shift of the profile of the relieved surface relative to the pitch line in the arc sections, which will be proved in the hob re-sharpening.

Since helical gears are usually made of thermally improved steels, the lateral rear angles of the hob teeth must be guaranteed not less than 5° and the angle φ_c of the turn of the relieving carrier is to be calculated on the basis of this circumstance [5, 6].

$$\sin (\alpha_{0\ min} + \varphi_{cR,L}) = 0.55\ r_F \cos \alpha_{0\ min} /(K\ z_ф) \tag{14}$$

where R, L are indices of the right and left sides of the hob tooth.

Pure axial relieving ($\varphi_c = 90°$) would have allowed to keep the profile of the generating surface during the re-sharpening. However, with this method it is necessary to check the adequacy of the lateral rear angle $\lambda_б \geq 5°$ at the tooth top on the radius r_a, where the profile angle $\alpha_{1a} > 70°$ using conditions:

$$\tan \lambda_{бa} = k \cos \alpha_{1a} / r_a \approx 0.342\ k / r_a > 0.0875, \text{ or } k / r_a > 0.24 \tag{15}$$

With radial-axial relieving, the axial component of the relieving parameter $k_x = k \sin(\alpha_{0i}+\varphi_c)/\cos\alpha_{0i}$, which enters in the formula (4), is variable along the profile. Therefore, strictly speaking, the distortion of the profile when the hob teeth are re-sharpened on the front surface is inevitable. However, at the profile the equal profile angles α_{0i} occur (for example, at the junction points of the large and small radii on the tooth addendum and dedendum), and therefore, regardless of the angle φ_c, at these points and between them, the profile deviations with the re-sharpening will be minimal.

The value of the angle φ_0 is chosen in such a way that the obtained value of the angle $\beta_и$ corresponds to the range of values:

$$\tan \gamma_{x01} > \tan \beta_и \cos\varphi_0 [1- \tan \varphi_0 \tan 0{,}5(\alpha_{01}+ \alpha_{02})] > \tan \gamma_{x02} \quad (16)$$

which provides a higher accuracy of profiling the ground surface. The limiting factor for assigning the angles φ_c and φ_0 is the condition that the grinding wheel entered the space between the threads without damaging the adjacent tooth. As a rule, for preliminary calculation, the angle φ_0 is chosen equal to φ_c.

6. Profiling the grinding wheels for worms and hobs with curvilinear profile

Significant difficulties in the manufacture of worms and hobs with a profile of substantial and variable curvature are due to the fact that the copying devices for dressing the grinding wheel included in the delivery set of the grinding machines are not suitable for reproducing such a profile. Therefore, it is necessary to develop either special rolling tool or dressing devices. At present, the dressing devices with diamond rollers and the program control of their movement relative to the working surface of the grinding wheel have received a certain distribution [5, 6].

The characteristic of the wheel surface, that is the line of its contact with the ground surface, is oriented along the axial section of the latter. Therefore, the profiling of the wheel in its axial section with sufficient degree of approximation reproduces the section of the ground surface normal to the helical line with the lead angle equal to $\beta_и$.

In general, the profile coordinates of this section and, consequently, the axial profile of the wheel are determined from the following expressions:

$$z_и = z_0 /(\cos \beta_и + \tan \gamma_i \text{ in } \beta_и);$$
$$x_и = x_0 + z_n^2 \sin 2\beta_и \tan \gamma_i /(4x_0). \quad (17)$$

If the value of the setting angle $\beta_и$ corresponds to the range (16), then the axial profile of the grinding wheel for single-thread hobs practically repeats the normal profile of the basic rack. In this case, the angle $\alpha_и$ of the wheel profile on the rectilinear section is determined from the expression:

$$\tan \alpha_и = \tan (\alpha_{0\,min} + \varphi_0) \cos\beta_и \quad (18)$$

7. Conclusion

- A technique is proposed for calculating the parameters of the setting-up, motion and profile of the grinding wheel which is used for relieving the tooth flanks of the hobs to cut the W-N gears.
- This technique allows for minimizing the organic error of grinding when processing on traditional equipment with profiling of the grinding wheel, which is as close as possible to the profile of the basic rack of the hob.
- The above procedure is proposed for use in the development of control programs for setting up the relieving machine and the mechanism for dressing the grinding wheel when processing the hobs on relieving machines with numerical program control.

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