Application of a vision system to determine the Rayleigh damping coefficients of materials used in stereolithography

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Abstract. It is a truism to say that vibrations can be a useful phenomenon and the reason of amage. Therefore, there is no doubt that when designing technical objects it is important to be able to evaluate its dynamic properties by appropriate modeling. One of the key properties of this type is damping. There are a number of methods to study this phenomenon, however, they require quite large resources. The main topic of this work is to propose an alternative, undemanding method to measure the damping property of any structure. The test is based on the analysis of the video recording of the vibrating element, on the basis of which the input quantities for further analytical calculations are determined. The method was presented on the example of the examination of damping properties of a photocurable resin used in stereolithography.

1. Introduction

Damping properties of all structures for obvious reasons are the object of interest of engineers and scientists. Despite this, the extensively dynamic development of materials engineering makes it very difficult to create databases on the dynamic properties of all materials, especially those used in Rapid Prototyping, where the final product's properties may depend on the conditions of the manufacturing process [1]. Therefore, it is important to develop a fast and simple method that allows the determination of proper coefficients, possible to be carried out also by private users, which would perfectly fit in the trend of amateur engineering exposed by Rapid Prortotyping methods.

The damping examination is usually performed by analyzing changes in the amplitude of the oscillating motion over time. For this, among others, they are used piezoelectric sensors [2] - devices that uses the piezoelectric effect, i.e. the phenomenon of generating by some materials (piezoelectrics) electric charges proportional to their deformation. Piezoelectric coupled with a vibrating element is subjected to cyclic accelerations which leads to cyclic deformations caused by inertia forces and thus the appearance of charges proportional to accelerations. Through a subsequent integration of acceleration over time, velocity and displacement waveforms are obtained. Another commonly used method is laser vibrometry [3]. It involves laser measurement of the distance between the emitter and the vibrating point.

Both of these methods allows for precise vibration analyze and quick measurement, however, they require the specialist equipment and the ability to use it, which is in contradiction with the mentioned trend. This paper presents method that allows to optically analyze vibrations at home and, based on it, to calculate the parameters describing the damping that can be implemented in the MES environment.

2. Theoretical basics

Damping can be defined as the phenomenon of decreasing the amplitude of vibration in the function of time. The general equation of movement with damping has a form:

$$m\ddot{x} + c\dot{x} + kx = 0 \tag{1}$$

where:

m – mass [kg]

c – viscous damping coefficient [kg/s]

k – stiffness coefficient [N/mm]

Dividing the equation by mass and substitution the modal frequency $\omega_0^2 = \frac{k}{m}$ and normalized damping coefficient $2b = \frac{c}{m}$ results:

$$\ddot{x} + 2b\dot{x} + \omega_0^2 x = 0 \tag{2}$$

Viscous damping c at which $b = \omega_0$, is called critical damping

$$c_{kr} = 2m\omega_0 \tag{3}$$

while the ratio of the damping coefficient c to the critical damping c_{kr} is called the damping ratio:

$$\gamma = \frac{c}{c_{kr}} = \frac{b}{\omega_0} \tag{4}$$

It is also important to note that as a result of damping the frequency of free oscillations of the system changes and takes the form:

$$\omega_d = (\omega_0^2 - b^2)^{\frac{1}{2}} \tag{5}$$

2.1. Damping decrement

The changes that taking place in the system as a result of damping are well described by ratio of successive amplitudes. It is called the damping decrement q:

$$q = \frac{x(t)}{x(t + T_D)} = e^{bT_D} \tag{6}$$

where $T_D = \frac{2\pi}{(\omega_0^2 - b^2)^{\frac{1}{2}}}$ is period of damped vibrations. After logarithming equation (6) and applying expressions (3) and (4) the logarithmic damping decrement δ can be represented as:

$$\delta = \ln(q) = bT_D = \frac{2\pi\gamma}{(1 - \gamma^2)^{\frac{1}{2}}} \tag{7}$$

2.2. Rayleigh damping

Above expressions are describing damping that characterize the phenomenon in a specific system, and therefore their application in the design of new structures requires the search for analogy with other objects and relying on ready-made models. In addition, it is not possible to include them in numerical calculations. Therefore, the introduction of material coefficients describing damping is crucial. This solution was proposed by Rayleigh, introducing material coefficients α and β describing the dependence of damping in the system on its mass and stiffness in the following form:

$$c = \alpha\, m + \beta\, k \tag{8}$$

Dividing equation (8) by M and substituting expressions for dynamic coefficients and damping ratio (4) it results:

$$\gamma = \frac{\alpha}{2\omega_0} + \frac{\beta\omega_0}{2} \tag{9}$$

In the above equation, there are two unknowns that are Rayleigh coefficients. Therefore, a system of equations should be arranged for two successive eigenvalues, assuming that the degree of damping is not dependent on the frequency of excitation.

3. Subject and methodology of research

The subject of the research is a beam with dimensions of 125x25x1 mm, made of photocurabe resin whose mechanical properties are shown in Table 1 and manufactured with the Stereolitography technology. In the manufacturing process has been used the XYZprinting Nobel 1.0A 3D printer with a working area of 280 x 345 x 590 mm and accuracy of 0.3 mm. In order to increase the accuracy of measurements, three identical samples were made. Each of them was irradiated with UV light for 24 hours to fully cure them.

Table 1. Mechanical properties of examined material.

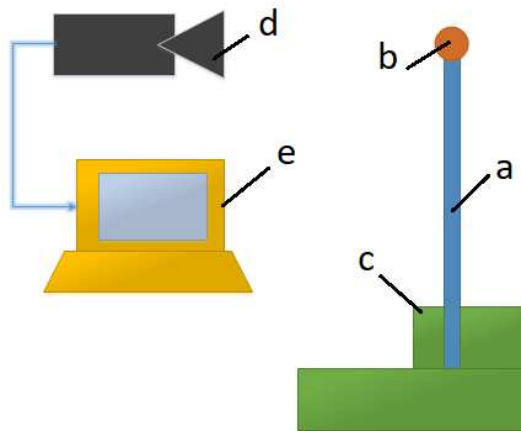| Young's modulus [MPa]	Tensile strength [MPa]	Poisson's ratio [-]	Density [kg/m^3]
246	15	0,41	1200

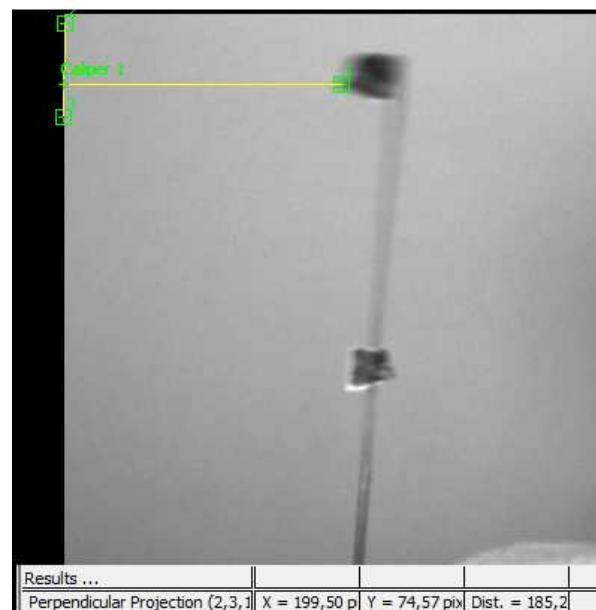**Figure 1.** Scheme of test facility.

Figure 2. Sample view from image analysis.

Figure 1 shows the diagram of the test facility. The sample (1a) with contrast marker (1b) was fixed in a vice (1c) so that its active length was 110 mm. Through the excitation by kinematic extortion, the beam was led to free damped vibrations. Its movement was recorded by the Basler Scout scA640-120gc camera (1d) connected to a PC (1e) with Basler pylon software until its full vibration expires. For each beam, the event was recorded three times.

Recorded video files were converted to a sequence of images in .jpg format, which were exported to the LabVIEW Vision Assistant module. There, on the way of image processing and analysis, deviation of the end of the beam from the equilibrium position for each of the images was determined (Figure 2). Obtained sets of points from nine measurements were averaged and used for used to interpolate a function that represents the beam's movement over time, which carries the information about damping occurring in the system and also about the first eigenvalue of vibrations (Figure 3).
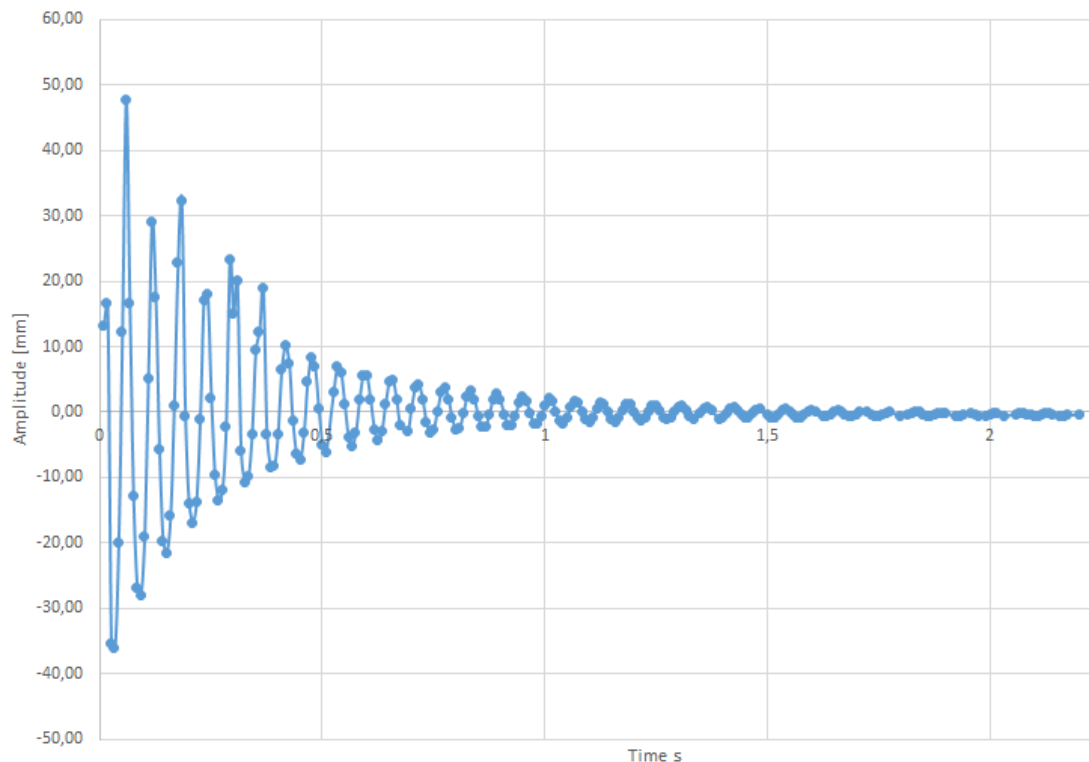
Figure 3. Interpolated function – amplitude versus time.

According to the formula (9), in these considerations the first two eigenvalues of vibrations are needed. The natural frequency of the n-th mode can be determined by means of analytical formula:

$$\omega_n = \frac{a_n^2}{l^2} \cdot \left(\frac{E \cdot I}{\rho \cdot A}\right)^{\frac{1}{2}} \qquad (10)$$

where:
$a_n = (n - 0,5)\pi$ [-]
l – beam length [m]
ρ – material density [kg/m^3]
A – cross section area [m^2]

To exclude the influence of deviations of geometrical quantities and mechanical properties of the beam, the value of the expression $\frac{1}{l^2} \cdot \left(\frac{E \cdot I}{\rho \cdot A}\right)^{\frac{1}{2}} = B$ will be determined experimentally. As the experiment shows, the first natural frequency equals $\omega_1 = 107.7$ rad / s, so it can be saved:

$$\omega_1 = (0,5\pi)^2 B \qquad (11)$$

$$B = \frac{107,7}{(0,5\pi)^2} = 43,6 \qquad (12)$$

therefore, second natural frequency:

$$\omega_2 = (1,5\pi)^2 B = 968,2 \; rad/s \qquad (13)$$

The value of the logarithmic damping decrement, that is required for further calculations, has been calculated based on experimental data. In order to minimize the effect of air resistance on the damping value, periods for which low vibration velocities have been selected - fourteen and fifteen.

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$$\delta = \ln\left(\frac{x(t + 14T_D)}{x(t + 15T_D)}\right) = \frac{3,30}{2,84} = 1,16 \tag{14}$$

Transformation of the formula (7), the expression for the attenuation level was obtained:

$$\gamma = \frac{\delta}{((2\pi)^2 + \delta^2)^{\frac{1}{2}}} = 0,42 \tag{15}$$

On the basis of equation (9), a system of equations can be derived:

$$\begin{pmatrix} \dfrac{1}{2\omega_1} & \dfrac{\omega_1}{2} \\ \dfrac{1}{2\omega_2} & \dfrac{\omega_2}{2} \end{pmatrix} \cdot \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \gamma \\ \gamma \end{pmatrix} \tag{16}$$

which solution are the values of Rayleigh's damping coefficients:

$$\alpha = 81,41, \ \beta = 0,00078$$

4. Results verification

In order to verify the correctness of calculations, a numerical experiment was carried out. In the Ansys Workbench environment, a single-sided fixed beam was modeled with dimensions corresponding to the dimensions of the tested element, and then it was given the properties of the tested material, including the calculated damping coefficients. As a excitation, the initial displacement of the free end of the beam with a value of 50 mm was applied. During the simulation a graph of changes of the positions of the free end of the beam was recorded. Figure 4 shows a comparison of results from experimental and numerical experiments. Convergence was considered as satisfying.
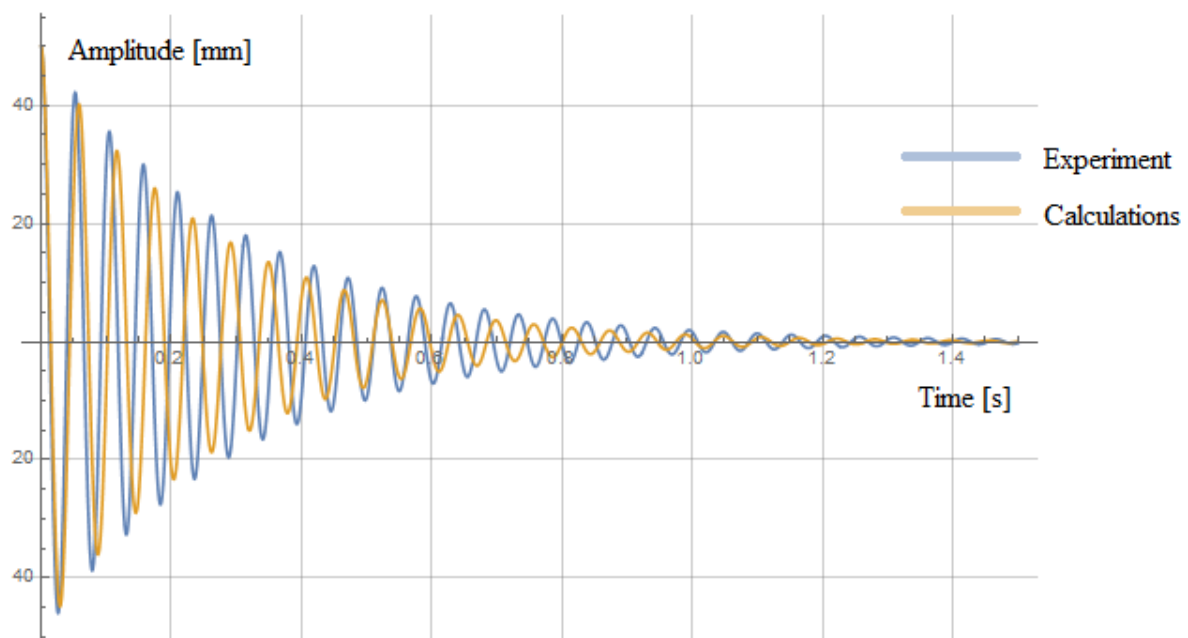

Figure 4. Damped vibration graphs comparison

5. Conclusions

The work describes a fast method of measuring damping properties, both in relation to material and whole structures. It is an excellent alternative to the demanding conventional studies, which may

be out of the reach of individuals who do not specialize in this type of research. The purposefulness of using the method to collect information about materials used in the Rapid Prototyping is evident - these methods are often used by private users who are not specialists. In addition, a simpler test method can result in increased information gain in databases with material damping properties and thus, make dynamical analysis even faster and easier. The disadvantage of the method may be its accuracy - during the measurement various disturbances may appear, such as the effect of air drag. Therefore, it is planned to carry out tests comparing the results obtained with the presented method with the results from laser vibrometry, which will be used to develop appropriate correction algorithms, increasing the accuracy of measurements.

References
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