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Graph model-based analysis of technical systems

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Abstract. Technical systems have a network structure. During mathematical model-based analysis of real technical systems, it is a key issue to determine the existence of interconnection between the subsystems and components. In cases of an integrated system the exposure of interconnections can be a difficult task because of their complexity of interconnections. Graph theory is a well-known mathematical tool; to study interconnections between components of network structure systems. The aim of this paper is to show an easy-usable algorithm for determining the existence of interconnection between the network structure system-components.

1. Introduction

During the analysis or synthesis of integrated network structure technical systems, it is a key issue to determine the existence of interconnection between the system components. Graphs are models; to study interconnections between components of network structure systems. Graph theory is the historical mathematical background of the modern network's science. The “Bridges of Königsberg” problem, first proposed by Leonhard Euler, is analysed using basic properties of graphs. As it turns out, the first graph in graph theory history is impossible to traverse without traversing one or more links repeated times, because the degree of all nodes in the Bridges of Königsberg graph are odd-valued. Thus, a traveller must visit at least one bridge twice, to cover the entire graph [1].

There is a vast literature on graph theory and its applications, including many articles and books. The mathematical background of graph theory can be read in books of Andrásfalvi [2], Finke [3], and Deistek [4]; handbook of Bronstejn [5], and textbook of Fazekas [6]. Lewis' book contains theoretical information and practical possibilities of using network science to investigate networks and network structure systems [1]. Pokorádi presented the mathematical models and their application in engineering [7].

Nagy has modified the well-known search algorithms and he has obtained a general heuristic algorithm to have a search algorithm with minimum total cost. The algorithm is related to the best-first graph-search algorithm and also to heuristic back trafficking algorithms uniting their advantages for the considered types of problems [8].

Jocić et.al proposed a novel algorithm for discovering similar nodes in very large directed graphs, with millions of nodes with billions of connections, which is based on the fuzzy set theory. The required input is a sample of representative nodes that are highly affiliated with some feature. This proposed approach is practically verified on Twitter social network case study to discover influential Twitter users in the field of science [9].

Péter has shown a new model developed for complex road networks [10]. So there is a universal network model, which is a powerful new tool not only for road networks, but also in the field of air

traffic network modelling and has all the specialties, which must be taken into account in case of the air traffic networks. With the model the dynamic operation of the air traffic network, specifically the basic questions of network development and management regarding the processes in a network, existing or under development, can be examined. They pointed out that the use of traditional modelling approach raises many unanswered questions and struggles always with size problems [11].
Kraus and Barsi presented a technique in junctions based on graph analysis to locate possible ghost driver spots [12]. The paper demonstrated a safety technical application in the case of different type of road crossings and gives an overview about the thematic data modelling and the applied technology. The developed algorithm is capable of providing potential ghost driving locations based on graph analysis. The algorithm gives solution in the case of particular junction structures; the others require further topological and geometric data analysis to determine possible ghost driving locations.

Chen et al proposed a variant of Nonnegative Matrix Factorization (NMF) on the basis of block strategy and graph theory [13]. The objective function is established by means of class label information and the local geometry information.

Tang and Chan have proposed an empirical framework of accuracy graphs and their construction that reveal the relative accuracy of formulas [14]. This work not only evaluates the association between certain assumptions and the theoretical relations among formulas, but also expands our knowledge to reveal new potential accuracy relationships of other formulas which have not been discovered by theoretical analysis. Using the proposed framework, we identified a list of formula pairs in which a formula is consistently statistically more accurate than or similar in accuracy to another, enlightening directions for further theoretical analysis.

Gandhi, Agrawal and Shishodia [15] have presented a procedure based on graph theory for reliability analysis of repairable systems.

Ševčíkova and Milková focused on two multimedia programs dealing with explanation of graph concepts and demonstration of graph algorithms serve as important means to support students' logical thinking development as well as broadening their knowledge and connection to practice [16].

Pálfi's aim is to reduce the length of the break-down periods and improve the work of the operation controllers by implementing a graph-based on-line automatic fault messages system [17].

The main contribution of paper [18] is to assess, prioritize and rank the drivers of sustainable manufacturing practices in the leather industries of Bangladesh. Moktadir et.al used graph theory and a matrix approach to examine the drivers. The results show that knowledge of the circular economy is paramount to implementing sustainable manufacturing practices in the leather industry of Bangladesh. This study will assist managers of leather companies to formulate strategies for the optimum utilization of available resources, as well as for the reduction of waste in the context of the circular economy.

Csiszér introduced the NTS network as a new way of analysing, verifying and improving quality-related risk assessment [19]. NTS is based on a network science approach that models complex systems by graphs, consists of failures or, in a broader sense, risk events as vertices and common occurrences or presumed causal connections among them as directed edges By using N, T and S-Graphs as the elements of NTS, risk events that play special role in the risk management system can be identified. Based on their characteristics, the strength of their potential causal connections can be recalculated, providing more precise predictions of the occurrence frequencies of events

Rosiński studied highway emergency communication system [22-21]. The system is presented in general and then the reliability-exploitation analysis was done. This enabled preparing a development graph of the relationship, under which is created a set of Kolmogorov-Chapman equations describing the system. On this basis, it was possible to set the relationships allowing determining the probability of the system's staying.

The aim of this paper is to show a new, easy-usable algorithm for the determination of existence of interconnection between the network structure system-components or states of technical processes. The proposed method is adaptable to the model sensor networks in automotive engineering and to the investigations of their uncertainties and reliability.

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The outline of this paper is the following: Section 1 contains the applied literatures and the main goals of the investigation. Section 2 shows the mathematical background of graph theory. Section 3 presents the proposed method to determine the connection matrix. Section 4 shows two case studies to demonstrate possibilities of use of proposed methods. Finally, the Author summarizes his work in Section 5.

2. Graph-theoretical background

A graph $G = (N,L,f)$ is a 3-tuple consisting of a set of nodes N, a set of links L, and a mapping function $f : L \rightarrow N \times N$, which maps links into pairs of nodes. Nodes directly connected by a link are called adjacent nodes.

When the node-pair order does not matter in linking the node pair, G is an undirected graph. In an undirected graph $p_i \sim p_j$ is equivalent to $p_j \sim p_i$. Figure 1 visualizes the topology of undirected graph.

A graph is a directed one, if $p_i \sim p_j$ is not $p_j \sim p_i$. Commuting a node pair does make a difference in the graph's topology. Links are directed, by definition, in a directed graph. A link defined by the node pair $(p_j; p_i.)$ is not the same as a link defined by node pair $(p_j; p_{ji})$. In fact, both links may exist in a directed graph.

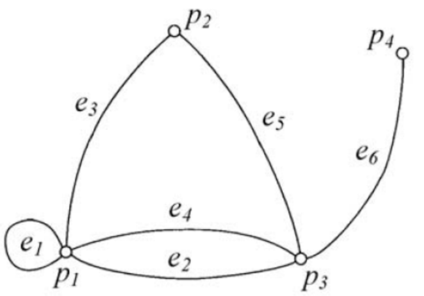

Figure 1. An undirected graph.

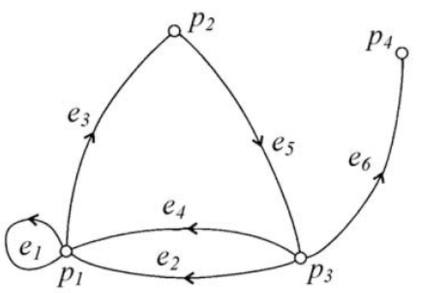

Figure 2. A directed graph.

Set theory is especially efficient for proving theorems and rigorously defining the properties of a graph, but for some forms of analysis a matrix representation is more effective. A graph's mapping function f can be represented by the adjacency matrix **A**, and the connection matrix **Z**.

The direct links are ignored in the graph's adjacency matrix **A** – which shows the number of links directly connecting node i to node j. This number is stored at row i, column j of the adjacency matrix. The adjacent matrix **A** of undirected graph shown by Figure 1:

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{1}$$

The adjacent matrix of directed graph shown by Figure 2:

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{2}$$

The connection matrix **Z** contains a 1 in row i column j if one or more links connect node i to node j. In other words, connection matrix shows that can we get at node j from node i. For example, the connection matrix can be used for troubleshooting or to determine “sink states” of technical processes in engineering practice.

3. Determination of connection matrix

In case of a big, size, integrated technical system it is a real practical problem to expose the connection of system elements, i.e. to determine the connection matrix easily. Therefore an easy-usable method is proposed which calculates connection matrix **Z** from adjunct one **A**.

Firstly, the adjunct matrix should be determined. This task is an easy one, because considering the system the neighbouring aggregates and their direct interconnection can be seen easily.

It is obvious that $a_{ij}^{[2]}$ element of the *k*-th power matrix $\mathbf{A}^k$ of matrix **A** shows the number of independent *k*-long paths from node *i* to node *j*. (Precise and exact mathematical proof of the proposition mentioned above can be seen in book of Fazekas [6].)

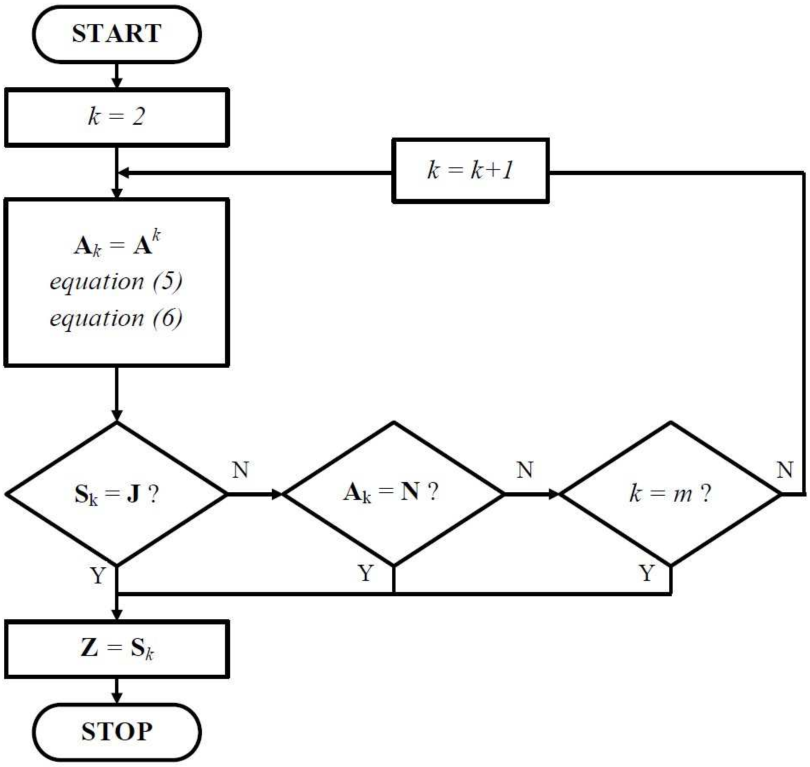

Figure 3. Block Diagram of Connection Matrix Determination.

The element **H** of

$$\mathbf{H}_{\mathrm{k}} = \sum_{n=1}^{\mathrm{k}} \mathbf{A}^n \tag{3}$$

summarized the matrix of power matrices shows the number of independent, maximum *k*-long paths from node *i* to node *j*. Let us generate signum matrix $\mathbf{S}_k$ of $\mathbf{H}_k$ using following function:

$$\mathbf{S}_{\mathrm{k}} = sign\ \mathbf{H}_{\mathrm{k}} \qquad s_{ij}^{[k]} = sign\ h_{ij}^{[k]} \tag{4}$$

where:

$$\mathrm{sign}\eta = \begin{cases} 1, & \text{ha } \eta > 0 \\ 0, & \text{ha } \eta = 0 \\ -1, & \text{ha } \eta < 0 \end{cases} \tag{5}$$

If

$$\mathbf{A}^{k} = \mathbf{N} \tag{6}$$

where **N** is zero (null) matrix, then the length of the longest path of graph is *k - 1*.
The equality

$$\mathbf{S}_{k} = \mathbf{J} \tag{7}$$

where **J** is matrix of ones, means that all nodes have connection with all other ones. It is obvious too, that the longest path (circuit which connects all nodes) of graph has equal number of its nodes of graph denoted *m*. Using the above mentioned statement and equations (6) and (7), the method of connection matrix determination can be depicted by block diagram of Figure 3. The connection matrix shows the interconnection between nodes. But, through the investigation of the connection matrix, the

$$\mathbf{e} = [e_k] \quad e_k = \sum_{j=1}^{m} z_{jk} \tag{8}$$

exposure vector, and

$$\mathbf{i} = [i_k] \quad i_k = \sum_{j=1}^{m} z_{kj} \tag{9}$$

impact vector can be determined. The exposure vector **e** represents the exposedness of nodes, in other words which node depend on the other ones mostly. The impact vector **i** shows which node(s) can be affected by other ones in the highest degree.

4. Case study

To demonstrate the method described above, let us determine the connection matrix of the helicopter Mi-8 Hip pneumatic brake system shown in Figure 4 by examining the braked state of the system.

When analysing the braked state of pneumatic brake system, it can be observable that the elements 10, 12 and 13 are considered passive. The filters do not play role in the braked steady state of the system; even the on-board refuelling device 11 has a role only during system maintenance. The one-way valves 9 can also be regarded as passive. But, as they determine the direction of the flow of

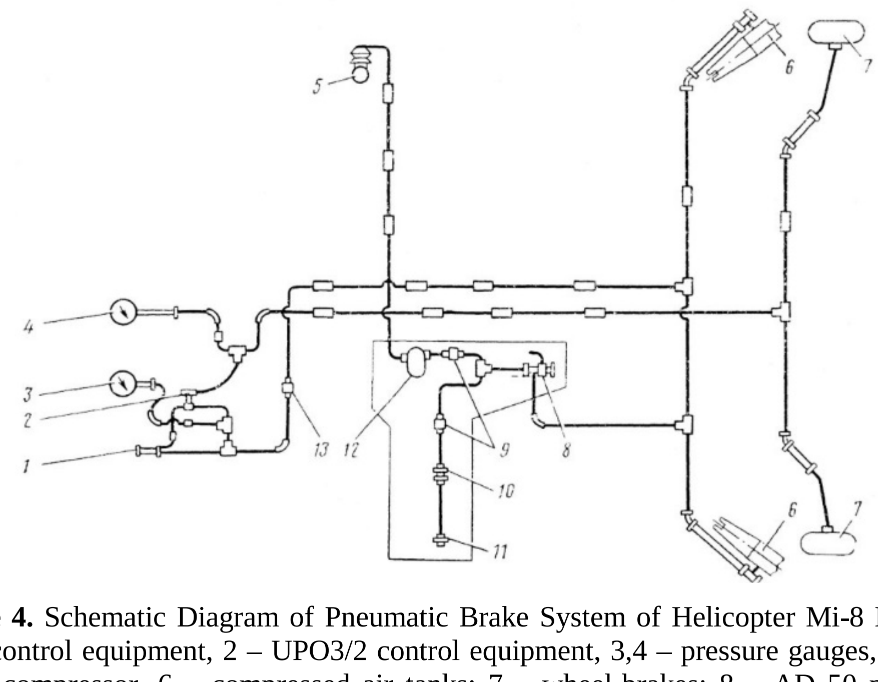

Figure 4. Schematic Diagram of Pneumatic Brake System of Helicopter Mi-8 Hip: 1 – UP-7 control equipment, 2 – UPO3/2 control equipment, 3,4 – pressure gauges, 5 – AK 50 air compressor, 6 – compressed air tanks; 7 – wheel-brakes; 8 – AD 50 pressure-adjusting knob, 9 – one-way valve, 10,13 – filters 11 – refuelling device, 12 –settler.

compressed air in a given pipe section, the graph describing the operation of the system is "directed".

Based on the above considerations and the analysis of the function of the components, the graph model of the system can be seen in Figure 5, using numbering of schematic diagram.

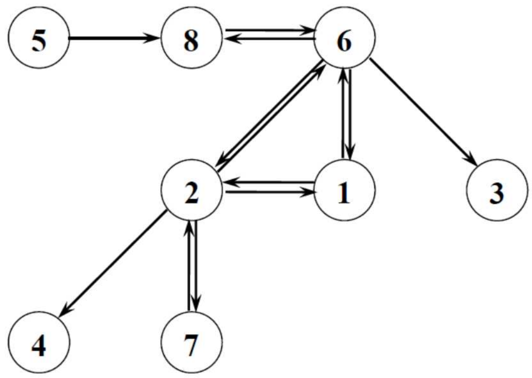

Figure 5. Graph Model of Pneumatic Brake System of Helicopter Mi-8 Hip.

The adjacent matrix of system:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{10}$$

The connection matrix:

$$\mathbf{Z} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \end{bmatrix} \tag{11}$$

Using equations (8) and (11) the exposure vector:

$$\mathbf{e}^{\mathrm{T}} = \begin{bmatrix} 6 & 6 & 6 & 6 & 0 & 6 & 6 & 6 \end{bmatrix} \tag{12}$$

Using equations (9) and (11) the impact vector:

$$\mathbf{i}^{\mathrm{T}} = \begin{bmatrix} 7 & 7 & 0 & 0 & 7 & 7 & 7 & 7 \end{bmatrix} \tag{13}$$

The following conclusions can be deduced from the results of graph modelling and analysis:

- the exposure vector **e** shows that the air compressor 5 is not affected by a failure of another element of the system, seeing that $e_5 = 0$ – see equation (12);
- exposedness of other elements are the same – see equation (12);
- the impact vector **i** also illustrates that derangement of the pressure gauges 3 and 4 has no effect on the operation of the other elements, because $i_3 = i_4 = 0$ – see equation (13);
- impact of other elements are the same – see equation (13).

In case of a more complex system, like a bigger graph, deducing conclusions is not easy, therefore the application of the proposed method is necessary.

5. Conclusions

The paper showed an easy-usable algorithm to determine the connections between the network structure system's components or states of technical processes. Using the proposed method it is possible to determine the connection matrix of investigated systems if its adjacent matrix is known.

The author proposed the application of exposure and impact vectors to characterize interconnections of network structure systems.

During prospective scientific research related to this field of the applied mathematics and the science of engineering management, the Author would like to develop other mathematical models and methods to investigate engineering systems such as sensor networks in automotive engineering and to investigate their uncertainties and reliability.

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