

# Out-plane stability safety factors of CFST arches using inverse finite element reliability method

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**Abstract.** Concrete filled steel tubular arch (CFST) bridges have been widely used in China since the 1990s. However, researches on the stability design of this type of bridges are limited, especially for the out-plane stability. Therefore, a numerical inverse finite element reliability method is presented to solve the problem. In this method, safety factors are introduced into the limit state equations. The cooperation of programming and general finite element software is utilized to implement the method. For different target reliability indices, out-plane stability safety factors of CFST arches considering nonlinearities are obtained via the method. The results show that: the method is of good efficiency and applicability; steel ratios and lateral loads have significant influences on the safety factors; the effects of resistance and load uncertainties on the factors are important which should be concerned in stability design of CFST arch bridges.

## 1. Introduction

Concrete-filled steel tubular (CFST) structures have been widely used in arch bridges over the past 20 years due to the advantages of light weight, high ultimate compressive strength, convenience of construction and good aesthetic appearance. According to incomplete statistics, more than 400 CFST arch bridges have been constructed worldwide and 300 of them are built in China [1]. Investigations on both theories and experiments of CFST arch bridges have attracted more attentions from researchers and some remarkable conclusions have been obtained.

However, the present investigations of CFST arch bridges are concentrated on the applications of new bridge types and new techniques. The gap between the rapid development of practical engineering and the slowly development of theoretical research becomes more and more serious [2-3].

Moreover, CFST arch bridges have become a member of large span bridges. For the properties of high strength and large span, the arch ribs are commonly quite slender, and the problem of stability becomes more significant. However, studies on the stability design of this type of bridges are limited, especially for the out-plane stability [4].

The inverse reliability problem is presented to directly seek the value of design parameters corresponding to specified reliability levels. The problem used to be solved by the trial and error method by using a forward reliability method and interpolating the parameters at the required reliability. For the complexity of the trial and error method, the Hasofer-Lind-Rackwitz-Fiessler (HLRF) iterative algorithm was proposed by Der Kiureghian et al [5]. The efficiency and applicability of the inverse reliability method have been proved by Li and Foschi, using several examples related to the earthquake and offshore engineering [6]. The inverse reliability method has also been applied in the assessment of main cable safety factors for long-span suspension bridges and the efficiencies were verified by some numerical examples [7].



In the present work, the inverse finite element reliability method is introduced into the investigations of safety factors for out-plane stability of CFST arches. Safety factors for different parameters are obtained using the inverse finite element reliability method (FERM).

## 2. Principles of inverse reliability analysis

The inverse reliability problem arises when one is seeking directly the value of design parameters corresponding to specified reliability levels. The inverse reliability problem is defined by the following set of equations.

$$\|\mathbf{u}\| - \beta_T = 0 \quad (1)$$

$$\mathbf{u} + \frac{\|\mathbf{u}\|}{\|\nabla_{\mathbf{u}} G(\mathbf{u}, \boldsymbol{\theta})\|} \nabla_{\mathbf{u}} G(\mathbf{u}, \boldsymbol{\theta}) = \mathbf{0} \quad (2)$$

$$G(\mathbf{u}, \boldsymbol{\theta}) = 0 \quad (3)$$

where  $\mathbf{u}$  is the standard normal vector,  $\beta_T$  is the target reliability index,  $\nabla_{\mathbf{u}}$  is the gradient operator with respect to  $\mathbf{u}$  and Eq.(2) states that the solution  $\mathbf{u}^*$  must be an origin-project point, which is the optimality condition for a fixed  $\boldsymbol{\theta}$  under the condition  $G(0, \boldsymbol{\theta}) > 0$ .

For a target reliability index  $\beta_T$ , the inverse problem can be stated as:

Given  $\beta_T$ ,

Find  $\bar{\theta}$  (mean value of  $\theta$ ) or  $\sigma_{\theta}$  (standard deviation of  $\theta$ ),

Subjected to:  $\min(\mathbf{u}^T \mathbf{u}) = \beta_T^2$  and

$G = G(\mathbf{u}, \boldsymbol{\theta}) = 0$ .

For the first order reliability method (FORM), the latter constructs a sequence of points according to the rule

$$\begin{pmatrix} \mathbf{u}_{k+1} \\ \boldsymbol{\theta}_{k+1} \end{pmatrix} = \begin{pmatrix} \mathbf{u}_k \\ \boldsymbol{\theta}_k \end{pmatrix} + \lambda_k \mathbf{d}_k \quad (4)$$

where,  $\mathbf{d}_k$  is the search direction vector,  $\lambda_k$  is the step size.

The solution of Eq. (1), Eq. (2) and Eq. (3) is

$$\mathbf{u} = -\beta_T \frac{\nabla_{\mathbf{u}} G(\mathbf{u}_k, \boldsymbol{\theta}_k)}{\|\nabla_{\mathbf{u}} G(\mathbf{u}_k, \boldsymbol{\theta}_k)\|} \quad (5)$$

$$\boldsymbol{\theta} = \boldsymbol{\theta}_k + \frac{[\nabla_{\mathbf{u}} G(\mathbf{u}_k, \boldsymbol{\theta}_k), \mathbf{u}_k] - G(\mathbf{u}_k, \boldsymbol{\theta}_k) + \beta_T \|\nabla_{\mathbf{u}} G(\mathbf{u}_k, \boldsymbol{\theta}_k)\|}{\nabla_{\boldsymbol{\theta}} G(\mathbf{u}_k, \boldsymbol{\theta}_k)} \quad (6)$$

Then  $\mathbf{d}_k$  can be expressed as

$$\mathbf{d}_k = \begin{pmatrix} -\beta_T \frac{\nabla_{\mathbf{u}} G(\mathbf{u}_k, \boldsymbol{\theta}_k)}{\|\nabla_{\mathbf{u}} G(\mathbf{u}_k, \boldsymbol{\theta}_k)\|} - \mathbf{u}_k \\ \frac{[\nabla_{\mathbf{u}} G(\mathbf{u}_k, \boldsymbol{\theta}_k), \mathbf{u}_k] - G(\mathbf{u}_k, \boldsymbol{\theta}_k) + \beta_T \|\nabla_{\mathbf{u}} G(\mathbf{u}_k, \boldsymbol{\theta}_k)\|}{\nabla_{\boldsymbol{\theta}} G(\mathbf{u}_k, \boldsymbol{\theta}_k)} \end{pmatrix} \quad (7)$$

The convergence criterion used in the inverse reliability analysis is

$$\frac{(\|\mathbf{u}_{k+1} - \mathbf{u}_k\|^2 + |\boldsymbol{\theta}_{k+1} - \boldsymbol{\theta}_k|^2)^{1/2}}{(\|\mathbf{u}_{k+1}\|^2 + |\boldsymbol{\theta}_{k+1}|^2)^{1/2}} \leq 10^{-3} \quad (8)$$

## 3. Limit state expressions for out-plane stability design of CFST arches

The limit state expression for stability can be expressed as

$$R / K - S > 0 \quad (9)$$

where  $R$  is the structural resistance,  $K$  is the stability safety factor and  $S$  is the load effect.

Then limit state expressions for elastic theory and nonlinear theory can be respectively expressed as

$$P_{cr} / K_3 - S > 0 \quad (10)$$

$$P_u / K_4 - S > 0 \quad (11)$$

where  $P_{cr}$  is the elastic stability bearing capacity,  $P_u$  is the nonlinear stability bearing capacity,  $K_3$  is the safety factor for elastic theory and  $K_4$  is the safety factor for nonlinear theory.

Consequently, the basic form of limit state equation for inverse reliability analysis can be expressed as

$$G(\mathbf{u}, y_{cap}) = g(\mathbf{x}, y_{cap}) = y_{cap} - r(\mathbf{x}) = y_{cap} - R(\mathbf{u}) \quad (12)$$

where  $y_{cap}$  is a deterministic design parameter.

By introducing the safety factors, the typical form of limit state equation for inverse reliability analysis is

$$G(\mathbf{u}, \lambda_{min}) = g(\mathbf{x}, \lambda_{min}) = r(\mathbf{x}) - \lambda_{min} = R(\mathbf{u}) - \lambda_{min} \quad (13)$$

Then Eq. (5) can be rewritten as

$$\mathbf{u} = -\beta_T \frac{\nabla_{\mathbf{u}} R(\mathbf{u}_k, \boldsymbol{\theta}_k)}{\|\nabla_{\mathbf{u}} R(\mathbf{u}_k, \boldsymbol{\theta}_k)\|} \quad (14)$$

For the explicit form of limit state functions (LSFs), the values and gradients of LSF could be easily obtained. While for the implicit form of LSFs, the finite element analysis is utilized [5]. The gradients are calculated by the central difference method:

$$\frac{\partial g}{\partial y} \approx \frac{g(y + \Delta y) - g(y - \Delta y)}{2\Delta y} \quad (15)$$

#### 4. Probability modelling

It is inevitable that the structural resistance is affected by various uncertainties, such as material properties, geometric parameters and calculation modes. For the reliability analysis of arch stability, the statistics of the basic random variables (RVs) are listed in Table 1.

Table 1. Statistics of basic RVs.

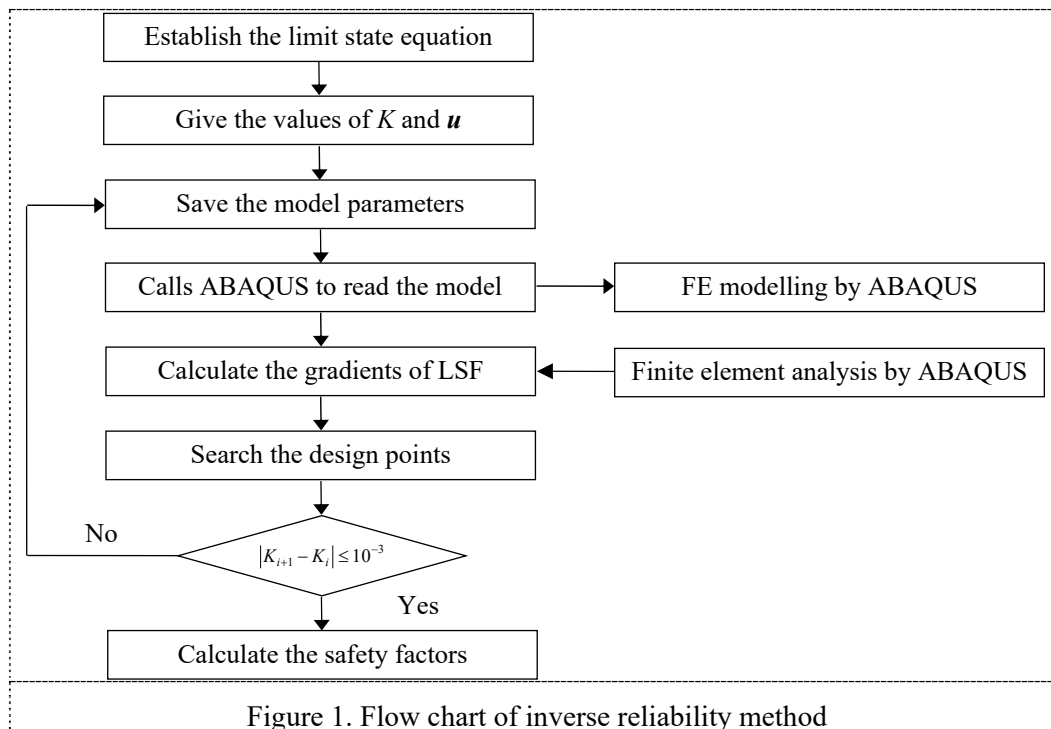
RV	Distribution type	Coefficient of variation
$f_{cu}/\text{MPa}$	Lognormal	0.13
$f_y/\text{MPa}$	Lognormal	0.083
$E_c/\text{MPa}$	Lognormal	0.10
$E_s/\text{MPa}$	Lognormal	0.06
$D/\text{mm}$	Normal	0.0135
$t/\text{mm}$	Normal	0.035
$y_0/\text{mm}$	Normal	0.5
$P/\text{kN}$	Normal	0.08

Notes:  $f_{cu}$  is the concrete compressive cube strength,  $f_y$  is the yield strength of steel tube,  $E_c$  means the elastic modulus of concrete,  $E_s$  means the elastic modulus of steel,  $D$  is the outer diameter of steel tube,  $t$  is the wall thickness of steel tube,  $y_0$  is the initial geometric imperfection of arches and  $P$  is the load effect [8]-[10].

#### 5. Procedures of inverse reliability method

While searching the design points in the inverse reliability method, the values of LSFs and its gradients are calculated in every iterative step via finite element analysis. Therefore, the efficiencies of the inverse reliability method rely on the efficiencies of finite element analysis and the times of

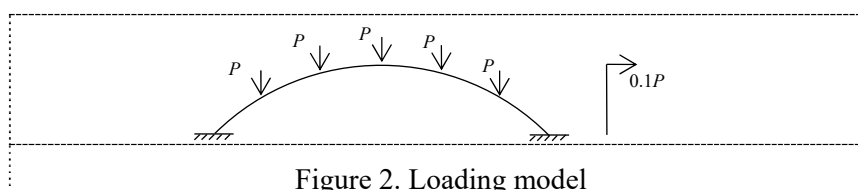
iteration. For nonlinear finite element analysis, the general FE software ABAQUS is utilized, and the cooperation of MATLAB and ABAQUS is realized via the API in MATLAB. The procedure of FERM is illustrated in Figure 1.



## 6. Case studies

### 6.1. Model parameters

A CFST parabolic arch model is selected as an illustrated example, as shown in Fig. 2. The span of the arch is 7.5m and the rise-to-span ratio is 1/5. The outer diameter of the steel tube is 121mm and the wall-thickness is 4.5mm. The elastic modulus of the concrete is 37.84GPa, and the elastic modulus of the steel is 206GPa.



## 6.2. Results

### 6.2.1. Elastic theory

For different lateral loads,  $K_3$  are summarized in Table 2.

For different steel ratios,  $K_3$  are summarized in Table 3.

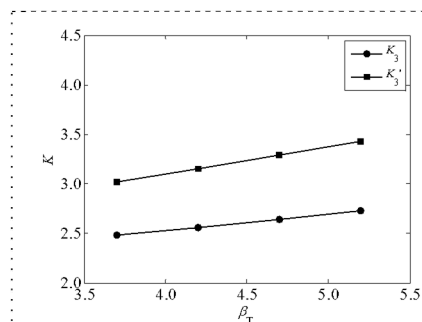
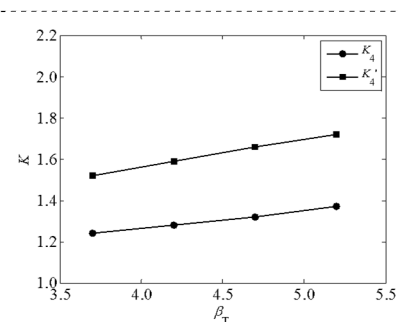
Comparisons of  $K_3$  and  $K_3'$  are shown in Figure 3.

Table 2.  $K_3$  for different lateral loads.

$\beta_T$	$\alpha$					
	3%	5%	8%	10%	20%	30%
3.7	1.60	1.74	1.90	1.99	2.48	2.96
4.2	1.65	1.79	1.96	2.06	2.56	3.05
4.7	1.70	1.84	2.03	2.13	2.64	3.14
5.2	1.76	1.89	2.09	2.21	2.73	3.24

Table 3.  $K_3$  for different steel ratios.

$\beta_T$	$\rho$			
	5%	10%	15%	20%
3.7	2.07	2.01	1.99	1.99
4.2	2.14	2.08	2.06	2.06
4.7	2.21	2.15	2.13	2.13
5.2	2.28	2.23	2.21	2.21

Figure 3. Comparisons of  $K_3$  and  $K_3'$ .Figure 4. Comparisons of  $K_4$  and  $K_4'$ .

### 6.2.2. Nonlinear theory

For different lateral loads,  $K_4$  are summarized in Table 4.

Table 4.  $K_4$  for different lateral loads.

$\beta_T$	$\alpha$					
	3%	5%	8%	10%	20%	30%
3.7	1.26	1.28	1.27	1.26	1.24	1.26
4.2	1.30	1.31	1.31	1.29	1.28	1.30
4.7	1.34	1.35	1.36	1.33	1.32	1.34
5.2	1.38	1.39	1.40	1.38	1.37	1.38

Comparisons of  $K_4$  and  $K_4'$  are shown in Figure 4.

## 7. Conclusions

The inverse finite element reliability method is presented to solve the safety factors for out-plane stability of CFST arches. The cooperation of programming and general finite element software is utilized to implement the method. For different design parameters, out-plane stability safety factors with and without considering nonlinearities are obtained based on arch models.

The results show that: the method is of good efficiency and applicability. Steel ratios and lateral loads have significant influences on out-plane stability safety factors. For elastic stability, safety factors increase with the lateral loads and decrease with the steel ratios. For nonlinear stability, safety factors keep constant with the increasing of the lateral loads. Effects of resistance and load uncertainties on out-plane stability factors are clear which should be concerned in design.

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