

Decentralized robust control of a hydraulic driven parallel manipulator based on disturbance estimation

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Abstract. Since the parallel robot manipulator is a nonlinear, coupling and time-varying dynamic system, it is difficult to synthesize a controller based on the dynamic model by using a centralized control method. Considering each driving unit as a subsystem and treating the coupling interconnection terms between the joints as a time-varying external disturbance of the driving unit, this paper presents a decentralized robust controller with extended state observer (ESO) in joint-space to implement the high-precision trajectory tracking of the robot manipulator. By constructing two extended state observers, not only matched uncertainties but also unmatched uncertainties can be estimated and then compensated in a feed-forward way. Consequently, high-gain feedback is avoided. In addition, the feedback robust laws ensure the global robustness of the proposed controller. Furthermore, the stability of the system is verified by constructing the Lyapunov function. The simulation results show that the proposed controller can estimate both matched and unmatched uncertainties of the driving unit accurately, and the system possesses high trajectory tracking performance and strong robustness.

1. Introduction

It is well known that hydraulic driven parallel manipulator is essentially a highly nonlinear, strongly coupled and time-varying dynamic system. Hence, it is challenging to design a high performance controller for the servo system [1-5]. In the laboratory environment, centralized control strategy (or so-called model-based multi-input-multi-output (MIMO) control) has been made to improve the tracking performance of robot manipulator [2-4]. However, since high computational loads as well as high sensitivity to the modeling errors are well-known shortcomings of these model-based schemes, their practical application is not very promising.

Differing from centralized control, since the time-varying coupling terms can be treated as an external disturbance of each joint, decentralized control strategy (or so-called independent joint control) decomposes the integrated dynamic model of robot manipulator into a set of interconnected subsystems with bounded uncertainties and synthesizes an independent high-performance controller for each joint using only local joint state feedback [5].

To handle the external disturbances and unmodeled dynamics, robust control schemes are applied to many plants. Sliding model control (SMC) [6-7] and adaptive robust control (ARC) [8-10] have been verified the effectiveness to handle the nonlinear uncertainties. However, in order to achieve high tracking performance, larger feedback gains are employed in practical implementations, which may cause instability of the system.



To overcome large time-varying coupling terms between joints, disturbance observer (DOB) can be introduced into the controller to estimate and cancel the disturbance terms. Early works of DOB-based decentralized control schemes can be found in [11-12]. In [13], an active disturbance rejection control (ADRC) strategy was proposed to deal with those plants with a large amount of uncertainties in both dynamics and external disturbances [14]. The core design of ADRC is that an extended state observer (ESO) is used to estimate the generalized disturbance and compensate them in a feed-forward way. The advantage of this control scheme has been verified by a lot of applications [15-16].

In this paper, a decentralized robust controller with ESOs is proposed for trajectory tracking of a hydraulic driven parallel manipulator. Unlike existing ESO-based controllers [15-16], in each local controller, two linear ESOs are constructed to estimate not only matched uncertainties but also unmatched uncertainties via full state back. Hence, uncertainties can be compensated in a feed-forward way and high-gain or high-frequency feedback is avoided. Then two robust terms are employed to handle the fast-changing components of the matched uncertainty and unmatched uncertainty beyond the pass-band of the ESOs respectively. Lyapunov function verifies the convergence and stability of the proposed controller. Simulation results are presented to confirm the established theoretical results.

The remainder of this paper is organized as follows. Section 2 gives the system models and problem formulation. Section 3 presents the proposed controller design procedure. Section 4 carries out the simulation results, and some conclusions can be found in Section 5.

2. System models and problem formulation

2.1. System layout and trajectory generation

As shown in Figure 1, it is a hydraulic driven 3-PSS parallel manipulator which is used for the rocket fuel filling. For the desired trajectory $r_d = [x_d, y_d, z_d]^T$ in Cartesian space, we can obtain the desired position trajectory q_{id} of each actuator through position inverse equation. The coordinate system definition and position inverse equation can be found in [17].

In order to guarantee the transient performance of the controller, the desired trajectory of each driving unit should be smooth enough. Thus, we can obtain the desired trajectory y_{id} and its derivatives $\dot{y}_{id}$, $\ddot{y}_{id}$, $\dddot{y}_{id}$ of each driving unit from the actual reference trajectory through a third-order filter given by [18-19]

$$\ddot{y}_{id} + \kappa_1 \ddot{y}_{id} + \kappa_2 \dot{y}_{id} + \kappa_3 y_{id} = q_{id} \quad (1)$$

where κ_1 , κ_2 , κ_3 are design parameters of the filter.

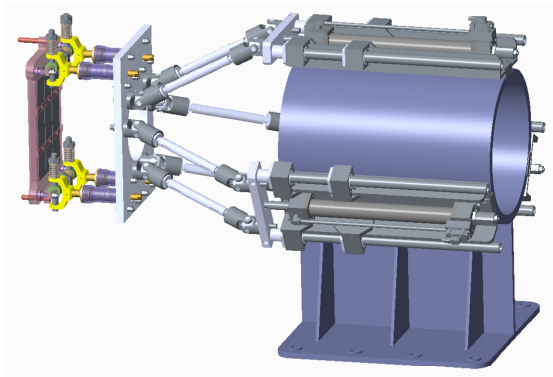


Figure 1. Mechanical structure of parallel manipulator

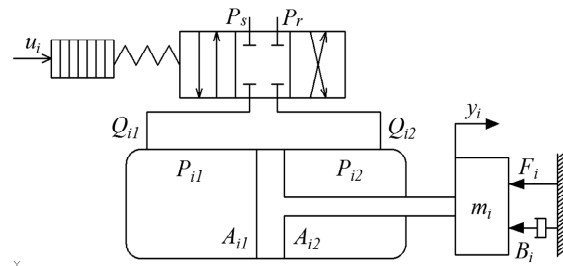


Figure 2. Schematic diagram of hydraulic driving unit

2.2. Model of hydraulic driving unit

The hydraulic driving unit under consideration is depicted in Figure 2. The dynamic equation of each driving unit can be described as

$$m_i \ddot{q}_i = A_{i1} P_{i1} - A_{i2} P_{i2} - B_i \dot{q}_i - \tau_i - D_{fi} \quad (2)$$

where m_i is the mass of piston rod, B_i represents the ideal viscous friction coefficient, P_{i1} and P_{i2} are the pressure inside the two chambers of the cylinder, A_{i1} and A_{i2} are the ram area of the two chambers respectively, τ_i represents the coupling disturbances between the joints, D_{fi} represents other disturbances, such as unmodeled friction, and/or unmodeled dynamics.

Neglecting the external leakage of hydraulic cylinder, the pressure dynamics of i th driving unit can be modeled as [9]

$$\dot{P}_{i1} = \frac{\beta_e}{V_{i1}} [-A_{i1} \dot{q}_i - C_{it} P_{iL} + Q_{i1} + p_{idn1}] , \quad \dot{P}_{i2} = \frac{\beta_e}{V_{i2}} [A_{i2} \dot{q}_i + C_{it} P_{iL} - Q_{i2} + p_{idn2}] \quad (3)$$

where $V_{i1} = V_{i01} + A_{i1} q_i$ and $V_{i2} = V_{i02} - A_{i2} q_i$ represent the effective volumes of the two chambers, respectively, V_{i01} and V_{i02} are the original volumes of two chambers, β_e is the effective bulk modulus of hydraulic oil, C_{it} is the coefficient of the internal leakage of the cylinder, Q_{i1} is the supplied flow rate of the forward chamber, and Q_{i2} is the return flow rate of the return chamber, $P_{iL} = P_{i1} - P_{i2}$ represents the load pressure, p_{id1} and p_{id2} are the modeling errors in the dynamics of P_{i1} and P_{i2} , respectively.

Q_{i1} and Q_{i2} are related to spool valve displacement of the servo valve x_{iv} by [9]

$$Q_{i1} = k_q x_{iv} [s(x_{iv}) \sqrt{P_s - P_{i1}} + s(-x_{iv}) \sqrt{P_{i1} - P_r}] , \quad Q_{i2} = k_q x_{iv} [s(x_{iv}) \sqrt{P_{i2} - P_r} + s(-x_{iv}) \sqrt{P_s - P_{i2}}] \quad (4)$$

where $k_q = C_d \omega \sqrt{2/\rho}$ is the flow gain, C_d is the discharge coefficient, ω is the spool valve area gradient, ρ is the density of oil, P_s and P_r are the supply pressure and return pressure of the fluid, $s(x_{iv})$ is defined as

$$s(x_{iv}) = \begin{cases} 1, & \text{if } x_{iv} \geq 0 \\ 0, & \text{if } x_{iv} < 0 \end{cases} \quad (5)$$

Selecting the high-response servo valve, it can be assumed that spool displacement x_{iv} is directly proportional to the control input u_i , i.e., $x_{iv} = k_m u_i$, where k_m is a positive constant. Therefore, (4) can be transformed to

$$Q_{i1} = k_u x_{iv} [s(x_{iv}) \sqrt{P_s - P_{i1}} + s(-x_{iv}) \sqrt{P_{i1} - P_r}] , \quad Q_{i2} = k_u x_{iv} [s(x_{iv}) \sqrt{P_{i2} - P_r} + s(-x_{iv}) \sqrt{P_s - P_{i2}}] \quad (6)$$

where $k_u = k_q k_m$ represents the total flow gain.

Defining the state variables as $\mathbf{x}_i = [x_{i1}, x_{i2}, x_{i3}]^T = [q_i, \dot{q}_i, A_{i1} P_{i1} - A_{i2} P_{i2}]^T$ and treating τ_i and D_{fi} as the total external disturbances, the hydraulic driving unit can be expressed in a state-space form as

$$\begin{cases} \dot{x}_{i1} = x_{i2} & ; \quad \dot{x}_{i2} = \frac{x_{i3}}{m_i} - b_i x_{i2} + d_{i2}(t) \\ \dot{x}_{i3} = k_u \beta_e \left(\frac{A_{i1} R_{i1}}{V_{i1}} + \frac{A_{i2} R_{i2}}{V_{i2}} \right) u_i - \beta_e \left(\frac{A_{i1}^2}{V_{i1}} + \frac{A_{i2}^2}{V_{i2}} \right) x_{i2} - \beta_e C_{it} \left(\frac{A_{i1}}{V_{i1}} + \frac{A_{i2}}{V_{i2}} \right) P_{iL} + p_{idn} \end{cases} \quad (7)$$

where $b_i = B_i / m_i$, $d_{i2}(t) = -(\tau_i + D_{fi}) / m_i$, $R_{i1} = s(u_i)\sqrt{P_s - P_{i1}} + s(-u_i)\sqrt{P_{i1} - P_r}$,
 $R_{i2} = s(u_i)\sqrt{P_{i2} - P_r} + s(-u_i)\sqrt{P_s - P_{i2}}$, $p_{idn} = \beta_e A_{i1} p_{idn1} / V_{i1} - \beta_e A_{i2} p_{idn2} / V_{i2}$.

To simplify the system state equations, we can define

$$\dot{x}_{i1} = x_{i2}, \quad \dot{x}_{i2} = \frac{x_{i3}}{m_i} - b_i x_{i2} + d_{i2}(t), \quad \dot{x}_{i3} = f_{i1} u_i + f_{i2} + d_{i3}(t) \quad (8)$$

where $f_{i1} = k_u \beta_e (A_{i1} R_{i1} / V_{i1} + A_{i2} R_{i2} / V_{i2})$, $d_{i3}(t) = p_{idn}$,
 $f_{i2} = -\beta_e (A_{i1}^2 / V_{i1} + A_{i2}^2 / V_{i2}) x_{i2} - \beta_e C_{it} (A_{i1} / V_{i1} + A_{i2} / V_{i2}) P_{iL}$.

Assumption 1: The desired trajectory $\mathbf{r}_d = [x_d, y_d, z_d]^T$ in Cartesian space is smooth enough and bounded. In practical hydraulic systems under normal working conditions, P_{i1} and P_{i2} are both bounded by P_s and P_r , i.e., $0 \leq P_r < P_{i1} < P_s$, $0 \leq P_r < P_{i2} < P_s$.

Assumption 2: The external disturbances of manipulator and the matched disturbance $d_{i3}(x, t)$ of each driving unit are both bounded with first-order derivative.

Since the unmatched disturbance $d_{i2}(x, t)$ and its derivative $\dot{d}_{i2}(x, t)$ are both the functions of the state variables and the external disturbances, meanwhile, the motions of hydraulic cylinders are all bounded, combining Assumption 2, we can obtain that the unmatched disturbance $d_{i2}(x, t)$ of each driving unit is bounded with its first-order derivative.

The design objective of decentralized controller is to synthesize an input u_i to make the output y_i track y_{id} as closely as possible.

3. Controller Design

3.1. Extended state observer design

In order to observe the uncertainties for each driving unit, by referring to [20], extended state disturbance observer can be designed for each driving unit.

Define $x_{ie1} = d_{i2}(t)$ and $x_{ie2} = \dot{d}_{i2}(t)$ as the traditional states of unmatched uncertainty and matched uncertainty. According to the Assumption 2, $s_{i1}(t)$ and $s_{i2}(t)$ can be defined as the derivatives of x_{ie1} and x_{ie2} , i.e., $\dot{x}_{ie1} = s_{i1}(t)$, $\dot{x}_{ie2} = s_{i2}(t)$. Hence, the original plant can be rewritten as

$$\begin{cases} \dot{x}_{i1} = x_{i2} \\ \dot{x}_{i2} = \frac{x_{i3}}{m_i} - b_i x_{i2} + x_{ie1} \\ \dot{x}_{ie1} = s_{i1}(t) \end{cases}, \quad \begin{cases} \dot{x}_{i3} = f_{i1} u_i + f_{i2} + x_{ie2} \\ \dot{x}_{ie2} = s_{i2}(t) \end{cases} \quad (9)$$

For the disturbances x_{ie1} and x_{ie2} , two linear ESOs can be constructed as

$$\begin{cases} \dot{\hat{x}}_{i1} = \hat{x}_{i2} + 3\omega_{ie1}(x_{i1} - \hat{x}_{i1}) \\ \dot{\hat{x}}_{i2} = \frac{x_{i3}}{m_i} - b_i x_{i2} + \hat{x}_{ie1} + 3\omega_{ie1}^2(x_{i1} - \hat{x}_{i1}) \\ \dot{\hat{x}}_{ie1} = \omega_{ie1}^3(x_{i1} - \hat{x}_{i1}) \end{cases}, \quad \begin{cases} \dot{\hat{x}}_{i3} = f_{i1} u_i + f_{i2} + \hat{x}_{ie2} + 2\omega_{ie2}(x_{i3} - \hat{x}_{i3}) \\ \dot{\hat{x}}_{ie2} = \omega_{ie2}^2(x_{i3} - \hat{x}_{i3}) \end{cases} \quad (10)$$

where $\hat{x}_{i1}$, $\hat{x}_{i2}$, $\hat{x}_{ie1}$, $\hat{x}_{i3}$ and $\hat{x}_{ie2}$ are the state estimates, $\omega_{ie1} > 0$ and $\omega_{ie2} > 0$ can be thought as the bandwidths of the linear ESOs.

Let $\tilde{x}_{ij} = x_{ij} - \hat{x}_{ij}$ ($j=1,2,3$) and $\tilde{x}_{iek} = x_{iek} - \hat{x}_{iek}$ ($k=1,2$) denote the estimation errors. The dynamic equations of the state estimation errors can be given as

$$\dot{\boldsymbol{\eta}}_i = \omega_{ie1} \mathbf{A}_1 \boldsymbol{\eta}_i + \mathbf{B} \frac{s_{i1}(t)}{\omega_{ie1}^2}, \quad \dot{\boldsymbol{\chi}}_i = \omega_{ie2} \mathbf{A}_2 \boldsymbol{\chi}_i + \mathbf{D} \frac{s_{i2}(t)}{\omega_{ie2}} \quad (11)$$

where $\boldsymbol{\eta}_i = [\eta_{i1}, \eta_{i2}, \eta_{i3}]^T = [\tilde{x}_{i1}, \tilde{x}_{i2} / \omega_{ie1}, \tilde{x}_{ie1} / \omega_{ie1}^2]^T$, $\boldsymbol{\chi}_i = [\chi_{i1}, \chi_{i2}]^T = [\tilde{x}_{i3}, \tilde{x}_{ie2} / \omega_{ie2}]^T$,

$$\mathbf{A}_1 = \begin{bmatrix} -3 & 1 & 0 \\ -3 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix}, \quad \mathbf{B} = [0, 0, 1]^T, \quad \mathbf{D} = [0, 1]^T.$$

Solving (11), we can obtain

$$\begin{cases} \boldsymbol{\eta}_i = e^{\omega_{ie1} \mathbf{A}_1 t} \boldsymbol{\eta}_i(0) + \int_0^t e^{\omega_{ie1} \mathbf{A}_1 (t-\tau)} \mathbf{B} \frac{s_{i1}(t)}{\omega_{ie1}^2} d\tau \\ \boldsymbol{\chi}_i = e^{\omega_{ie2} \mathbf{A}_2 t} \boldsymbol{\chi}_i(0) + \int_0^t e^{\omega_{ie2} \mathbf{A}_2 (t-\tau)} \mathbf{D} \frac{s_{i2}(t)}{\omega_{ie2}} d\tau \\ \tilde{x}_{ie1} = \omega_{ie1}^2 \eta_{i3}, \quad \tilde{x}_{ie2} = \omega_{ie2} \chi_{i2} \end{cases} \quad (12)$$

Lemma 1 [21]: Since $s_{i1}(t)$ and $s_{i2}(t)$ are all bounded, then the estimated states are always bounded and there exists a constant $\sigma_{ij} > 0$ and a finite time $T_1 > 0$ such that

$$|\tilde{x}_{ij}| \leq \sigma_{ij}, \quad \sigma_{ij} = O\left(\frac{1}{\omega_o^c}\right), \quad \forall t \geq T_1 \quad (i=1,2,3, \quad j=1,2,3, e_1, e_2) \quad (13)$$

for some positive integer c .

Remark 1: By increasing the bandwidths of ω_{ie1} and ω_{ie2} , the estimation errors can be compressed to a prescribed range after a finite time.

According to Lemma 1, there exist

$$|\tilde{x}_{iek}| \leq D_{ik}, \quad (k=1,2) \quad (14)$$

where D_{ik} is a bounded positive define function which is known.

3.2. Controller design based on ESOs

Step 1. Define the following errors

$$z_{i1} = x_{i1} - x_{i1d}, \quad z_{i2} = \dot{z}_{i1} + k_{i1} z_{i1} = \dot{x}_{i2} - \alpha_{i1} \quad (15)$$

where z_{i1} is the output tracking error, $\alpha_{i1} = \dot{x}_{i1d} - k_{i1} z_{i1}$ is the virtual control law of the state x_{i2} , k_{i1} is a positive feedback gain.

Differentiating z_{i2} and considering (8), (15), we have

$$m_i \dot{z}_{i2} = m_i (\dot{x}_{i2} - \dot{\alpha}_{i1}) = x_{i3} - m_i b x_{i2} - m_i d_{i2} - m_i \dot{\alpha}_{i1} \quad (16)$$

Based on the unmatched uncertainty estimation by ESOs, the virtual control law α_{i2} of the state x_{i3} can be designed as

$$\alpha_{i2} = \alpha_{i2a} + \alpha_{i2s}, \quad \alpha_{i2a} = m_i (\dot{\alpha}_{i1} + bx_{i2} - \hat{x}_{ie1}), \quad \alpha_{i2s} = \alpha_{i2s1} + \alpha_{i2s2}, \quad \alpha_{i2s1} = -k_{i2s1} z_{i2} \quad (17)$$

where $k_{i2s2} > 0$ is a linear feedback gain.

In (17), α_{i2a} is a model compensation term, and α_{i2s} is a robust control law consisting of two terms: α_{i2s1} is the proportional feedback part of z_{i2} and α_{i2s2} is used as the robust feedback part to inhibit the affection of model uncertainties.

Define $z_{i3} = x_{i3} - \alpha_{i2}$ to be the input discrepancy and substitute (17) into (16), we obtain

$$m_i \dot{z}_{i2} = z_{i3} - k_{i2} z_{i2} + \alpha_{i2s2} - m_i \tilde{x}_{ie1} \quad (18)$$

Design α_{i2s2} to suffice the following conditions [10]

$$z_{i2} (\alpha_{i2s2} - m_i \tilde{x}_{ie1}) \leq \varepsilon_{i2}; \quad z_{i2} \alpha_{i2s2} \leq 0 \quad (19)$$

where $\varepsilon_{i2} > 0$ is a design parameter. One example of α_{i2s2} satisfying (19) is given by

$$\alpha_{i2s2} = -k_{i2s2} z_{i2} \triangleq -h_{i2}^2 z_{i2} / (4\varepsilon_{i2}) \quad (20)$$

where $k_{i2s2} > 0$ can be thought as a nonlinear feedback gain and h_{i2} can be any smooth function satisfying $h_{i2} \geq D_{i1}$.

Step 2: Differentiating z_{i3} and noting the last equation of (8) yields

$$\dot{z}_{i3} = \dot{x}_{i3} - \dot{\alpha}_{i2} = f_{i1} u_i + f_{i2} + d_{i3} - \dot{\alpha}_{i2c} - \dot{\alpha}_{i2u} \quad (21)$$

where $\dot{\alpha}_{i2c} = \frac{\partial \alpha_2}{\partial t} + \frac{\partial \alpha_2}{\partial x_1} x_2 + \frac{\partial \alpha_2}{\partial x_2} \hat{x}_2 + \frac{\partial \alpha_2}{\partial \hat{x}_{ie1}} \dot{\hat{x}}_{ie1}$, $\dot{\alpha}_{i2u} = \frac{\partial \alpha_2}{\partial x_2} \tilde{x}_{ie1}$, $\dot{\hat{x}}_{i2} = (x_{i3} - bx_{i2} - m_i \hat{x}_{ie1}) / m_i$, $\tilde{\hat{x}}_{i2} = -\tilde{x}_{ie1} / m_i$

in which $\dot{\alpha}_{i2c}$ represents the calculable part of $\dot{\alpha}_{i2}$, $\dot{\alpha}_{i2u}$ is the incalculable part of $\dot{\alpha}_{i2}$, $\tilde{\hat{x}}_2 = \dot{\hat{x}}_2 - \hat{\hat{x}}_2$ is the estimation error of $\hat{x}_2$.

Similar to (17), the actual control input of each driving unit can be synthesized as

$$u_i = u_{ia} + u_{is}, \quad u_{ia} = \frac{1}{f_{i1}} [-f_{i2} - \hat{x}_{ie2} + \dot{\alpha}_{i2c}], \quad u_{is} = \frac{1}{f_{i1}} (u_{is1} + u_{is2}), \quad u_{is1} = -k_{i3s1} z_{i3} \quad (22)$$

where k_{i3s1} is a positive feedback gain.

In (22), u_{ia} is the model compensation term, u_{is} is a robust control law consisting of two terms: u_{is1} is the proportional feedback part of z_{i3} , and u_{is2} is used to inhibit the affection of model uncertainties.

By applying the resulting control law (22) into (21), we have

$$\dot{z}_{i3} = -k_{i3} z_{i3} + u_{is2} + \tilde{x}_{ie2} - \dot{\alpha}_{i2u} \quad (23)$$

u_{is2} is designed to satisfy the following conditions

$$z_{i3} \left(u_{is2} + \tilde{x}_{ie2} + \frac{\partial \alpha_{i2}}{\partial x_{i2}} \frac{\tilde{x}_{ie1}}{m_i} \right) \leq \varepsilon_{i3}; \quad z_{i3} u_{is2} \leq 0 \quad (24)$$

where $\varepsilon_{i3} > 0$ is a design parameter that may be arbitrarily small. One example of u_{is2} satisfying (24) is given by

$$u_{is2} = -k_{i3s2}z_{i3} \triangleq -h_{i3}^2 z_{i3} / (4\varepsilon_{i3}) \quad (25)$$

where $k_{i3s2} > 0$ can be thought as a nonlinear feedback gain and h_{i3} can be any smooth function satisfying $h_{i3} \geq \left| \frac{\partial \alpha_{i2}}{\partial x_{i2}} \frac{D_{i1}}{m_i} \right| + D_{i2}$.

3.3. Main results

Theorem 1: The proposed control law (22) guarantees that all closed loop system signals are bounded, and the positive define Lyapunov function

$$V_i = \frac{1}{2} (z_{i1}^2 + m_i z_{i2}^2 + z_{i3}^2) \quad (26)$$

is bounded by

$$V_i \leq e^{-\lambda_i t} V_i(0) + \frac{\xi_i}{\lambda_i} [1 - e^{-\lambda_i t}] \quad (27)$$

where $\lambda_i = 2 \min \left\{ k_{i1} - \frac{1}{2}, k_{i2} - 1, k_{i3} - \frac{1}{2} \right\}$.

Proof: Differentiating (26) and substituting (18), (23) into it, we can obtain

$$\dot{V}_i = z_{i1} (z_{i2} - k_{i1} z_{i1}) + z_{i2} (z_{i3} - k_{i2} z_{i2} + \alpha_{i2s2} + \tilde{x}_{ie1}) + z_{i3} (-k_{i3} z_{i3} + u_{is2} + \tilde{x}_{ie2} - \dot{\alpha}_{i2u}) \quad (28)$$

Integrating (19), (24) and noting the definition of λ_i , we can obtain

$$\begin{aligned} \dot{V}_i &\leq -k_{i1} z_{i1}^2 + |z_{i1}| |z_{i2}| + |z_{i2}| |z_{i3}| - k_{i2} z_{i2}^2 + \varepsilon_{i2} - k_{i3} z_{i3}^2 + \varepsilon_{i3} \\ &\leq -\left(k_{i1} - \frac{1}{2}\right) z_{i1}^2 - (k_{i2} - 1) z_{i2}^2 - \left(k_{i3} - \frac{1}{2}\right) z_{i3}^2 + \xi_i \leq -\lambda_i V_i + \xi_i \end{aligned} \quad (29)$$

where $\xi_i = \varepsilon_{i2} + \varepsilon_{i3}$,

The solution of (29) is

$$V_i \leq e^{-\lambda_i t} V_i(0) + \frac{\xi_i}{\lambda_i} [1 - e^{-\lambda_i t}] \quad (30)$$

Integrating (30) and noting (26), it is clear that as the Lyapunov function V_i above is globally bounded, then z_{i1} , z_{i2} and z_{i3} are all bounded. According to Lemma1, the estimated state $\hat{x}_{ij}$ is bounded. From Assumption 1, it can be inferred that the state x_{ij} is bounded. Combining Assumption 2 and noting (22), we obtain that the control input u_i is bounded. Hence, the Theorem 1 is proved.

4. Simulation results

4.1. Simulation parameters

In the simulation, the hydraulic system parameters are: $m_i = 25\text{kg}$, $B_i = 45\text{N} \cdot \text{m} / \text{s}$, $V_{i01} = 1.2 \times 10^{-4} \text{m}^3$, $V_{i02} = 2.96 \times 10^{-4} \text{m}^3$, $P_s = 12\text{MPa}$, $P_r = 0\text{MPa}$, $\beta_e = 2 \times 10^8 \text{Pa}$, $A_1 = 1.26 \times 10^{-3} \text{m}^2$, $A_2 = 6.4 \times 10^{-4} \text{m}^2$, $C_t = 9 \times 10^{-12} \text{m}^5 / (\text{N} \cdot \text{s})$, $k_u = 4 \times 10^{-8} \text{m}^4 / (\text{s} \cdot \text{V} \cdot \sqrt{\text{N}})$.

Meanwhile, in order to verify the observation performance of ESOs and tracking performance of

proposed controller, a joint simulation scheme based on Matlab and Adams is adopted. Adams calculates and provides the load force for each hydraulic cylinder in simulation.

4.2. Simulation analysis

The following three controllers are compared.

1) **DRCESO**: This is the decentralized robust controller with ESO. The controller parameters are chosen as $k_1 = 800$, $k_2 = k_{2s1} + k_{2s2} = 500$, $k_3 = k_{3s1} + k_{3s2} = 300$. The observer gains are given by $\omega_{ie1} = 800$, $\omega_{ie2} = 500$.

2) **DRC**: This is the decentralized robust controller, which is obtained by using the same control law as in DRCESO but without disturbance estimation, i.e., $\omega_{ie1} = \omega_{ie2} = 0$.

3) **VFPID**: This is the velocity feed-forward PID controller. The controller gains tuned via error-and-try method are chosen as $k_p = 180$, $k_i = 25$, $k_d = 2$, $k_v = 10$.

The desired trajectory in Cartesian space is given by $x_d = 0.05 \sin(2\pi t(1 - \exp(-0.1t^3)))$, $y_d = 0$, $z_d = 0.05 - 0.05 \cos(2\pi t(1 - \exp(-0.1t^3)))$.

In order to test the tracking performance of the controller mentioned above, at 8 seconds, a sinusoidal external disturbance $D_x = 300 \sin(2\pi t)$ N is applied in x-axis, meanwhile, the matched uncertainty $p_{idn1} = 5 \times 10^{-6} \sin(\pi t)$ m³/s is added into the actuator 1 at 5 seconds.

Comparative simulation results are shown in Figure 3-6. As seen, the proposed DRCESO controller has better tracking performance than the other two controllers in terms of both transient and final tracking error. When the manipulator is subjected to a sinusoidal disturbance in x-axis (8-20s), the tracking error in x-axis which the DRCESO controller is used is amplified slightly. Since the DRC controller doesn't have the ability to estimate and compensate the disturbances, comparing with DRCESO, the tracking errors of DRC controller are larger. In addition, it can be seen that, although VFPID controller has some robustness against uncertainties, it has significantly poorer tracking performance than the other two model-based controllers.

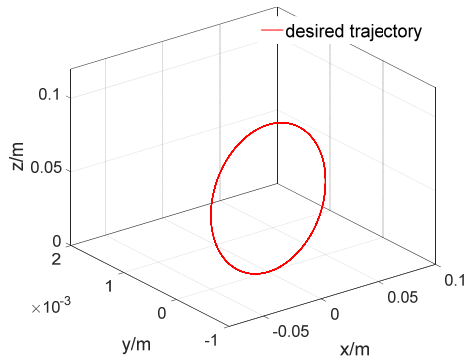


Figure 3. Desired trajectory in Cartesian space

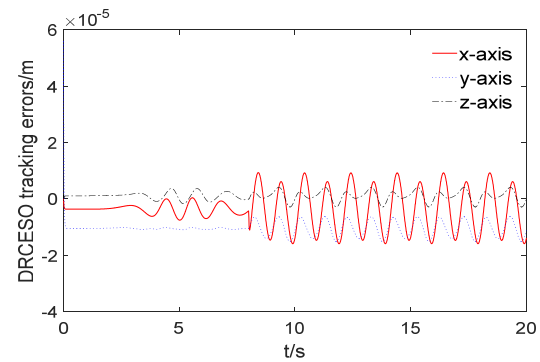


Figure 4. Tracking errors with DRCESO

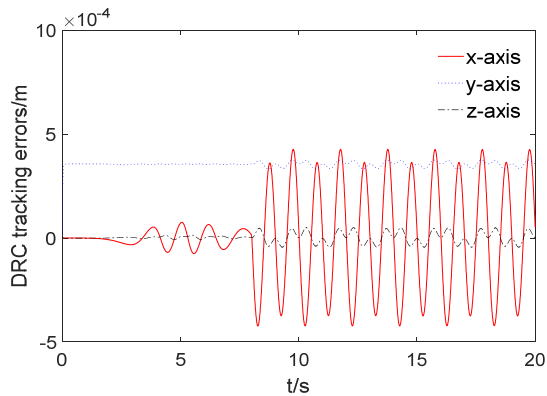


Figure 5. Tracking errors with DRC

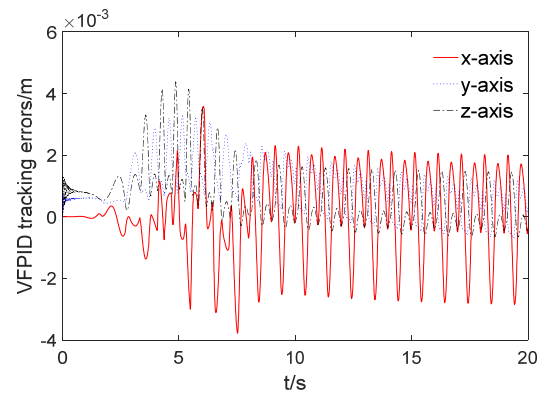


Figure 6. Tracking errors with VFPID

In Figure 7-9, the external disturbance (unmatched uncertainty) estimation and its error of each driving unit is shown respectively, which presents pretty good observation performance of ESOs. As seen, most of the time, the estimation errors are nearly close to zero which can be almost neglected. Figure 10 shows the estimation error of matched uncertainty of actuator 1. Obviously, the ESOs can estimate the matched uncertainty with high accuracy.

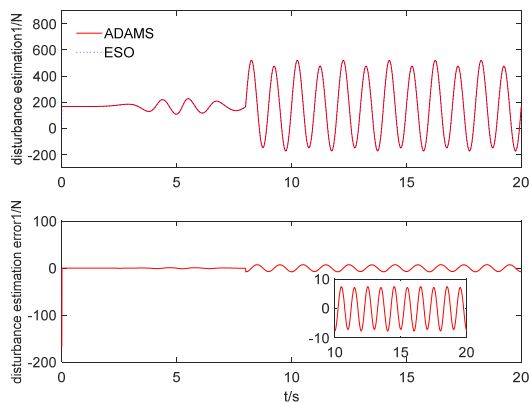


Figure 7. Disturbance estimation and its error of actuator 1

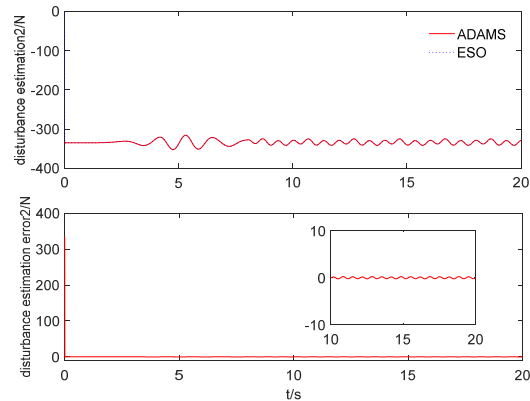


Figure 8. Disturbance estimation and its error of actuator 2

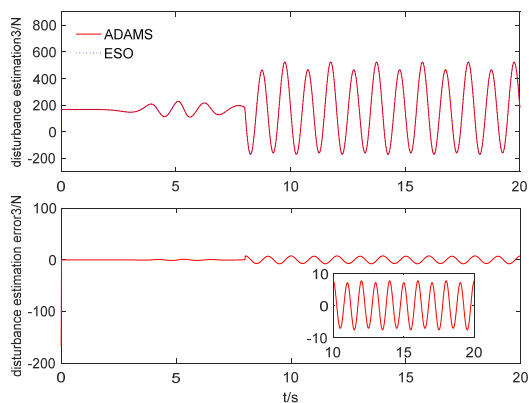


Figure 9. Disturbance estimation and its error of actuator 3

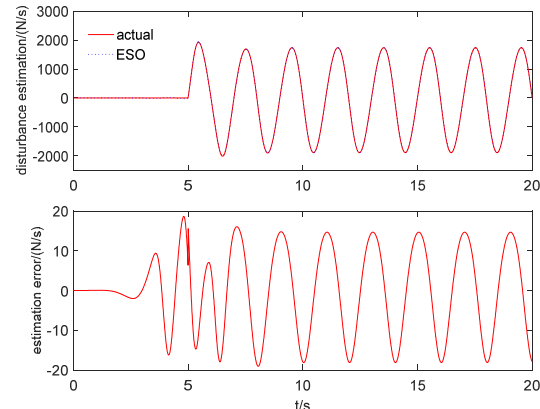


Figure 10. Matched uncertainty estimation and its error of actuator 1

5. Conclusions

In this paper, a decentralized robust control strategy for trajectory tracking has been proposed for a hydraulic driven parallel manipulator. A third-order filter is used to generate the desired trajectory of each driving unit in joint-space. In each local joint, treating the large time-varying coupling terms as the external disturbances, the controller uses two linear ESOs to estimate and compensate both matched and unmatched uncertainties without high feedback gains. The convergence and stability of the control scheme are analyzed via a Lyapunov method. Extensive comparative simulation results show that the higher tracking accuracy and stronger robustness are obtained due to the introduction of the ESOs into the robust controller, which reveal the effectiveness of the proposed controller.

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