Energy-efficient train driving strategy considering the on-board energy storage

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Abstract. Energy-efficient train driving strategy is an effective way to reduce the energy consumption of train operations. Based on the classic energy-efficient driving strategy approach, this paper studies the influence of the on-board energy storage on the optimal train driving strategy. Firstly, this paper applies the dynamic programming algorithm to obtain the energy-efficient driving strategy for a single train in one interval, especially with considering the variety of conditions of line as well as the traction and braking characteristic of the vehicle. Then, based on the result of the energy-efficient driving strategy, an optimal control approach of the capacitor discharging in train operations is proposed in this paper to analyze the influence of the usage of regenerative energy of on-board capacitor on the net energy consumption of the train. Furthermore, a numerical algorithm is designed to solve the problem of the optimal discharge strategy of the vehicle capacitor. Finally, a case study is conducted based on the actual line and vehicle data. The simulated results show that the utilization of regenerative braking energy can reduce the energy consumption by 17.4% by applying the proposed approach

1. Introduction

With the given trip time, the energy consumption of train operations is related to the train driving strategy. On the basis of whether the traction and the braking forces in the model are continuous variables between zero and the maximum, the optimal control problem can be divided into discrete and continuous control problems. For the discrete control problem, Benjamin et.al summarized the control principle of diesel electric locomotive[1]. Cheng [2], Howlett [3] and Leizarowitz [4] proposed a discrete optimal control model and presented an analytic method to find an idealized strategy by solving the optimal duration of the control sequences. Howlett [5],[6] also proved that the energy-efficient driving strategy of trains with the continuous control can be approximated with a discrete control sequence which consumes the same time and energy. A lot of researchers also have carried on the research to solve the continuous problem. Milroy [7] proposed the energy-efficient driving strategy on a short line which consists of maximum traction, coasting, and maximum braking. Howlett [8] theoretically proved that the optimal driving strategy for a general line should use maximum traction, cruising, coasting, and maximum braking. Su et al. [9],[10] used the iterative algorithm to get the train driving strategy on the whole line, which integrates the optimization of driving strategy and the timetable.   Albrecht et al. [11], [12] proved the existence and uniqueness of the solution for the continuous control problem.

Utilization of regenerative braking energy is another effective method to reduce the energy consumption of trains [13]. Hellgren [14] studied the factors which associated with regenerative braking,

such as the motor efficiency, battery level, transmission ratio, etc. And he designed a genetic algorithm to obtain the maximum utilization of the regenerative braking energy. The overall optimization model is established to get the acceleration / braking state and current demand of the train at any point by Miyatake [15], [16].

In the previous works, the maximum traction and braking forces are usually regarded as two fixed values. However, the practical maximum traction and braking forces will decrease with the increase of velocity. Moreover, the riding comfort demand from passengers is considered in this paper. On the other hand, the regenerative braking energy is utilized to change the energy-efficient driving strategy which can achieve a better energy saving effect.

The remainder of this paper is as follows: In the second section, the energy consumption model based on the kinetic energy equation is presented. Then, the model of the energy consumption with consideration of regenerative braking is developed. In the third section, the dynamic programming algorithm is introduced, and a numerical algorithm is designed to solve the optimal discharge positions for trains. In the fourth section, a case study is conducted by using the actual line and vehicle data of Yizhuang line.

2. The Proposed model
This paper firstly gives a solution to obtain the energy-efficient driving strategy of a single train in one interval at a given time with considering of various operation conditions. Based on the solution, the discharging control strategy of regenerative braking energy is proposed.

The basic assumptions are as follows:

1.   We only consider the energy consumption of traction. The energy consumption of lighting, air conditioning and other electricity will not be calculated in this paper;

2.   The train is regarded as a particle, and the interaction between successive trains is not considered.

The parameters that will be used in the model are as follow.

2.1. Train Energy Consumption Model
For the train operation, the train movement is subjected to the combined action of the traction force, the braking force, the basic resistance and the additional resistance of the ramp, which follows the Newton's second law:

$$Ma = \eta F(v) - \mu B(v) - r(v) + g(s) \tag{1}$$

In the equation (1), $g(s)$ is the gradient additional resistance, while it is negative for the uphill section and positive for the downhill section. $F(v)$ and $B(v)$ are the maximum traction force and the maximum braking force, respectively. $r(v)$ is the train basic resistance. η and μ are the efficiency of the traction force and the efficiency of the braking force, respectively.

The energy consumption of train is all due to the traction force exerted by the power grid on the train. Therefore, the energy consumption of the train can be expressed as the integral of the traction force and the distance. The objective function can be written in the form of equation (2)

$$\min E = \int_0^S F \, ds \tag{2}$$

The speed of the train at the stations is zero. So, we have $v(0) = v(S) = 0$.

Train speed in the range cannot exceed the line and the conditions of the vehicle itself. At the same time, in order to ensure the efficiency of operation, the speed of the train should be kept at a higher speed except for the line of middle section, i.e.,

$$V_{\min} \leq v \leq V_{\max} \tag{3}$$

The error between the train running time in the interval and the time specified in the timetable should be controlled within a certain range.

$$T_{total} - T_{\det} \leq T(S) \leq T_{total} + T_{\det} \tag{4}$$

In the equation (4), T_{total} is the running time regulated by the timetable.

At any moment, the train cannot apply both accelerating and braking. We have

$$\eta \cdot \mu = 0 \tag{5}$$

Moreover, the traction and braking force cannot exceed the maximum value provided by the vehicle. Hence,

$$\eta \in [0,1] \quad \mu \in [0,1] \tag{6}$$

The displacement of the train at beginning is zero. When the train arrives at the stop, the distance should equal to the length of the line.

$$S(0) = 0 \quad S(T) = S \tag{7}$$

By referring to the experience in the actual operation, we assumed that the train cannot change its working regime from traction to braking directly and vice versa. According to this constraint, we have:

$$a_s \cdot a_{s+\Delta s} \ge 0 \tag{8}$$

The comfort of passenger, which is mainly related to the derivative of acceleration and deceleration is also an important aspect that should be considered in the operation, so the maximum acceleration and maximum deceleration should be limited in a certain range

$$\frac{\eta F(v) - r(v) + g(s)}{M} \le a_{\max} \tag{9}$$

$$\frac{-\mu B(v) - r(v) + g(s)}{M} \le b_{\max} \tag{10}$$

2.2 Energy Consumption Model with Regenerative Braking

When the regenerative braking energy is utilized, the force equation of the train can be written as follow,

$$Ma = \eta F(v) + \gamma F_c(v) - \mu B(v) - r(v) + g(s) \tag{11}$$

The objective function and the boundary conditions remains unchanged. Moreover, the utilization of regenerative braking energy and the braking cannot occur synchronously. Hence, we also have

$$\gamma \cdot \mu = 0 \tag{12}$$

3 Solution Approach

In this paper, dynamic programming is applied to solve the optimization problem if the train operation is seemed to be a multi-stage decision problem. The specific dynamic programming algorithm in this paper is described as follows

3.1 Dynamic Programming

1.   In this paper, we choose the location of the train as the basis for the discretization and the stage division. Set a step every D meters. And, set state points on each step with the velocity interval Δv.

2.   Determine the driving strategy from one step to the next. And, calculation the transition cost between the two state points on the adjacent steps.

3.   Generate the driving strategy from the departure station to the following steps.

4.   After all the driving strategy to all steps are generated, the energy-efficient driving strategy can be obtained.

3.2 Numerical Algorithm of the Capacitor

1.   The numerical algorithm that proposed below aimed at maximizing the utilization of the regenerative energy. The specific algorithm steps are as follows:

2.   Calculate the regenerative energy that can be utilized by the train with the given parameters, and the result is set as E_B. Then, divide E_B into n copies.

3.   Allocate the one piece of regenerative energy on each segment, calculate the corresponding new energy-efficient driving strategy with respect to the optimal solution which is obtained by the

dynamic programming.

    4.   Select the driving strategy as the new optimal solution which have the minimum energy consumption, record the corresponding segment and the running time.

    5.   Repeat the steps 2 to 3 until all the pieces of regenerative are allocated. The final optimal solution will be the energy-efficient driving strategy when utilization of the regenerative braking energy.

4 Case Study

This paper chooses the interstation between Jiugong and Yizhuangqiao of Yizhuang subway line as an example and the actual vehicle data and line data is used to do the model simulation experiment with MATLAB. The line data is shown in Table 1.

Table 1 Operation Data of YiZhuang Subway Line

Position(m)	Gradient(‰)	Speed limit (km/h)	Position(m)	Gradient(‰)	Speed limit (km/h)
0-130	0	54	1029-1115	0	80
130-466	0	80	1115-1405	-2	80
466-543	-4	80	1405-1768	-3	80
543-700	-6	80	1768-1802	-2	80
700-761	0	80	1802-1840	-1	80
761-851	7	80	1840-1975	0	54
851-1029	12	80			

*The sign '-' means the line is downhill in this part.

During the train operation, the maximum value of traction force $F(v)$ and braking force $B(v)$ are the piecewise functions with respect to the train speed.

$$F(v) = \begin{cases} 310\ kN, & v \le 10m/s \\ 310-(v-10)\cdot 10\ kN, & 10m/s<v \end{cases} \quad (13)$$

$$B(v) = \begin{cases} 260\ kN, & v \le 15m/s \\ 260-(v-15)\cdot 15\ kN, & v>15m/s \end{cases} \quad (14)$$

The calculation method of the running resistance of train is shown in equation (15)

$$r(v) = 0.005\times v^2 + 0.23\times v + 2.965\ kN \quad (15)$$

The mass of the train is assumed to be 250 tons. The maximum acceleration and the maximum deceleration are defined as $a_{max}=0.8m/s^2$, $b_{max}=0.6m/s^2$. Allowable range of the error of the train running time is $\Delta T=\pm 2.5s$. Restricted of minimum speed is $V_{min}=60km/h$. Velocity interval between the adjacent state points of the same step is $\Delta v=2km/h$. The parameters that $D=50m$.

We choose the Maxwell c 3000F cell (medium) capacitor as the on-board energy storage device. The maximum usable energy of the capacitor is $4.53kw\cdot h$ and the mass of it is $1275kg$.

The conversion efficiency from the braking kinetic energy to the regenerative braking energy is $\xi=0.8$. Charge efficiency φ and discharge efficiency λ are equal and are assumed to be 0.9 in the case study. The regenerative energy E_B is divided into $n=10$ copies. When the velocity of train is below 10km/h, the train can only use mechanical brake. Therefore, we assume that v_{B2} is 10km/h and v_{B1} is 72km/h.

According to the above conditions, by utilizing of the dynamic programming algorithm in the 3.1 section, the energy-efficient driving strategy can be solved. The energy consumption of the energy saving strategy is 23.3475kwh, and the running time is 137.17 seconds. (Figure 1, Line 1). When the energy storage device is equipped and the regenerative braking energy is utilized, the energy-saving

strategy will be changed. Energy consumption of the new energy-efficient driving strategy is 19.2757kw*h, and the running time is 133.604 seconds. (Figure 1, Line 2).
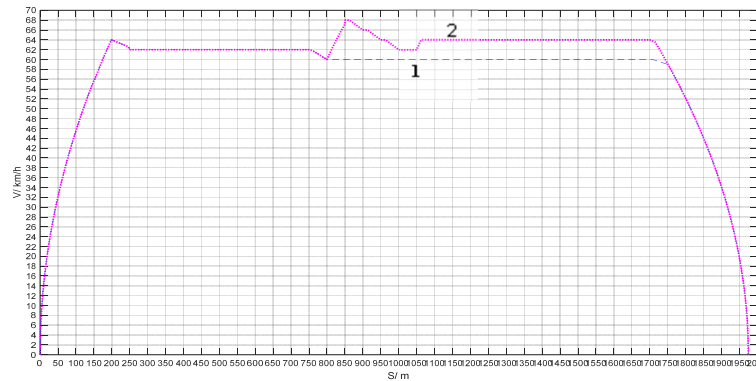

Figure 1 Speed Profile

As the red rectangle shows in Figure 2 that the energy-efficient driving strategy has been changed on that segment when the regenerative braking energy is utilized. It is obvious that the average speed of line 2 is higher than the average speed in line 1.

Table 2 shows the cumulative distance of each regime for these two speed profiles.
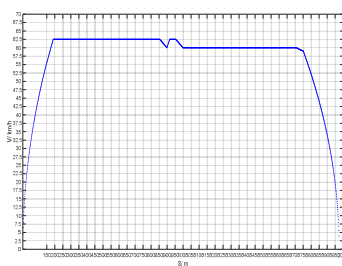
Table 2 Cumulative distance of each regimes (m)

|	Traction(m)	Cruising(m)	Coasting(m)	Braking(m)	Total(m)
Without Regenerative Energy	198	1412	136	229	1975
Utilize Regenerative Braking Energy	259	1232	219	265	1975

Furthermore, we made some experiment to test the effect of the parameters on the simulation results. When the velocity interval Δv changes its value to $2.5km/h$, $1.25km/h$, $1km/h$, and other conditions are fixed, the energy consumption and running time under different velocity intervals are shown in Table 3. And the speed profiles are shown in Figure 2 (a) to (c).
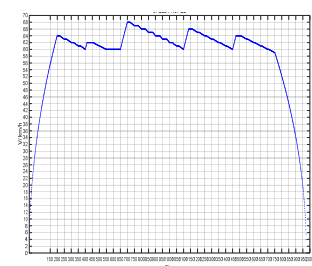
Table 3   Energy consumption and running time

	Energy Consumption（kwh）	Running Time（s）
$2.5km/h$	19.6446	136.64
$1.25km/h$	16.5237	136.69
$1km/h$	14.9990	134.47
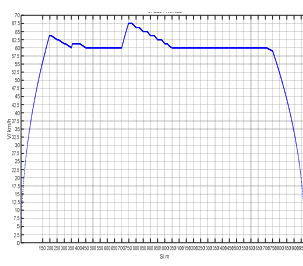
(a)   (b)   (c)

Figure 2 Speed Profile under Different Conditions

The cumulative distance of each regime under the condition of different velocities intervals are shown in Table 4.

Table 4 Cumulative distance of each regime (m)

	Traction	Cruising	Coasting	Braking	Total
$2.5km/h$	203	1412	135	225	1975
$1.25km/h$	249	1134	367	225	1975
$1km/h$	317	566	867	225	1975

5 Conclusion

When the regenerative energy is utilized or the velocity interval decreased, the total distance of coasting and braking reduced. Generally, the train only consumes energy when traction or cruising regimes are applied. And, the traction energy consumption is zero when the train uses coasting or braking regime. Due to the limitation of storage capacity and conversion efficiency, the whole regenerative braking energy that the capacitor can discharge is $4.077kw \cdot h$. However, by taking the regenerative energy as the auxiliary power, the original energy-efficient driving strategy of the train can be changed. Therefore the energy saving effect is $4.0955kw \cdot h$.

The utilization of regenerative braking energy can decrease the energy consumption by 17.4%. The parameter adjustment can significantly influence the energy consumption and running time of the energy-efficient driving strategy. Since this paper used a discrete model, the driving strategy getting close to the actual situation as the velocity interval getting smaller.

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