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The Location Method of Logistics Facilities Based on Efficiency and Fairness Preferences

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Abstract: In location problems for logistics sector such as emergency logistics system, the fairness is an important goal for facility design. This paper considers an extension to the p-median model by changing the weights of individuals based on their utilities. Also, we define and formulate performance metrics of efficiency and fairness in facility location. A method is proposed for decision makers selecting their efficiency-fairness preference from the perspective of quantitative arguments. The method was tested on a real world data set from the distribution system at Shanghai provided by Alibaba data center.

1. Introduction
Typical location models are concerned with efficiency goals, rarely considering the gaps of service levels between individuals. Since both the decision makers and customs have the fairness preference, Morrill [1] presented that the fairness is an important criteria when estimating a location solution in public facility and medical facility location sectors. The tradeoff between efficiency and fairness based on the decision maker's preference is a core problem in view of the antimony between those two goals.The traditional way to achieve those two goals is constructing multi-objective model in which the efficiency is measured by cost or time and the fairness is measured by Gini coefficient or variance. The related researches are focus on public service field. The minimum envy model was first introduced in location problems by Espejo [2], among which they defined the envy as the differences in service quality between all possible pairs of customs. Drezner [3] analyzed minimizing both objectives, the variance of the distances and the range of distance in the plan. The Gini coefficient is also a usual measure metric of fairness [4]. Tzeng [5] considered fairness by maximizing the minimum service satisfaction among demand points. Since the measure of fairness highly influences the optimal result in multi-objective model and it's difficult to balance the efficiency and fairness, it makes more sense to find a simple method for achieving the tradeoff between those two goals. In our model, the weights of demand points are changed following their utilities such as cost or distances. The application of variable weight avoids the negative effect of choosing improper measure metric of fairness. Meanwhile, we define the loss in efficiency and fairness by comparing the performances of a solution with the optimal one. In this way, the decision makers could choose from the feasible solutions by estimating their tolerance of loss.

2. Variable Weight Principle
The variable weight approach was first introduced by PZ Wang [6]. The integral theory system was built by HX Li [7] and DQ Li [8] and they presented the axiomatic definition of variable weight, variable weight vectors, balance functions, etc. The variable weight vector is the Hadamard product of constant weight vector and state variable weight vector.

$$V(X) = \frac{(w_1S_1,...,w_mS_m)}{\sum_{j=1}^{m}(w_jS_j)} \tag{1}$$

Firstly, the state variable weight S_j is the function of the utility x_j, $S_j=g(x_1,x_2,..,x_m)$. This function is called the balance function. Besides, the constant weight reflects the original importance of individuals. Then the variable weight V_j could be obtained by normalize the product of constant weight and variable weight through Eq. (1). The forms of balance functions include incentive type, disciplinal type and mixed type. Different types have different effects of variable weight. For example, the mixed type $S_j(x_1,x_2,...,x_m)=e^{a(x_j-\bar{x})}$, assuming the range of the parameter $\alpha=[-0.005,0,0.005]$ and the value of utility $X=(0,10,...,500)$, the results of variable weight vectors are shown in Figure 1. The variable weights equals the constant weights for α=0. And the balance function is incentive for α=0.05. In this situation, the weights of individuals with higher utilities ($X_i \geq 410$) will increase. If α=-0.05, the function is the disciplinal type and the individuals with lower utilities ($X_i \leq 330$) become more important in decision making.

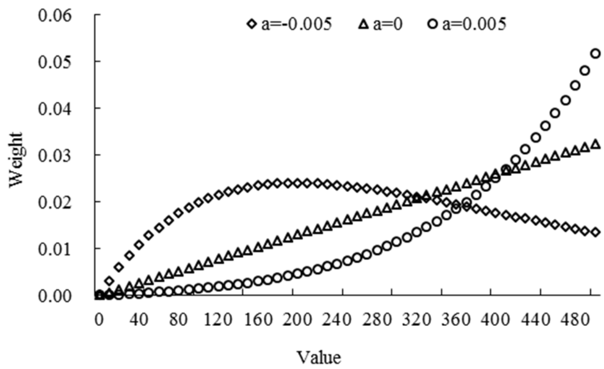

Figure 1 The effects of variable weight for various values of α

3. The Location Model

The decision makers need to select p facilities from m potential stations to satisfy n demand points. The demand amount in point i is h_i and the distance between demand point i and potential station j is d_{ij}. If a facility is located at station j, the value of x_j equals 1, otherwise $x_j=0$. If a facility at station j assigned to serve point i, the value of y_{ij} equals 1, otherwise $y_{ij}=0$. The p-median location model combined with variable weights:

$$\min \sum_{i=1}^{n}\sum_{j=1}^{m} v_i h_i d_{ij} y_{ij} \tag{1}$$

Subject to:

$$y_{ij}-x_j \leq 0 \quad \text{for i=1,...,n;j=1,...,m} \tag{2}$$

$$\sum_{j=1}^{m} x_j = p \tag{4}$$

$$\sum_{j=1}^{m} y_{ij} = 1 \quad \text{for i=1,...,n} \tag{3}$$

$$v_i = \frac{h_i S_i(u_1,u_2,...,u_n)}{\sum_{i=1}^{n} h_i S_i(u_1,u_2,...,u_n)} \quad \text{for i=1,2,...,n} \tag{4}$$

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$$u_i=\sum_{j=1}^{m} d_{ij} y_{ij} \quad \text{for } i=1,2,\dots,n \tag{5}$$

The p-median model is introduced as an integer programming. The objective of the location model is to minimize the sum of weighted total service distance, as shown in Eq. (2). Eq. (3) requires that a demand point i can be served by a station j if a facility is located at station j. Eq. (4) limits the number of facilities that are available to be located. Eq. (5) ensures that a demand point must be served by exactly one facility. Eq. (6) and Eq. (7) works together to calculate the variable weight of demand point i. And the constant weights are counted by demand amounts. The balance function $S_i=g(u_1,u_2,\dots,u_n)$ has different forms under various purposes.

4. The Efficiency and Fairness Preferences

4.1 Measure Functions

Let $\psi_e(Y)$ denote the measure of efficiency which calculate total service distance.

$$\psi_e(Y)=\sum_{i=1}^{n}\sum_{j=1}^{m} h_i d_{ij} y_{ij} \tag{6}$$

Actually, scholars haven't reach an agreement about the most ideal approach of measuring the fairness. In this paper, we analyze the Gini coefficient of distances as the measure of fairness $\psi_f(Y)$. Perfect fairness is achieved when distances to the closet facility are the same for all demand points. The Gini coefficient $\psi_f(Y)$ is the ratio of the area between the Lorenz curve and the straight "equity" line to the entire area below the equity line with $0 \le \psi_f(Y) \le 1$.

$$\psi_f(Y)=\frac{\sum_{i=1}^{n}\sum_{i'=1}^{n}\left(\sum_{j=1}^{m}\left|d_{ij}y_{ij}-d_{i'j}y_{i'j}\right|\right)}{2n\sum_{i=1}^{n}\sum_{j=1}^{m} d_{ij}y_{ij}} \tag{7}$$

4.2 The Preferences of Decision Makers

To describe the preferences of decision makers, this paper use the concepts named "the loss in efficiency (fairness)". It represents the change rate of $\psi_e(Y_\alpha)$ or $\psi_f(Y_\alpha)$ of a certain solution with the variable weight parameter α compared with the efficiency optimal solution Y_e^* and fairness optimal solution Y_f^*. We can calculate the loss in efficiency $P_e^\alpha=\frac{\psi_e(Y_\alpha)}{\psi_e(Y_{e^*})}-1$ and the loss in fairness $P_f^\alpha=\frac{\psi_f(Y_\alpha)}{\psi_f(Y_{f^*})}-1$.

Then the curves can be constructed which present a distinct way of visualizing the loss in efficiency and the loss in fairness. The decision maker varies his choice of objective by varying the variable weight parameter α and choose α via a tolerance on efficiency or fairness. The decision maker might decide that he is willing to be as efficient or fair as possible while allowing no more than a certain degradation in fairness or efficiency.

Thus, the location method under efficiency and fairness preferences can be described as following steps:

(1) Defining the form of balance function and the primary range of variable weight parameter α.
(2) Calculating the variable weight of individuals for all solutions.
(3) Obtaining the feasible solution set with various value of α.
(4) Calculating the performance of efficiency and fairness through the measure functions.
(5) Portraying the Pareto front of those two goals.
(6) Choosing the value of α according to the tolerance of efficiency or fairness loss and resulting in the optimal location solution.

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5. Illustrative Example

This paper applies a real world data set from the distribution system at Shanghai provided by Alibaba data center to demonstrate the feasibility of this method. Fig. 2 presented 50 demand points with their demand amounts and 20 potential facility stations in Shanghai. The number of facilities to be located is $p = 5$.

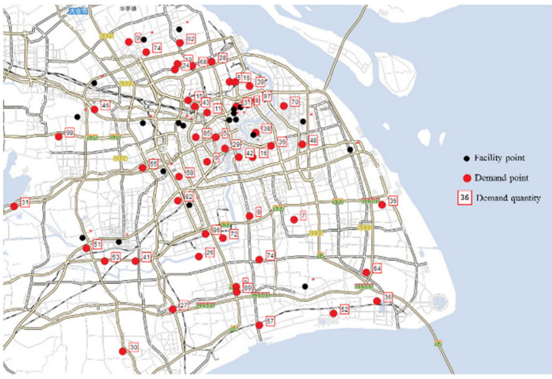

Figure 2 The demand points and potential facility stations

The form of balance function is the mixed type where the range of α is [-2,2], and the interval is 0.2.

$$S_i(Y)=e^{\alpha(\sum_{j=1}^{m} d_{ij}y_{ij} - \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{m} d_{ij}y_{ij})} \tag{10}$$

The p-median problem has been proved as NP problem. In order to solve the Large-scale problem, the tabu search algorithm is applied. Table 1 displays 19 solutions with various value of α. Some α results in the same location choices.

Table 1 Results of location for different value of α

Value of α	Location result	Value of α	Location result
[-2.00,-1.92]	{2,3,4,8,11}	[0.04,-0.04]	{6,11,14,16,18}
[-1.90,-1.04]	{2,4,6,8,11}	[0.06,0.12]	{6,11,16,17,18}
[-1.02,-1.00]	{2,4,6,11,18}	[0.14,0.26]	{2,3,11,18,20}
[-0.98,-0.96]	{2,4,6,9,18}	[0.28,0.30]	2,8,11,16,18}
[-0.94,-0.50]	{2,9,11,12,18}	[0.32,0.42]	{2,7,8,11,16}
[-0.48,-0.40]	{2,7,9,12,18}	[0.44,0.46]	{2,7,11,16,19}
[-0.38,-0.24]	{7,9,12,14,18}	[0.48,1.26]	{2,5,7,11,16}
[-0.22,-0.18]	{4,6,9,10,14}	[1.28,1.46]	{1,2,3,7,11}
[-0.16,-0.14]	{4,6,10,14,16}	[1.48,2.00]	{1,2,3,11,18}
[-0.12,-0.06]	{9,13,14,16,18}		

Figure 3 presents the efficiency and fairness performances of different solutions. For $\alpha \le 0$, the value of efficiency experience three stages. The value decreases sharply when $\alpha \in [-1.02,-0.96]$, and then increases slowly reaching the peak when $\alpha \in [-0.38,-0.24]$. On the third stage, the efficiency value decreases again until the minimum appears ($\alpha \in [-0.04,0]$). The change of fairness performance is much complicated, and roughly shows the tendency to decrease first and then increase. The optimal fairness performance is obtained when $\alpha \in [-0.94,-0.5]$. For $\alpha > 0$, the value of efficiency function continuously reduces which reflects the promotion of efficiency performance. As to the fairness, the value decreases when $\alpha \in (0,1.26]$ and reach the optimal performance for $\alpha \in [0.48,1.26]$. After that, the performance become worse. From the tendencies of those performances, we can figure out that the change of the individual weight has great influence on the location decision in the discrete location problem.

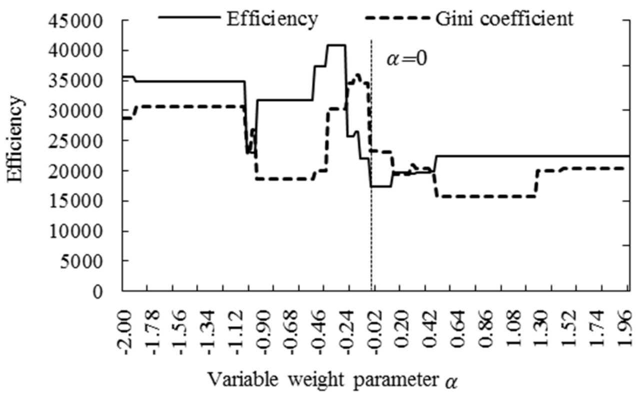

Figure 3 The efficiency and fairness performances with various value of α

Since the efficiency and fairness are two important objectives with the antinomy relationship, we depict the Pareto solution set to visualize the alternative solutions. Assuming two alternative solutions Y_A and Y_B in the Pareto solution set, if $\psi_e(Y_A) < \psi_e(Y_B)$, then there must be $\psi_f(Y_A) > \psi_f(Y_B)$ and vice versa. Figure 4 displays the Pareto front of 6 alternative solutions. The solution with $\alpha \in [-0.04, 0.04]$ is of the optimal efficiency and the worst fairness. With the increase of α, the performance of efficiency reduces while the fairness increases. When the value of α satisfies $\alpha \in [0.48, 1.26]$, the fairness reaches the optimum and the efficiency drop to the minimum. The location solutions whose $\alpha < 0.14$ $a > 1.26$ are both out of the Pareto solution set. This result reflects that excessive motivation or punishment could damage the efficiency and fairness. The decision maker could vary the value of α within proper range to achieve different performances of two goals.

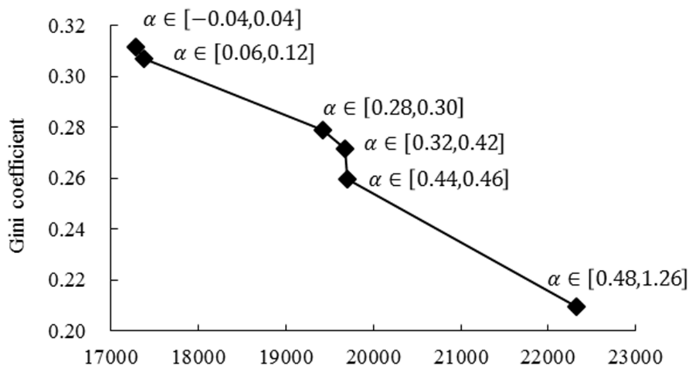

Figure 4 The Pareto solution set

Figure 5 shows the efficiency optimal solution ($\alpha \in [-0.04, 0.04]$), the fairness optimal solution ($\alpha \in [0.48, 1.26]$) and a trade-off solution ($\alpha \in [0.32, 0.42]$). In the efficiency optimal solution, the distances between facilities and demand points with larger demand amounts are much shorter. And in the optimal solution, although the average distance between facilities and demand points increases, the gaps between demand points decrease. Also some of the points with poor demands have improved their situations.

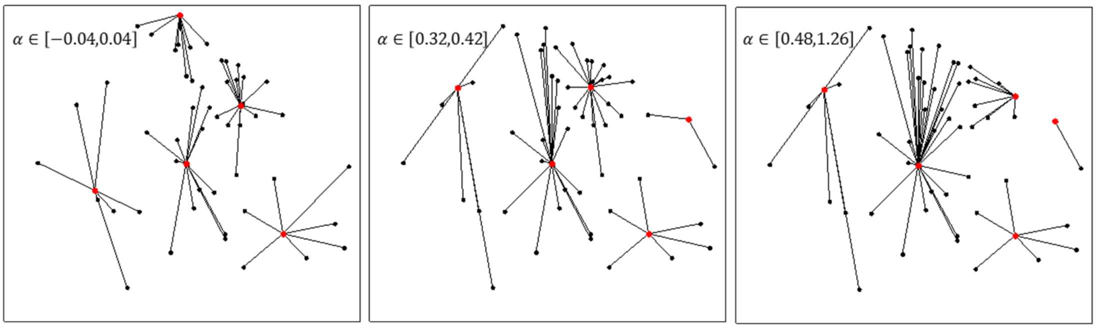

Figure 5 The location results under various preferences

The efficiency and fairness loss of the Pareto solutions are shown in Figure 6. The maximal value of the efficiency loss is 48.8%. At this time, the loss of fairness equals 0. On the contrary, the maximal value of the fairness loss is 29.1% with none loss of fairness. If the decision maker is willing to tolerate at most a 20% drop in the efficiency, the optimal choice is the 18th solution and the facilities located at stations {1,2,3,7,11}. If the decision maker is willing to tolerate at most a 20% drop in the fairness, he can choose the 16th solution, namely the stations {2,7,11,16,19}.

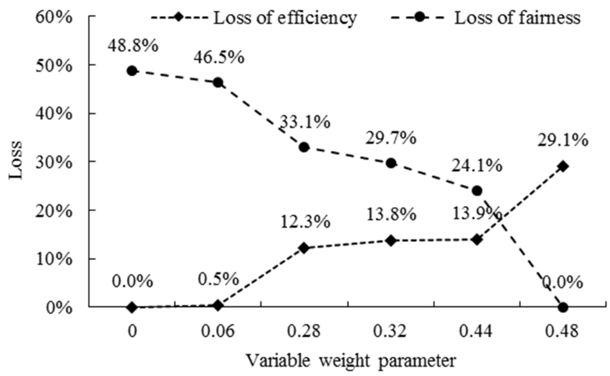

Figure 6 The efficiency and fairness loss of the Pareto solutions

6. Conclusion

To extend the method of logistics facility location considering both efficiency and fairness goals, this paper constructs a new p-median model combined the variable weight. We discuss the way of changing individual weight based on its utility and analyze its influence on the efficiency and fairness. The efficiency metric is the total service distance while the fairness metric is Gini coefficient. When the variable weight parameter is varied within the appropriate range, this method could realize different performance of efficiency and fairness. Also, we propose the process for decision makers choosing the optimal solution based on their preference which can be estimated by the tolerance of efficiency or fairness loss. A real world experiment demonstrate the validity and practicability of this method.

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