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Application of Parzen window in filter back projection algorithm

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Abstract: Image reconstruction is the core of CT technology, and the key factor to determine the quality of image reconstruction is the reconstruction algorithm. The algorithm has a direct impact on the accuracy of the image reconstruction results. In the CT image reconstruction algorithm, the filter back projection algorithm has a low time complexity and a high spatial resolution. At the same time, this algorithm is easily realized by parallel computing, which makes the speed of the reconstruction further improved, so this algorithm is a mainstream rebuilt algorithm. According to the principle of filtered back projection algorithm, the selection of filter function has a nonnegligible influence on image reconstruction results and reconstruction speed. First, based on the analysis of classic R-L and S-L filter functions, this paper introduces Parzen window filter function. Then this paper analyzes the quality of reconstructed image of Parzen window filter function in the absence of noise and noise conditions. Through the simulation experiment, it is verified that the Parzen window filter function has high spatial resolution while maintaining good image reconstruction quality.

1. INTRODUCTION

The core technology of image reconstruction in computed tomography (CT) is algorithm. Currently, there are two main categories: algebraic method and analytic method. FBP Filter Back Projection algorithm is one of the most commonly used analytic image reconstruction algorithms. FBP algorithm has low time complexity, and its reconstructed image has high spatial resolution. Therefore, the FBP algorithm is widely applied in CT imaging [1] . In the back projection algorithm, all the projections in all directions are accumulated to form reconstructed images. After accumulating, the point with zero pixel value in the infinite space becomes non-zero. Therefore, there will be obvious "Star" artifacts in reconstructed images. In order to eliminate this artifact, we need to introduce filter function in the reconstruction process. R-L (Ram-Lak) filter function and S-L (Shepp-Logan) filter function are two typical filtering functions in filtered back projection algorithm. R-L filter function is a simple and easy filtering function. Its reconstructed image quality is high, but it is accompanied by a serious Gibbs phenomenon [2,3] . In the filter back projection algorithm, the selection of filter function has always been the focus of research. The Parzen window function is introduced in this paper. It is proved by experiments that the quality of the Parzen window reconstruction image is higher than that of the R-L filter function and the S-L filter function.

2. REALIZATION OF FILTERING BACK PROJECTION ALGORITHM

According to the projection theorem, the projection data can be filtered before the back projection reconstruction, and then the filtered data can be back projected, so that the "Star" artifact can be eliminated. The concrete steps are as follows:

1)Conduct one-dimensional Fourier transform on projection data $P_\theta(t)$ at a certain angle, and the result is recorded as $S_\theta(\omega)$.

$$S_\theta(\omega)=\int_{-\infty}^{\infty} p_\theta(t)\ e^{-j2\pi\omega t}dt \tag{1}$$

2) Multiply $S_\theta(\omega)$ by one-dimension weight factor $|\rho|$.

3)Conduct one-dimensional inverse Fourier transform on the second step result, and the result is recorded as $Q_\theta(t)$.

$$Q_\theta(t)=\int_{-\infty}^{\infty} S_\theta(\omega)H(\omega)\ e^{j2\pi\omega t}d\omega \tag{2}$$

4)The corrected projection function $Q_\theta(t)$ in 0° to 180° is calculated by direct back projection, and the fault image $f(x, y)$ is obtained.

$$f(x,y)=\int_0^\pi Q_\theta(t)d\theta=\int_0^\pi Q_\theta(x\cos\theta+y\sin\theta)d\theta \tag{3}$$

3. DESIGN PRINCIPLE OF FILTER

Filtering back projection algorithm solves the problem of image reconstruction from projection in theory, but the selection of filtering plays a key role in the implementation of the algorithm, and the quality of the reconstructed image will be directly affected by the quality of the filter design. In the implementation of the filter back projection reconstruction algorithm, the system function of the filter is theoretically required to be $H_R(\rho)=|\rho|$, but because the filter function is a function of infinite bandwidth, it is difficult to realize the ideal filter in practice according to the Paley-Wiener criterion. However, the resolution of the actual CT device is limited, and its collected projection is a density distribution of the spectrum energy distributed in the low frequency region[4] . Therefore, projection data beyond a cut-off frequency can be ignored and the filter function can be selected as:

$$H(\rho)=|\rho|W(\rho) \tag{4}$$

The essence of realizing the function of filter is the selection of window function $W(\rho)$. In order to get better resolution of reconstructed image, we should follow the following principles:

1) The width of the main lobe should be narrow to get a steepest transition zone.

2) The maximum side lobe relative main lobe should be very small, so as to improve the usual stationarity and increase the attenuation of stopband.

But generally, the high and narrow central prominent window function has prominent sidelobes. Therefore, in practice, we cannot blindly request high resolution, otherwise it will cause serious Gibbs phenomenon. In addition, the selection of window function is also determined by the actual components and the reconstruction objectives.

In recent years, many scholars have put forward many filter functions, and have analyzed and improved some filtering functions. The most typical filtering functions are R-L filtering function, S-L filtering function and so on.

A. Common filter

1) R-L filter

The expression of the R-L filter function is as follows:

$$H_{R-L}(\rho)=|\rho|W(\rho)=|\rho|\mathrm{rect}(\rho/2B) \tag{5}$$

In the expression:

$$\mathrm{rect}(\rho/2B)=\begin{cases}1, & |\rho|<B=\dfrac{1}{2d}\\ 0, & \text{otherwise}\end{cases} \tag{6}$$

The convolution function $h_{R-L}(R)$ corresponding to the filter function $H_{R-L}(\rho)$ is:

$$\begin{aligned} & h_{R-L}(R) = 2B^2 \sin c(2RB) - B^2 \sin c^2(RB) \\ & B = 1/(2d), \sin c(x) = \sin(x)/x \end{aligned} \tag{7}$$

Its discrete expression is:

$$h_{R-L}(nd) = \begin{cases} 1/(4d^2), n = 0 \\ 0, n = \text{Nonzero even number} \\ -1/(n^2\pi^2 d^2), \ n=\text{Odd} \end{cases} \tag{8}$$

The form of R-L filter is simple and practical. The reconstructed image is clear, but it has obvious Gibbs phenomenon. It shows a significant concussion response. In addition, if the projection data contains noise, the quality of the reconstruction will be worse. The reason is that when $\rho=1/(2d)$, $H(\rho) = |\rho|$, the ideal rectangle window is the root cause of the Gibbs phenomenon.

2) S-L filter

In order to mitigate the oscillation of R-L filter function, Shepp and Logan proposed S-L filter function.

$$H_{S-L}(t) = \frac{1}{2}\left(\frac{4B}{\pi}\right)^2 \frac{1-4Bt\sin(\frac{\pi}{2}4Bt)}{1-(4Bt)^2} \tag{9}$$

Its discrete form is:

$$h_{S-L}(nd) = \frac{-2}{\pi^2 d^2(4n^2-1)}, n = 0, \pm1, \pm2, \cdots \tag{10}$$

Using S-L filter function to reconstruct image, its oscillation response decreases, and the reconstruction quality of projection data containing noise is also better than that of R-L filter function [5] . However, because $h_{S-L}(\rho)$ also deviates from $|\rho|$ at low frequency, the quality of image reconstruction at low frequency is not as good as that of R-L filter function.

Besides, there are Hanning window filters, Kaiser window filters, triangular window filters and so on[6] .

B. Image evaluation index

In order to evaluate the quality of each image, the following two image evaluation indices are used to represent:

1) Normalized mean square distance d

$$d = \left[\frac{\sum_{u=1}^{N}\sum_{v=1}^{N}(t_{u,v} - r_{u,v})^2}{\sum_{u=1}^{N}\sum_{v=1}^{N}(t_{u,v} - \bar{t})^2} \right]^{\frac{1}{2}} \tag{11}$$

Among them, $t_{u,v}$ and $r_{u,v}$ denote the pixel density of u and v columns in object models and reconstructed images, respectively. $\bar{t}$ is the average value of the density of the object model, and the pixel of the image is $N * N$. $d=0$ indicates that the reconstructed image can truly reflect the original object model image. The greater the d value, the greater the error of the two. Normalized mean square distance sensitively reflects the error of a few points. Larger deviation of individual points will make d larger.

2) Normalized mean absolute distance r

$$\mathrm{r} = \frac{\sum_{u=1}^{N}\sum_{v=1}^{N}\left|t_{u,v} - r_{u,v}\right|}{\sum_{u=1}^{N}\sum_{v=1}^{N}\left|t_{u,v}\right|} \tag{12}$$

$r = 0$ shows that there is no error. The larger the r, the greater the error. The normalized mean absolute distance r reflects many small errors in many points. According to the above indicators, we compare the mentioned part of window functions, as shown in Table 1.

Table 1 Error comparison of window function filters

Pixels	Error	Rectangular window	S-L window	Hanning window	Kaiser window（β =7）
128×128	d r	0.5933 0.8024	0.6080 0.8470	0.62242 0.33278	0.61974 0.33909
256×256	d r	0.5743 0.8466	0.5426 0.7794	0.45767 0.19082	0.45635 0.19401
360×360	d r	0.5832 0.8903	0.5572 0.8401	0.38594 0.15303	0.38467 0.15263
512×512	d r	0.6760 1.0786	0.6559 1.0446	0.32643 0.13962	0.32472 0.13533

From table 1, we can see that the reconstruction error of Hanning window is less than the reconstruction error of rectangular window and S-L window.

4． PARZEN FILTER FUNCTION

A. Analysis of the property of Parzen window function

In this paper, we choose the Parzen window function as the research object, and its expression is as follows.

$$W_P(\rho)=\begin{cases}1-\dfrac{6\rho^2}{B^2}(1-\dfrac{|\rho|}{B}) & |\rho|\le\dfrac{B}{2}\\ 2(1-\dfrac{|\rho|}{B})^3 & \dfrac{B}{2}<|\rho|\le B\\ 0 & \text{otherwise}\end{cases} \tag{13}$$

Impulse response function:

$$\begin{aligned}&\mathrm{h}_P(\mathrm{t})=\frac{1}{32\pi^5B^3\mathrm{t}^5}(48\pi Bt\cos 2\pi Bt-96\sin 2\pi Bt-96\pi Bt\cos\pi Bt+384\sin\pi Bt\\&-16\pi^3B^3t^3-144\pi Bt)\\&\mathrm{h}_P(0)=0.175\mathrm{B}^2\end{aligned} \tag{14}$$

Parzen window, also known as kernel density estimation, is one of the nonparametric methods used to estimate the unknown probability density function in probability theory. Parzen windows are widely applied in pattern recognition and image processing. Figure 1 is the time domain waveform and frequency domain waveform of Parzen window function.

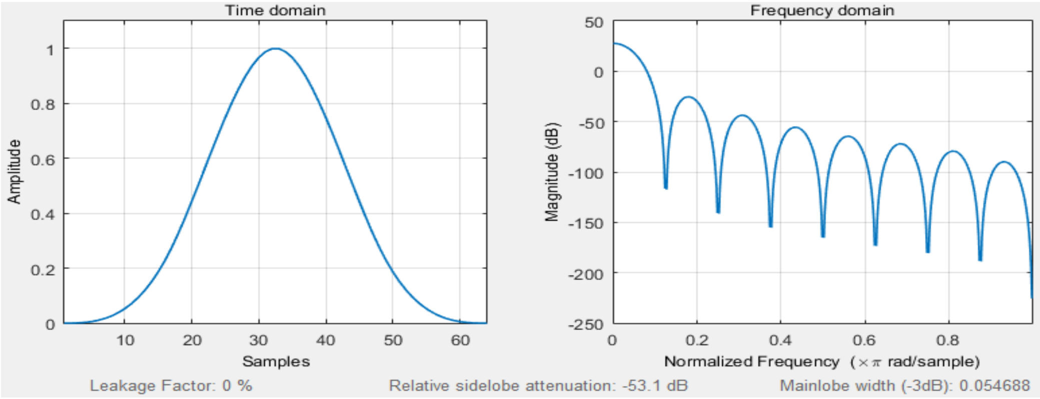


Figure 1 The time domain and frequency domain waveforms of the Parzen window function

The main lobe width and the first sidelobe attenuation values of each function are listed in Table 2.

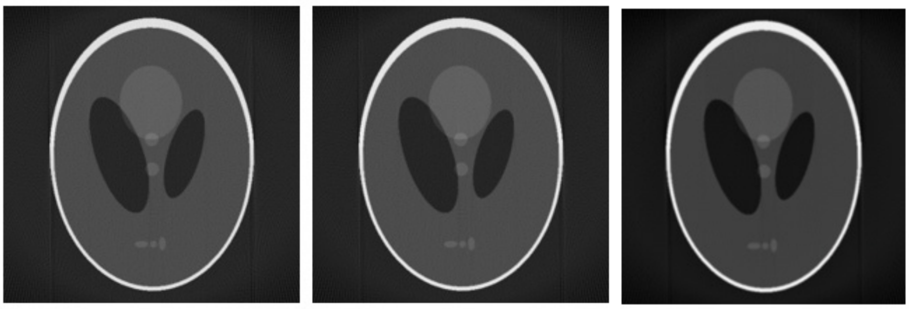
Table 2 Main lobe width and attenuation value of the first sidelobe relative to the main lobe

Window function	Main lobe width (-3dB)	Attenuation value of the first sidelobe relative to the main lobe（dB）
Rectangular window	0.027344	-13.3
Trigonometric window	0.039063	-26.6
Hanning window	0.042969	-31.5
Parzen window	0.054688	-53.1
Kaiser window（β =7）	0.046875	-50.9

From Figure 1 and table 2, we can see that the main lobe width of Parzen window is larger, but the sidelobe amplitude is smaller than the other four window functions. According to the analysis of the influence factors of filter function for image reconstruction error, the smaller the sidelobe amplitude, the weaker the Gibbs effect is. Therefore, it can be concluded that the Parzen window filter function can ensure better spatial resolution while improving the density resolution relatively, and thus the reconstructed image with this function has good characteristics.

B. Analysis of experimental results of Parzen window function

In order to illustrate the effectiveness of Parzen window filter function for image reconstruction, two-dimensional parallel beam filter back projection algorithm is used in the experiment. The model used is typical Shepp-Logan. The projection data is 512* 512, and the reconstructed image is 512*512. The result is shown in Figure 2.



(a) R-L (b) S-L (c) Parzen

Figure 2 Reconstruction results of three filter functions

Table 3 The d and r values of the reconstruction of three filter functions

Filter function	Normalized mean square distance d	Normalized mean absolute distance r
R-L filter function	0.6760	1.0786
S-L filter function	0.6559	1.0446
Parzen window filter function	0.4604	0.6470

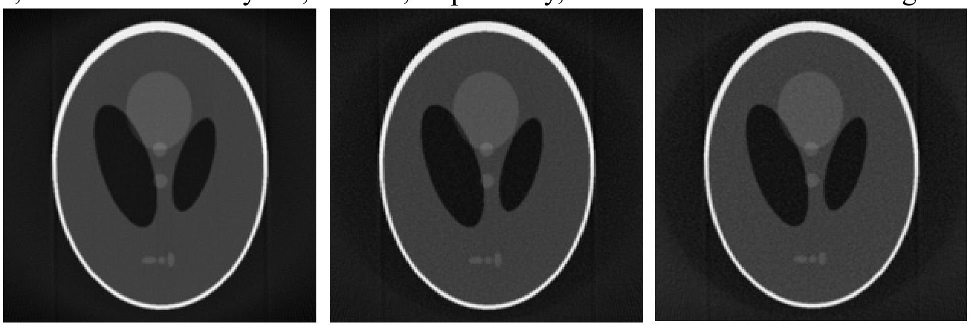
As can be seen from table 3, the normalized mean square distance and the normalized absolute distance of the reconstructed image of the Parzen window filter function are smaller than the usual R-L and S-L filtering functions, thus the better image reconstruction results can be obtained.

C. Analysis of the reconstruction results of Parzen window under noise environment

In analyzing the effect of filter function on image reconstruction, we often need to simulate noise and analyze its influence on the quality of reconstructed image, so the Poisson noise is added to the experiment.

$$P^{'}(t,\theta) = P(t,\theta) + g(\lambda) \tag{15}$$

By changing the Poisson noise parameter λ, the intensity of simulated noise is changed. In the experiment, λ is simulated by 0.1, 1 and 2, respectively, and the result is shown in Figure 3.



λ=0.1 λ=1 λ=2

Figure 3 Reconstruction results of Parzen window filter function under Poisson noise

Table 4 is normalized filter mean square distance d and normalized absolute distance r value of Parzen filter function under Poisson noise.

Table 4 d and r values of images reconstructed by Parzen filter function under Poisson noise.

λ	Normalized mean square distance d	Normalized mean absolute distance r
0.1	0.4552	0.6341
1	0.4596	0.6270
2	0.5056	0.7118

From Figure 3 and table 4, we can see that Parzen window filter function can still get high-definition images under Poisson noise.

5 . CONCLUSION

In the filtered back projection algorithm, the function of filtering is completed through the use of window function. By comparing the Parzen window filter function with the common R-L and S-L functions, this paper proves that the normalized mean square distance and the normalized absolute distance of the Parzen window are smaller than the commonly used R-L and S-L filtering functions, so

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it can obtain better image reconstruction results. In addition, the Parzen window has good anti-noise ability. The selection of window function in filter back projection algorithm has always been a hot topic, and people are constantly exploring to find more suitable filtering functions to obtain better quality reconstructed images.

REFERENCE

[1] Luo yanhong. Research on exponential filter in CT image reconstruction filtered back projection algorithm[J]. Computer science,2014,41(S1):220-223.(in Chinese)

[2] Qiao zhiwei. Research on high speed and high precision analytical image reconstruction algorithm [D]. Beijing Jiaotong University,2011. (in Chinese)

[3] Wang jue, Huang suhong, Cai yufang. Ring artifact correction for industrial CT image [J]. Optical precision engineering,2010,18(05):1226-1233. (in Chinese)

[4] Ma. chenxin Research on the key technology of CT image reconstruction [D]. The PLA Information Engineering University,2011. (in Chinese)

[5] Wei huitian. Filter design based on Hanning window function [J]. Sensor world,2011,17(12):27-29. (in Chinese)

[6] Zhang bin, Pan jinxiao. A new hybrid filter for CT image reconstruction [J]. Microcomputer information,2009,25(09):298-299+308. (in Chinese)