Cross diffusion effects on combined bioconvection of nanofluid in a flat channel along with microorganisms

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Abstract. This paper investigates the cross diffusion effects on fully developed mixed bioconvection of gyrotactic microorganisms through horizontal channel filled with nanoliquid under the influence of Soret and Dufour effects. The governing nonlinear partial differential equations are converted into set of nonlinear ordinary differential equations using appropriate similarity variables. The equations together with the boundary conditions are solved by using homotopy analysis method (HAM). The effects of thermophoresis, Brownian motion, Soret and Dufour parameters on mass and heat transfer are analysed graphically. It was found that Soret and Dufour effects was highly influenced in the temperature and the movement of microorganisms.

Keywords: Bioconvection, Soret effect, Dufour effect, Nanofluid, Channel, Microorganisms.

1. Introduction

A great amount of studies on convection of nanoliquid has been reported in recent years due to their energy transport control potential in different energy systems [1, 2, 3]. Bioconvection pattern happen due to up-swimming of microorganisms which are a slightly denser than water in suspensions. The movable microorganisms are commonly larger than water and they swim skyward when stimuli such as light and chemical attraction exist, which move under the variable density concept and hydro-dynamic inconstancy. Combinations of nanoliquid and microorganisms in convective flows is influenced by many researchers due to its significant in microfluidic devices. The nanofluid bioconvection of gyrotactic microorganisms was examined by several authors [4, 5, 6]. Geetha et al. [6] explored the bioconvection of nanoliquid in the presence of Joule heating in a channel. Soret and Dufour effects along with various effects on convection was discussed by [7, 8, 9, 10, 11].

The present investigation extends the work of Xu and Pop[12] proposed by Kuznetsov and Nield by considering the reactions of Dufour and Soret over doubly diffusive combined convection with gyrotactic microorganisms.

2. Mathematical Modelling

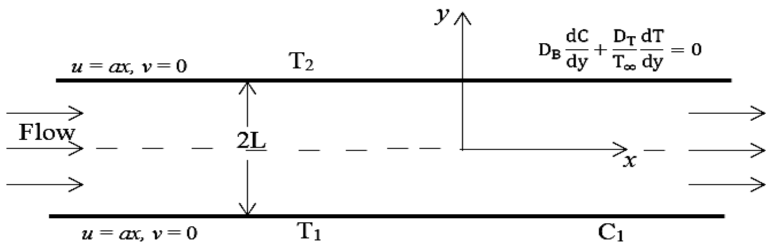

Figure 1. Physical configuration and co-ordinate systems.

Consider the combined bio-convection of nanofluid in a horizontal channel of length $2L$ filled along gyrotactic microorganisms as shown in Figure 1. The axes of the horizontal and vertical directions to the channel walls are along the axes of x and y respectively. Let the velocity of upper and lower walls are $u_1 = ax$. Let T_1 and T_2 are the constant temperatures lie on the top and the bottom of the walls. Suppose N_1 and N_2 are constant densities of the movable microorganisms. Two different solutal concentrations are taken on the walls. Let us consider on the upper wall, the nanoparticle volume fraction satisfies the passively controlled model, but on the lower wall, it is constant. The equations of continuity, momentum, heat energy, nanoliquid concentration, solutal concentration and the microorganisms density are:

$$u_{1x} + v_{1y} = 0, \tag{1}$$

$$u_1\zeta_x + v_1\zeta_y = \nu\left(\zeta_{xx} + \zeta_{yy}\right), \tag{2}$$

$$u_1T_x + v_1T_y = \alpha\left(T_{xx} + T_{yy}\right) + \frac{Dk}{c_s c_p}\left(\Gamma_{xx} + \Gamma_{yy}\right) + \tau\left\{D_B\left(C_xT_x + C_yT_y\right) + \frac{D_T}{T_0}\left[\left(T_x\right)^2 + \left(T_y\right)^2\right]\right\}, \tag{3}$$

$$u_1C_x + v_1C_y = D_B\left(C_{xx} + C_{yy}\right) + \frac{D_T}{T_0}\left(T_{xx} + T_{yy}\right), \tag{4}$$

$$u_1\Gamma_x + v_1\Gamma_y = D\left(\Gamma_{xx} + \Gamma_{yy}\right) + \frac{Dk}{T_m}\left(T_{xx} + T_{yy}\right), \tag{5}$$

$$u_1N_x + v_1N_y + (N\hat{v})_y = D_nN_{yy}. \tag{6}$$

where p and T be the pressure and heat energy, C is the nanoparticle volume fraction, Γ - the solutal concentration, $\hat{v} = (u_1, v_1)$ being the velocity components of nanoliquid in the Cartesian coordinates, ρ, μ and α are respectively density, viscosity and thermal diffusivity of the nanofluid, c_s be the concentration susceptibility and c_p is the specific heat. The ratio of the efficient heat energy measure of the nanoparticle to the basic liquid is represented by $\tau = (\rho c_p)_p/(\rho c_p)_f$. D_B and D_T are called the two diffusion coefficients.

The conditions at wall and centre of the channel are as follows:

$$u_1 = ax, \quad v_1 = 0, \quad T = T_2, \quad D_B\frac{dC}{dy} + \frac{D_T}{T_\infty}\frac{dT}{dy} = 0, \quad \Gamma = \Gamma_1, \quad N = N_2 \quad \text{as} \quad y = L;$$

$$u_{1y} = 0,\ v_1 = 0 \ \text{ as } \ y = 0; \qquad u_1 = ax,\ v_1 = 0,\ T = T_1,\ C = C_1,\ \Gamma = \Gamma_2,\ N = N_1 \ \text{as}\ y = -L.$$

By using the successive similarity transformations

$$\psi(x,y) = axLf_1(\eta), \quad \eta = \frac{y}{L}, \quad \theta(\eta) = \frac{T - T_0}{T_2 - T_0}, \quad \phi(\eta) = \frac{C - C_0}{C_0}, \quad \gamma(\eta) = \frac{\Gamma - \Gamma_0}{\Gamma_2 - \Gamma_0}, \quad S(\eta) = \frac{N}{N_2}.$$

in Eqs. $(1)-(6)$, continuity equation (1) is obviously true and the simplified form of other equations are given by

$$f_1'''' + Re(f_1 f_1''' - f_1' f_1'') = 0, \tag{7}$$

$$\theta'' + (RePr)f_1\theta' + Nb\theta'\phi' + Nt(\theta')^2 + Df\gamma'' = 0, \tag{8}$$

$$\phi'' + \frac{Nt}{Nb}\theta'' + (ReLe)f_1\phi' = 0, \tag{9}$$

$$\gamma'' + ReSc_\gamma f_1\gamma' + SrSc_\gamma\theta'' = 0, \tag{10}$$

$$s'' - Pe_b(\phi's' + s\phi'') + (ReSc)f_1s' = 0, \tag{11}$$

along with the following conditions at boundary

$$\theta = \delta_\theta, \quad \phi = \delta_\phi, \quad \gamma = \delta_\gamma, \quad s = \delta_s \quad \text{as} \quad \eta = -1; \quad f_1 = 0, \quad f_1'' = 0 \quad \text{as} \quad \eta = 0;$$

$$f_1 = 0, \quad f_1' = 1, \quad \theta = 1, \quad Nb\phi' + Nt\theta' = 0, \quad \gamma = 1, \quad s = 1 \quad \text{as} \quad \eta = +1.$$

The dimensionless numbers are given by

$$Re = \frac{aL^2}{\nu}, \quad Pr = \frac{\nu}{\alpha}, \quad Nb = \frac{\tau D_B C_0}{\alpha}, \quad Nt = \tau\left(\frac{D_T}{T_0}\right)\frac{T_2 - T_0}{\alpha}, \quad Le = \frac{\nu}{D_B}, \quad Pe_b = \frac{bW_c}{D_n},$$

$$Sc = \frac{\nu}{D_n}, \quad Df = \frac{Dk(\Gamma_2 - \Gamma_1)}{\alpha c_s c_p (T_2 - T_0)}, \quad Sc_\gamma = \frac{\nu}{D}, \quad Sr = \frac{Dk(T_2 - T_0)}{\nu T_m (\Gamma_2 - \Gamma_0)}, \quad \delta_\theta = \frac{T_1 - T_0}{T_2 - T_0},$$

$$\delta_\phi = \frac{C_1 - C_0}{C_0}, \quad \delta_\gamma = \frac{\Gamma_1 - \Gamma_0}{\Gamma_2 - \Gamma_0}, \quad \delta_s = \frac{N_1}{N_2},$$

where Re-Reynolds number, Pr-Prandtl number, Nb-Brownian motion parameter, Nt-thermophoresis parameter, Le-Lewis number, Df-Dufour number, Sr-Soret number, Sc-Schmidt number, Sc_γ-Schmidt number for mass transfer and Pe_b-Peclet number.

In convection flow problems, very important to aware the amount of heat transfer between the solid wall and the liquid. To calculate the heat and mass transfer rates over the channel, it is essential to give description of the local Nusselt number and the local Sherwood number along the boundary. The definition of the local Nusselt number and local Sherwood number are listed by

$$Nu = \frac{q_w x}{k(T_2 - T_0)}, \quad \text{where the heat flux} \quad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0},$$

$$Sh = \frac{j_w x}{D(\Gamma_2 - \Gamma_0)}, \quad \text{where the mass flux} \quad j_w = -D\left(\frac{\partial \Gamma}{\partial y}\right)_{y=0}.$$

From the above equation, the dimensionless form of the Nusselt and the Sherwood numbers of this model are given by $Nu = -\theta'(\eta)$ and $Sh = -\gamma'(\eta)$ at $\eta = 0$.

3. Results and analysis

The set of non-linear differential equations $(7)-(11)$ are evaluated by using HAM technique subject to the values of parameters $h_f = h_\phi = -1.5, h_\theta = h_\gamma = -0.2, h_s = -0.5$. The fixed values of other parameters are taken as $Pr = Sc = Le = Pe_b = Sc_\gamma = Sr = Df = Nb = Nt = 1$, $\delta_\theta = \delta_\phi = \delta_\gamma = 0.5, \delta_s = 1$. The Soret number ($Sr$) and Dufour number ($Df$) lie between 0 to 4, $-10 \le Re \le 10$ for Reynolds number, Brownian motion and Thermophoresis parameters lie between 0 to 2.

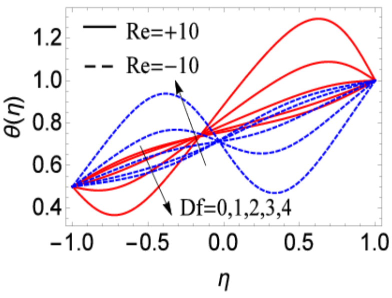

Figure 2. The temperature for different entries of Df with $Re = \pm 10$, $Sr = 1$.

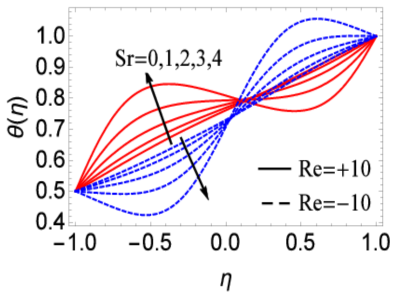

Figure 3. The temperature for various entries of Sr with $Re = \pm 10$, $Df = 1$.

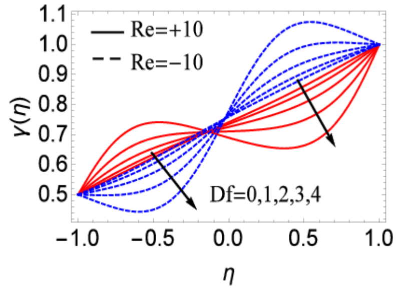

Figure 4. The concentration for various entries of Df including $Re = \pm 10$, $Sr = 1$.

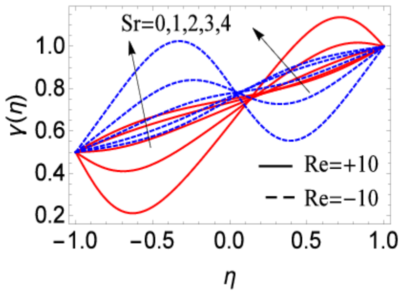

Figure 5. The concentration for various entries of Sr including $Re = \pm 10$, $Df = 1$.

Fig 2-3 analyse the temperature profile under Sr and Df respectively. The thermal energy of the liquid enhances with the rising values of Dufour and Soret numbers. These two effects provide opposite influence on the nanoliquid temperature. This is because of the Soret effect reflects reverse effect of Dufour number. Figures 4-5 discuss the impact of Sr and Df for the solute concentration. It is seen that the variation of Sr and Df can change concentration to a large extent. The solutal concentration get opposite behaviour in the upper and lower parts of the channel. The Soret number more pronounces on solutal concentration than the Dufour number. Figures 6-7 deal the effect of microorganisms density under Sr and Df. It is detected that $S(\eta)$ enhances with Df in the lower portion of the channel and it raises with Sr values in the upper portion of the channel. The reverse trend is discovered on growing the values of Df and Sr in the upper and lower portions of channel respectively.

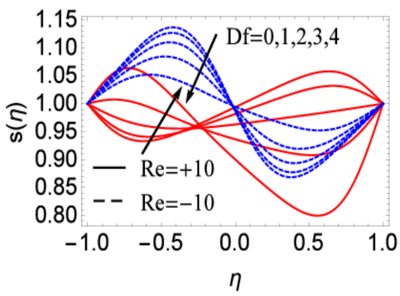

Figure 6. The microorganisms density for various Df values with $Re = \pm 10$, $Sr = 1$.

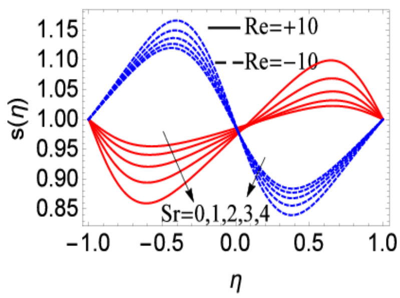

Figure 7. The microorganisms density for various Sr values with $Re = \pm 10$, $Df = 1$.

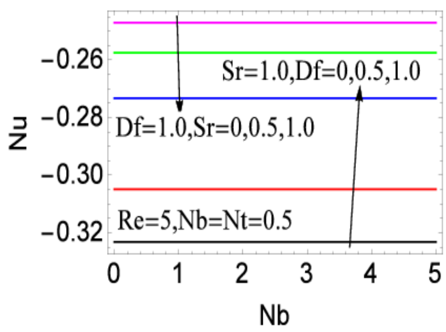

Figure 8. Nusselt number versus Nb.

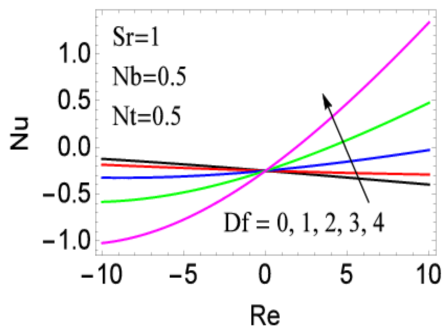

Figure 9. Nusselt number versus Re.

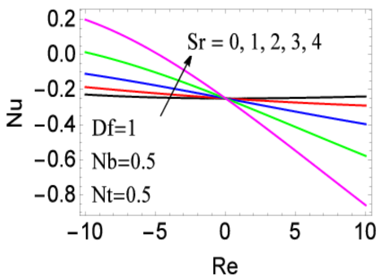

Figure 10. Nusselt number versus Re.

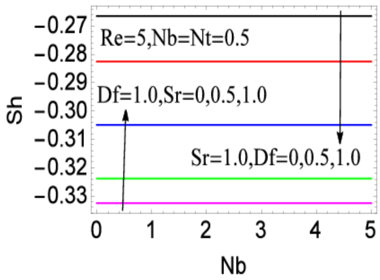

Figure 11. Sherwood number versus Nb.

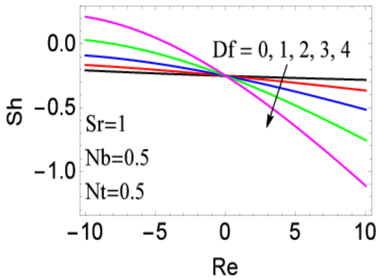

Figure 12. Sherwood number versus Re.

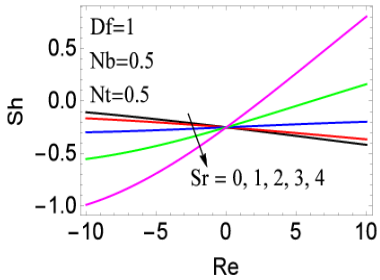

Figure 13. Sherwood number versus Re.

The effect of energy transport inside the channel is demonstrated by Nusselt number and it is pictured in the Figures 8-10 for different entries of Sr and Df along with Reynolds number. The energy transport enhances on raising the values of Soret number and it diminishes by growing the values of Dufour number. Figures 11-13 demonstrated the influence of solutal transport inside the channel for several combinations of Sr, Df and Re. The Dufour number reduces the mass transfer rate inside the channel whereas the mass transport raises when increasing the values of Sr. The local Sherwood number behaves in the opposite manner with respect to Nu in front of Soret and Dufour number, Brownian number and Reynolds number.

4. Conclusion

In this paper, mixed bioconvection of nanofluid with microorganisms in the horizontal channel in front of Brownian motion, diffusion-thermo and thermo-diffusion reactions are examined. The important concepts are noted in the following lines:

- The temperature and solute concentration enhance for raising the values of Dufour and Soret numbers.
- The local Nusselt number enhances whereas the local Sherwood number diminishes on raising the values of Dufour number. The rate of heat transport diminishes whereas the mass transport rate enhances when the Soret number raises.
- The density of microorganisms enhances with Soret and Dufour numbers.
- In overall, it is discovered that the physical quantities behaves opposite manner in the upper and lower portion of the channel.

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