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Prediction of fatigue limit for unidirectional carbon fibre/epoxy composites

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Abstract. A micromechanics model is used for the prediction of the fatigue limit of unidirectional carbon fibre /epoxy composite materials. The model is based on the hypothesis that failure of a fibre will result in fibre/matrix debonding of the broken fibre. The associated debond crack tip stress fields will raise the stress in the neighbour fibres as the debond crack tips move along the broken fibre and can thus cause failure of the neighbouring fibres. The fatigue limit is defined from the maximum applied cyclic stress that does not induce failure of any neighbour fibres. Effects of microscale mechanical properties are investigated. The model predicts that the fatigue limit, expressed in terms of stress, increases with fibre volume fraction until 50-60 %, whereafter the fatigue limit decreases with increasing fibre volume fraction. With other parameters held fixed, the fatigue limit increases with increasing interfacial frictional sliding shear stress and with decreasing interfacial fracture energy.

1. Introduction

Fatigue - material degradation under cyclic loading - is a phenomenon that greatly influences the design limits of composite materials for load-carrying structures. Examples are wind turbine rotor blades that are designed to be in operation for 20-25 year, during which they will be subjected to cycling varying loads as the blades rotate. During one rotation, each blade will go from a low aerodynamical load (in the bottom position in front or behind the tower) to a high aerodynamical load (at the top position) as well as will undergo a complete change in the direction of the gravitation force. With a rotation time of about 5 seconds per rotation, 25 years of constant use corresponds to more than 150 million rotations. For such high number of load cycles, the use of traditional S-N data may not be attractive, since it is very time consuming and thus expensive to conduct cyclic experiments and establish lifetimes up to 150 million cycles (or higher). Instead, it may be more appropriate to focus on which conditions the material might possess a fatigue limits, i.e., whether a maximum load (or strain) value exists, below which the material would sustain an infinite number of load cycles.

Frequently, the run-out of 1 million cycles has been used as an engineering fatigue limit. Over the past few years, more experiments have been conducted far beyond 1 million cycles. Mandell *et al.* [1] loaded specimen cyclically up to 10^{10} cycles. They found that fatigue failure did happen after 1 million cycles at strain values below the engineering fatigue limit corresponding to run-out at 1 million cycles. These finding do not support the hypothesis of the existence of a true fatigue limit. However, neither do they rule out the possibility that a true endurance fatigue limit can exist.

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One hypothesis could be to claim that a true fatigue limit can exist if the material does not develop any damage at all during cyclic loading. While such a hypothesis might be correct, is could also come out being very conservative. For instance, Gamstedt and Talreja [2] investigated the microstructural damage state and found that isolated fibre failures were always presented in specimens having been subjected to cyclic loading. This could lead to the discouraging conclusion that a fatigue limit may never exist. Another hypothesis, therefore, could be that a fatigue limit can exist if the situation of isolated fibre failures is stable in the sense that the failure of one isolated fibre does not lead to failure of neighbor fibres. It is therefore of interest to investigate the influence of one broken fibre on the nearest neighbour fibres. The effect of a broken fibre on neighbour fibres was investigated by use of Raman spectroscopy [3]. It was found that the stress in neighbour fibres was increased at the position of the debond crack tip of a broken fibre. This suggests that the debond crack tip stress field of the broken fibre plays an important role in the occurrence of fibre failure in neighbour fibres. An analytical micromechanical model that accounts for this has recently been developed [4]. The model enables the prediction of a fatigue limit as a a function of microscale parameters. In the present paper we will use this model for the prediction of a fatigue limit for unidirectional carbon fibre composites using available litterature data for AS4 carbon fibre in epoxy matrix.

2. Model summary

2.1. Interface degradation laws

In this section we will briefly describe the main assumptions behind the analytical model. The situation we analyse is a broken fibre surrounded by intact fibres. It is not of importance exactly at which load level the fibre has broken. It is assumed that debonding and frictional sliding occur along a part of the fibre/matrix interfaces in connection with the breakage of the fibre. The mechanics of the fibre/matrix interface is described in terms of an interfacial fracture energy (debond energy), G_c^i, and a frictional sliding shear stress, τ_s. When subjected to cyclic loading, the crack tip can advance in accordance with a Paris-Erdogan law. A threshold value is assumed to exist for the crack tip stress intensity range $(\Delta K)_{th}$; if the crack tip stress intensity range, ΔK_{tip}, is below this value, no crack tip advancement will occur under cyclic loading. Furthermore, it is assumed that during repeated forward and reverse slip, the interfacial frictional sliding shear stress decreases to a lower value denoted τ_s^c due to changed along the fibre/matrix interface e.g. by wear of asperities or plastic deformation. We take the assumption that the fibers do not degrade during cyclic loading. The fatigue model thus assumes that the fatigue mechanism is primarily driven by interface properties.

2.2. First loading

In the first load cycle, following fibre failure, the debond length will increase until the maximum applied strain value, $\bar{\varepsilon}_{max}$. At the maximum applied strain, the debond length, ℓ_d, is given by [4]

$$\frac{\ell_d}{r} = \frac{E_f}{2\tau_s}\left(\bar{\varepsilon}_{max} - \frac{\bar{\sigma}_i}{E_c}\right), \tag{1}$$

where r is the fibre radius, E_f is the Young's modulus of the fibres and E_c is the Young's modulus of the composite given by the rules of mixture

$$E_c = V_f E_f + (1 - V_f)E_m, \tag{2}$$

where E_m is the Young's modulus of the matrix and V_f is the volume fraction of fibres in the composite. The parameter $\bar{\sigma}_i$ denotes the a so-called initiation stress given by

$$\frac{\bar{\sigma}_i}{E_c} = \frac{(1-V_f)E_m}{E_c}\Delta\varepsilon^T + 2\sqrt{\frac{(1-V_f)E_m}{E_c}\left(\frac{G_c^i}{E_f r}\right)}, \tag{3}$$

where $\Delta\varepsilon^T$ is the so-called mismatch strain, i.e, the difference in the in-elastic strains of the fibre and matrix occurring during processing of the composite. As an example, in case $\Delta\varepsilon^T$ is due to differences in thermal expansion coefficients and the cool-down from processing, $\Delta\varepsilon^T$ is given by $\Delta\varepsilon^T = (\alpha_f - \alpha_m)(T - T_0)$, where α_f and α_m are the thermal expansion coefficients of fibre and matrix, respectively, T is the present temperature and T_0 is the processing temperature.

2.3. After many load cycles

We now consider the situation after so many load cycles that the interfacial sliding shear stress has decreased to τ_s^c along the part of the interface experiencing cyclic reverse and forward slip. It turns out [4] that cyclic slippage along the fibre/matrix interface occur along only a part of the debonded fibre; the region near the debond crack tip will experience sticking friction with τ_s. As a result, the crack tip stress intensity factor will remain at the same value as it was at the maximum applied stress for the very first load cycle. The stress intensity range ΔK_{tip} is thus zero, i.e. below the threshold value, $(\Delta K)_{th}$ [4]. Consequently, the debond crack tip remains stationary. The length of the debond crack under cyclic loading is predicted to be:

$$\frac{\ell_d^c}{r} = \frac{E_f}{2\tau_s}\left\{\frac{\bar{\varepsilon}_{max}}{2}\left[(1+R) + (1-R)\frac{\tau_s}{\tau_s^c}\right] - \frac{\bar{\sigma}_i}{E_c}\right\}, \tag{4}$$

where r is the fibre radius, E_f is the Young's modulus of fibre, $\bar{\varepsilon}_{max}$ is the maximum applied strain, R is the R-ratio, i.e. the ratio between the minimum and maximum applied strain, $R = \bar{\varepsilon}_{min}$ / $\bar{\varepsilon}_{max}$.

2.4. Stress from debond crack tip

Next, the stress induced to the nearest neighbour fibres from the debond crack tip stress field is found as [4]

$$\hat{\sigma}_f^K = 0.435\frac{E_f}{E_m}\sqrt{\frac{G_c^i \bar{E}^*}{d^*}}, \tag{5}$$

where d^* is the fibre spacing given by (assuming perfect hexagonal fibre packing)

$$\frac{d^*}{r} = \sqrt{\frac{2\pi}{\sqrt{3}V_f} - 2}, \tag{6}$$

and $\bar{E}^*$ is given as

$$\frac{1}{\bar{E}^*} = \frac{1}{2}\left(\frac{1}{\bar{E}_f} + \frac{1}{\bar{E}_m}\right). \tag{7}$$

2.5. Prediction of fatigue limit

Finally, the fibre strength variation is described in terms of a Weibull distribution, with σ_0 being the characteristic strength, L_0 the reference length and m being the Weibull modulus. As mentioned, it is assumed that the fibres do not undergo any degradation due to the cyclic loading.

When the debond crack tip propagates along the interface of the broken fibre, its crack tip stress field will add an additional stress to its nearest fibres. More precisely, the debond crack tip

stress field will influence the nearest 6 fibres (assuming a hexagonal fibre packing arrangement) as the singular crack tip stress field moves along the broken fibre, from the locus of fibre breakage, $z = 0$ to the final positions of the debond crack tip, $z = \pm\ell_d^c$. Using the Weibull model with the condition that the probability equal to 1/6 (corresponding to failure of one of the six neighbour fibres), the condition for the maximum strain level becomes:

$$A_1 \left\{ \frac{\tau_s L_0}{E_f r} \left[\left(\frac{\sigma_f^+ (\bar{\varepsilon}_{fl}) + \hat{\sigma}_f^K}{\sigma_0} \right)^m + 5 \left(\frac{\sigma_f^+ (\bar{\varepsilon}_{fl})}{\sigma_0} \right)^m \right]^{-1} ln\left(\frac{5}{6}\right) + \frac{\bar{\sigma}_i}{E_c} \right\} - \bar{\varepsilon}_{fl} = 0 \qquad (8)$$

where the parameter A_1 depends on both the decrease in the interfacial frictional shear stress and the R-ratio:

$$A_1 = \frac{2\tau_s^c}{(1+R)\,\tau_s^c + (1-R)\,\tau_s}. \qquad (9)$$

Generally, A_1 lies in the range of 0 to 1. Decreasing τ_s^c/τ_s and decreasing R lead to a decrease in A_1. The stress $\sigma_f^+ (\bar{\varepsilon}_{fl})$ is the stress in fibres remote from the fibre break at the strain level corresponding to the fatigue limit. $\sigma_f^+ (\bar{\varepsilon}_{fl})$ is given by

$$\frac{\sigma_f^+ (\bar{\varepsilon}_{fl})}{E_f} = -\Delta\varepsilon^T \frac{(1-V_f)\,E_m}{E_c} + \bar{\varepsilon}_{fl}. \qquad (10)$$

Furthermore, the parameter $\hat{\sigma}_i$ in Eq. (8) is given by Eq. (3), and the parameter $\hat{\sigma}_f^K$ is given by Eq. (5).

Eq. (8) is not easy to solve explicitly for $\bar{\varepsilon}_{fl}$ - recall that the parameter σ_f^+ depends on $\bar{\varepsilon}_{fl}$. However, Eq. (8) can be solved by numerical methods. In the remainder of this paper, the fatigue limit $\bar{\varepsilon}_{fl}$ is found by solving Eq. (8) numerically using a Newton-Raphson method and a convergence criterion based on the difference between two successive iteration values of $\bar{\varepsilon}_{fl}$ being less than 10^{-9}.

3. Results: Predictions of fatigue limit for carbon/epoxy composites

3.1. Material parameters

In the following we present predictions of the fatigue limit for an AS4 carbon fibre/epoxy composites. The following material parameters were used: E_f = 231 GPa, r = 3.55 μm, σ_0 = 4680 MPa, L_0 = 20 mm, m = 8.86 (data from Jimenes and Pellegrini [5]) and E_m = 4 GPa. Values for $\Delta\varepsilon^T$ =0.00407 (estimated from thermal expansion mismatch and cool down from processing temperature). In addition, values for τ_s = 128 ± 30 MPa and G_c = 6 J/m^2 were estimated from the data for debond length as a function of applied strain given by Kim and Nairn [6] using the procedure described elsewhere [7]. However, in the following we will investigate the effects of τ_s, τ_s^c, G_c^i and the parameter A_1 on the predicted fatigue limit, $\bar{\varepsilon}_{fl}$.

3.2. Model predictions

Fig. 1 shows the predicted fatigue limit as a function of fibre volume fraction and the parameter A_1 for τ_s = 128 MPa. The fatigue limit, expressed in terms of strain, $\bar{\varepsilon}_{fl}$, is seen to decrease with increasing V_f and to increase with increasing A_1.

Next, the fatigue limit expressed in terms of stress, $\bar{\sigma}_{fl}$, is calculated using (2) and is shown in Fig.2. Starting from a low value of V_f, $\bar{\sigma}_{fl}$ is seen to increase with V_f attains a maximum value and decrease with V_f. For $A_1 = 1.0$, the maximum fatigue limit (1830 MPa) is predicted for V_f= 0.54, while for $A_1 = 0.25$, the maximum value of $\bar{\sigma}_{fl}$ is 1327 MPa is attain at $V_f = 0.47$.

Fig. 3 shows $\bar{\varepsilon}_{fl}$ as a function of the interfacial fracture energy, G_c^i for various values of A_1. Initially, $\bar{\varepsilon}_{fl}$ decreases from a value around 0.020 (weakly dependent on A_1) for vanishing G_c^i,

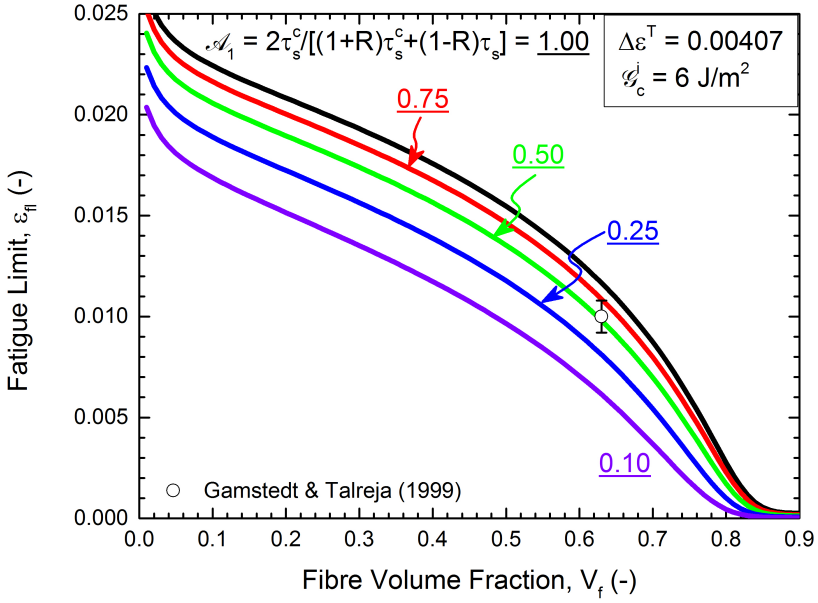


Figure 1. The predicted fatigue limit (in terms of strain) for unidirectional carbon fibre/epoxy composites is shown as a function of fibre volume fraction for various values of A_1 ($\tau_s = 128$ MPa). The fatigue limit found experimentally by Gamstedt and Talreja [2] is included as a point.

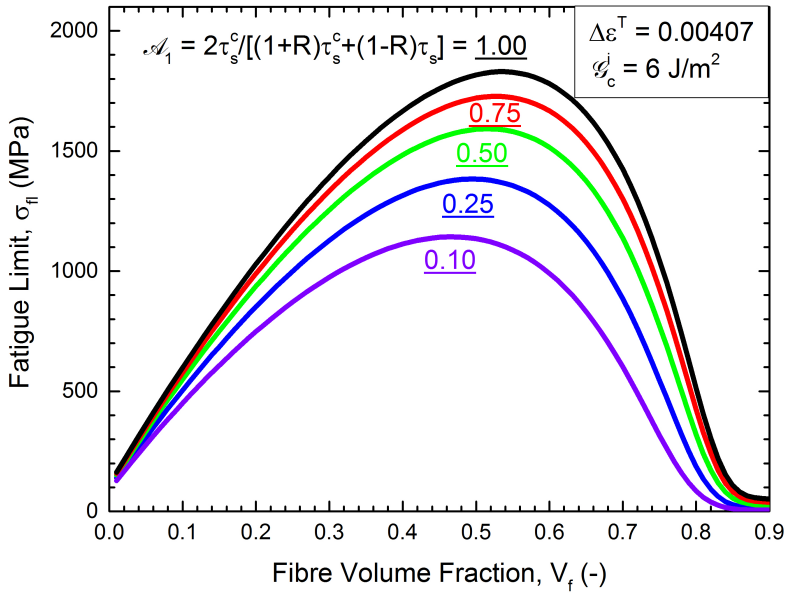


Figure 2. The predicted fatigue limit (in terms of stress) for unidirectional carbon fibre/epoxy composites is shown as a function of fibre volume fraction for various values of A_1 ($\tau_s = 128$ MPa).

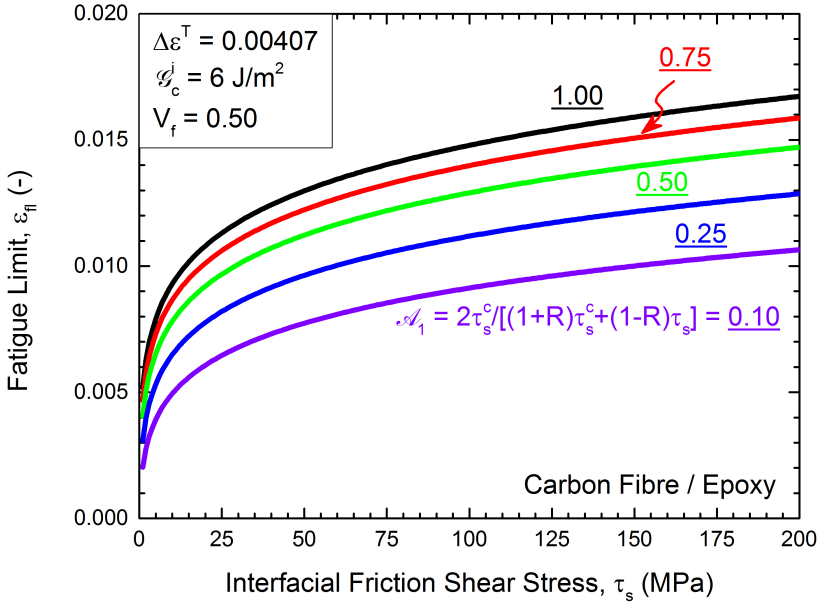


Figure 3. The predicted fatigue limit (in terms of strain) for unidirectional carbon fibre/epoxy composites is shown as a function of frictional shear stress for various values of A_1.

to a minimum value around (occurring at $G_c^i \approx 50$ J/m^2), and then slowly increases (almost linearly) with G_c^i reaching 0.0012 to 0.0046 (depending on the value of A_1) for $G_c^i = 250$ J/m^2.

The predicted fatigue limit in terms of strain, $\bar{\varepsilon}_{fl}$, is shown as a function of the initial interfacial frictional shear stress, τ_s, for various values of A_1 in Fig. 4. $\bar{\varepsilon}_{fl}$ increases monotonically with increasing τ_s; the increase is particularly strong for small values of τ_s. For large value of τ_s, say larger than 100 MPa, the increase in $\bar{\varepsilon}_{fl}$ is more moderate.

4. Discussion

4.1. Determination of τ_s^c, G_c^i and τ_s^c

In the following we sketch how the model can be applied for the prediction of the fatigue limit. First, interface properties must be determined from experiments. For instance, ℓ_d can be obtained from in-situ observations of composites or single filament specimens subjected to monotonic loading. Then, τ_s and G_c^i can be calculated by the use of micromechanical models, e.g. Sørensen [7]. Next, and ℓ_d^c can be measured from cyclic experiments as the maximum debond length that is reached after many load cycles. The analysis for determination of τ_s^c is as follows: We subtract (1) from (4) and rewrite:

$$\frac{\tau_s}{\tau_s^c} = 1 + 4\frac{\tau_s}{E_f}\frac{1}{\bar{\varepsilon}_{max}\,(1-R)}\left(\frac{\ell_d^c - \ell_d}{r}\right). \tag{11}$$

Eq. (11) thus enables the calculation of τ_s^c.

4.2. Comparison with experiments

The fatigue limit of 1.0% ± 0.08% (run-out for 10^6 cycles, $R = 0.1$) for a unidirectional AS4 carbon fibre/epoxy composite is superimposed as a point in Fig. 1. The data point lies comfortably within the range predicted by the model, just slightly above the curve for A_1=0.50,

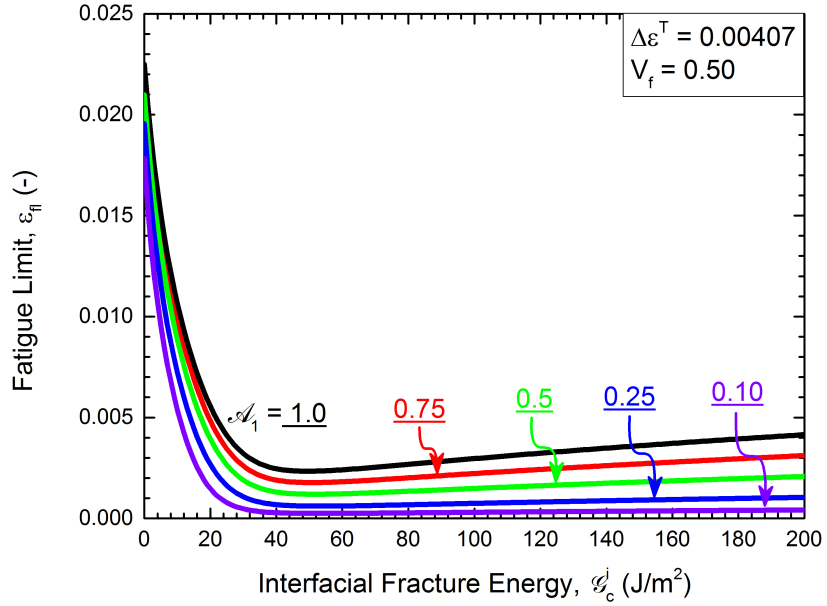


Figure 4. The predicted fatigue limit (in terms of strain) for unidirectional carbon fibre/epoxy composites is shown as a function of interfacial fracture energy for various values of A_1 (τ_s = 128 MPa).

lending confidence in the model predictions. However, this may be somewhat fortuitous since the curves for the predicted fatigue limit depends strongly on the chosen value of G_c^i, as indicated in Fig. 4.

We proceed to test the model predictions further. First, Gamstedt and Talreja [2] found that the debond length after millions of cycles was approximate 23 μm. Next, from the data from the monotonic single fibre fragmentation tests of Kim and Nairn [6], ℓ_d/r was around 1.5 for a strain value of 1.3-1.5 %. Extrapolating to a strain value of 1% gives ℓ_d/r =0.5 or ℓ_d =1.78 μm. Next, knowing ℓ_d and ℓ_d^c, we use (11) to calculate τ_s^c = 51.5 MPa. Finally, these values, together with $R = 0.1$ are inserted into (9) to give $A_1 = 0.60$. This value is within the error bars of the experimental point from Gamstedt and Talreja [2], see Fig. 1. Recall that the prediction is made using data from two un-related studies, this finding is encouraging.

It can be noted that a similar comparison between model predictions and experimental data for fatigue limit of glass-fibre composites also gives a good agreement.

4.3. Raising the fatigue limit

One of the main reasons for the development of micromechanical models is that they can give model predictions to guide the development of future materials. The parameter study included in this paper suggests that the fatigue limit of unidirectional carbon fibre/epoxy composites can be raised by microstructural optimization. More specifically, the model predictions suggest that the fatigue limit (expressed in strain) can be raised by:

- decreasing V_f
- preventing a decrease (or minimizing the decrease) in the τ_s^c/τ_s ratio (preventing a decrease in A_1)

- increasing τ_s
- decreasing G_c^i, preferable below a few J/m^2.

Decreasing V_f may not be of great practical use, since a certain minimum value of V_f is often chosen in order to obtain a certain stiffness value. It is therefore also relevant to consider the fatigue limit in terms of stress. Then the situation is a bit more complicated since the maximum value of $\bar{\sigma}_{fl}$ as a function of V_f depends on a number of additional parameters, as can be seen from Fig. 2. A more detailed parameter study might be needed to determine the optimum set of interface parameters for a given material system.

Microstrutural optimization also offers a number of experimental challenges. First, improved experimental methods and data analysis approaches may be needed for characterisation of the interface properties obtained under a given set of processing parameters (see [7] for a more detailed discussion). Next, when the interfacial parameters of a given composite material have been well characterized, the next step will be to alter the processing conditions such that the optimal microscale parameters (e.g. optimal values for G_c^i, τ_s and τ_s^c) can be obtained. The results of the present study suggest that the interface fracture energy should be as low as possible whereas the friction should be as high as possible. It is recommended that more weight should be put in the design of fibre topology (e.g. roughness) and less on chemistry - the goal of the chemistry should be to decrease the fibre/matrix debond energy.

In a more broader sense, the model results suggest that the main issue in maximizing the fatigue limit is in controlling parameters that lowers $\hat{\sigma}_f^K$, the stress on neighbour fibres from the debond crack tip stress fields of the broken fibre.

Finally, it is noted that a decrease in the R-ratio will give a lower value of $\bar{\varepsilon}_{fl}$ since the parameter A_1 decreases with decreasing R, see eq. (9).

5. Concluding remarks

A micromechanical model was used for the prediction of the fatigue limit of unidirectional fibre composites. The effect of interface properties were investigated for a unidirectional AS4 carbon fibre/epoxy composite. The model predicts that the fatigue limit, expressed in terms of strain, decreases with increasing fibre volume fraction, decreases with decreasing τ_s^c/τ_s ratio, and increases with increasing interfacial frictional sliding shear stress. When expressed in terms of stresses, the maximization of the fatigue limit is more complicated. With other parameters held fixed, $\bar{\sigma}_{fl}$, attains a maximum at intermediate values of V_f. A further increase in the fibre volume fraction will then lead to a lower fatigue limit.

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