

# Program spatial movement of high-speed vehicles

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**Abstract.** Program spatial movement of a high-speed vehicle has been synthesized with the help of hodograph in terms of spiral paths. A mathematical model of a primary road in the form of a line over ruled bearing surface in the neighbourhood of the program trajectory has been developed. Vector equations of the elements of ruled surface and bearing surfaces oriented orthogonally to inertia forces have been set up. The inertia forces effecting a mass of a high-speed vehicle in the context of non-uniform motion along spiral program path has been demonstrated in a vector form.

## 1. Introduction

The task of a vehicle is to transport certain freight from one specified spatial point to another while taking into consideration additional conditions. In terms of ground transport, the boundary points and motion route are on the Earth's surface. As for the railway transport, motion route is determined by a rigid route being a single nominal calculation program trajectory. In the context of motor transport, motion route determines the neighbourhood of the admissible trajectories [1, 2]. Neighbourhood of program trajectory in the shape of a line on the bearing surface forms a highway. The development of mathematical model of the bearing surface of a high way as a constituent part of calculation experiment technique providing dynamic design of a “vehicle – road” system is proposed [3-7].

## 2. Statement of the problem

It is assumed that at initial time and final time (boundary conditions), phase state of a vehicle has been specified. Time for the vehicle transport from one spatial point to another as well as a shape of the spatial trajectory compatible with probable motion of the vehicle is among the heuristically specified additional conditions. In this context, program motion of the vehicle is considered as the motion of a material point in terms of the required mode (i.e. accelerated, decelerated, and uniform) along the selected spatial curvilinear trajectory. The spatial curvilinear trajectory is represented with the help of vector-radius of a material point depending on time as a parameter that is program notion of the vehicle is determined by means of a hodograph of the radius-vector. The hodograph of program motion is taken as a bearing surface. The bearing surface is found in the class of ruled surfaces. It is required to develop mathematical model of a bearing surface corresponding to program spatial transport of the high-speed vehicle.



### 3. Hodograph of the program motion

It is proposed to form a variety of program spatial motions of vehicles in the class of spiral lines with the help of following hodograph [8, 9]:

$$\bar{r}(t) = \|\rho_0 \rho_1 \rho_2 \rho_3\| \begin{Bmatrix} 1 \\ t \\ t^2 \\ t^3 \end{Bmatrix} \left( \bar{i} \cos \omega t + \bar{j} \sin \omega t \right) + \bar{k} \|h_0 h_1 h_2 h_3\| \begin{Bmatrix} 1 \\ t \\ t^2 \\ t^3 \end{Bmatrix}, \quad (1)$$

where:  $\rho_i h_i (i=0, 1, 2, 3)$  are running parameters being determined according to the specified boundary conditions;  $t$  is time (independent parameter);  $\omega$  is average angular turning velocity, i.e.  $\omega = \frac{\varphi_0}{t_0}$ ;  $\varphi_0$  is full turning angle;  $t_0$  is the required spatial transport time;  $\bar{i}, \bar{j}, \bar{k}$  is the basis of ground Cartesian reference system.

Method of the program motion hodograph development within the considered spiral trajectory class is to determine the introduced running parameters according to the required boundary conditions. Additional conditions also include such limitations on technical applicability as the geometry of the trajectory shape as well as the velocity motion mode along the trajectory.

### 4. Natural trihedral of the program motion hodograph

Natural trihedral of the program motion hodograph is movable orthogonal coordinate system connected with a vehicle being considered as a material point of mass  $m$ . In this context, basis of the natural trihedral is convenient to be determined according to the hodograph (1) of the program motion in the form of:

$$\begin{aligned} \bar{\tau} &= \frac{1}{(\dot{\bar{r}} \cdot \dot{\bar{r}})^{\frac{1}{2}}} \dot{\bar{r}} \\ \bar{n} &= -\frac{1}{(\dot{\bar{r}} \cdot \dot{\bar{r}})^{\frac{1}{2}} |\dot{\bar{r}} \times \ddot{\bar{r}}|} \dot{\bar{r}} \times (\dot{\bar{r}} \times \ddot{\bar{r}}) \\ \bar{b} &= -\frac{1}{(\dot{\bar{r}} \cdot \dot{\bar{r}})^{\frac{1}{2}} |\dot{\bar{r}} \times \ddot{\bar{r}}|} \dot{\bar{r}} \times [\dot{\bar{r}} \times (\dot{\bar{r}} \times \ddot{\bar{r}})] \end{aligned} \quad (2)$$

where:  $\bar{\tau}$  is a unit vector of a tangent;  $\bar{n}$  is a unit vector of a principal normal;  $\bar{b}$  is a unit vector of a binormal;  $\dot{\bar{r}}$  and  $\ddot{\bar{r}}$  are 1<sup>st</sup> and 2<sup>nd</sup> time derivatives from  $\bar{r}(t)$ .

Represent the equation of a normal plane passing through the current point  $\bar{r}_M(t)$  of hodograph, in a vector form:

$$[\bar{\rho} - \bar{r}_M(t)] \cdot \bar{\tau} = 0, \quad (3)$$

where  $\bar{\rho}$  is a radius-vector of a random point of the normal plane within the earth coordinate system.

### 5. Bearing surface

Bearing surface is being formed as a ruled surface (Shukhov surface) which guiding line is determined with the help of program motion hodograph and rectilinear generators are within the cross of normal and bearing surfaces corresponding to the current hodograph point. Vector equation of bearing surfaces of the hodograph is:

$$[\bar{\rho} - \bar{r}_M(t)] \cdot \bar{g}_n = 0, \quad (4)$$

where  $\bar{g}_n$  is a unit vector of a projection of the resulting inertia forces on a normal plane. Then, the generators of the bearing surface in space can be specified as the cross of two planes (3), (4):

$$\begin{cases} [\bar{\rho} - \bar{r}_M(t)] \cdot \bar{\tau} = 0 \\ [\bar{\rho} - \bar{r}_M(t)] \cdot \bar{g}_n = 0 \end{cases} \quad (5)$$

or with the help of the equation of a straight line lying within the normal plane (3) and passing through the hodograph point  $\bar{r}_M(t)$  (1) in the direction being perpendicular to the projection of resulting inertia forces on a normal plane, i.e.

$$[\bar{\rho} - \bar{r}_M(t)] \times \bar{s} = 0, \quad (6)$$

where  $\bar{s}$  is a unit guiding vector of the forming bearing surface. In this context,

$$\bar{s} = \bar{\tau} \times \bar{g}_n. \quad (7)$$

## 6. Inertia forces

Inertia forces [10, 11] include the Earth gravity force as a vector directed to a center of a spherical surface:

$$\bar{G}_0 = -\gamma \frac{M_0 m}{R_0^2} \cdot \frac{\bar{R}_0}{R_0}. \quad (8)$$

Acceleration

$$\bar{g}_0 = \gamma \frac{M_0}{R_0^2} \cdot \frac{\bar{R}_0}{R_0} \quad (9)$$

Corresponds to the force (8), where:  $M_0$  and  $R_0$  are mass of the Earth and its radius;  $\gamma$  is gravitational constant;  $\bar{R}_0$  is a radius-vector of a vehicle at the spherical earth's surface; the radius-vector starts in the center of the Earth. Then, (8) with allowance for (9) can be represented in the form:

$$\bar{G}_0 = -\bar{g}_0 m. \quad (10)$$

Inertia force, depending upon the Earth rotation (centrifugal force) is determined by means of the active formula:

$$\bar{G}_C = -\bar{\Omega}_0 \times (\bar{\Omega}_0 \times \bar{R}_0) m. \quad (11)$$

Acceleration

$$\bar{g}_C = \bar{\Omega}_0 \times (\bar{\Omega}_0 \times \bar{R}_0). \quad (12)$$

Corresponds to the force (11), where  $\bar{\Omega}_0$  is a vector of angular velocity of the Earth rotation. Then, (11) with allowance for (12) can be represented in the form:

$$\bar{G}_C = -\bar{g}_C m. \quad (13)$$

Inertia force depending upon the Earth rotation and high-speed motion of the vehicle (Coriolis force) is determined by means of the vector formula:

$$\bar{G}_{Cor} = -2(\bar{\Omega}_0 \times \dot{\bar{r}}_M) m. \quad (14)$$

Coriolis acceleration

$$\bar{g}_{Cor} = 2(\bar{\Omega}_0 \times \dot{\bar{r}}_M). \quad (15)$$

Corresponds to the force (14). Then, (14) with allowance for (15) can be represented in the form

$$\bar{G}_{Cor} = -\bar{g}_{Cor} m. \quad (16)$$

Inertia force depending upon high-speed motion of the vehicle along the spherical earth's surface (centrifugal inertia force) is:

$$\bar{N}_C = \frac{(\dot{\bar{r}}_M \cdot \dot{\bar{r}}_M)}{(\bar{R}_0 \cdot \bar{R}_0)} \bar{R}_0 m. \quad (17)$$

Centripetal acceleration

$$\bar{g}_N = -\frac{(\dot{\bar{r}}_M \cdot \dot{\bar{r}}_M)}{(\bar{R}_0 \cdot \bar{R}_0)} \bar{R}_0 \quad (18)$$

Corresponds to the force (17). Then, (17) with allowance for (18) can be represented in the form

$$\bar{N}_C = -\bar{g}_N m. \quad (19)$$

Inertia force depends upon the non-uniform transport motion along the curvilinear program trajectory. Tangential inertia force is:

$$\bar{N}_\tau = -\frac{(\dot{\bar{r}}_M \cdot \ddot{\bar{r}}_M)}{(\dot{\bar{r}}_M \cdot \dot{\bar{r}}_M)} \dot{\bar{r}}_M. \quad (20)$$

Tangential acceleration

$$\bar{w}_\tau = \frac{(\dot{\bar{r}}_M \cdot \ddot{\bar{r}}_M)}{(\dot{\bar{r}}_M \cdot \dot{\bar{r}}_M)} \dot{\bar{r}}_M \quad (21)$$

Corresponds to the force (20). Then, (20) with allowance for (21) can be represented in the form

$$\bar{N}_\tau = -\bar{w}_\tau m. \quad (22)$$

Normal inertia force is:

$$\bar{N}_n = \frac{[\dot{\bar{r}}_M \times (\dot{\bar{r}}_M \times \ddot{\bar{r}}_M)]}{(\dot{\bar{r}}_M \cdot \dot{\bar{r}}_M)} m. \quad (23)$$

Normal acceleration

$$\bar{w}_n = -\frac{[\dot{\bar{r}}_M \times (\dot{\bar{r}}_M \times \ddot{\bar{r}}_M)]}{(\dot{\bar{r}}_M \cdot \dot{\bar{r}}_M)}. \quad (24)$$

Corresponds to the force (23). Then, (23) with allowance for (24) can be represented in the form

$$\bar{N}_n = -\bar{w}_n m. \quad (25)$$

Resulting inertia force is being determined as a vector sum (10), (13), (16), (19), (22) and (25)

$$\bar{I} = \bar{G}_0 + \bar{G}_C + \bar{G}_{Cor} + \bar{N}_C + \bar{N}_\tau + \bar{N}_n \quad (26)$$

or within projections of the natural trihedral axis (2):

$$\bar{I} = \bar{I}_\tau + \bar{I}_n + \bar{I}_b. \quad (27)$$

In this context,  $(\bar{I}_n + \bar{I}_b)$  determines the projection of the resulting inertia forces on a normal plane (3).

Then, a unit normal vector of the bearing surface (4) is:

$$\bar{g}_n = \frac{\bar{I}_n + \bar{I}_b}{|\bar{I}_n + \bar{I}_b|}. \quad (28)$$

It should also be noted that a unit vector of the direction of local vertical line of the earth coordinate system is determined in the form of:

$$\bar{k} = -\frac{\bar{G}_o + \bar{G}_C}{|\bar{G}_o + \bar{G}_C|}. \quad (29)$$

## 7. Summary

It is proposed to consider program spatial motion of a high-speed vehicle with the help of a hodograph in the class of spiral trajectories. Mathematical model of a primary road is built in the form of a line at a ruled bearing surface in the neighbourhood of a program trajectory. Spiral program trajectory is a guideline of a ruled bearing surface; rectilinear generators in each point of a hodograph are found at the cross of normal and bearing surfaces. Equations of normal and bearing surfaces of a program hodograph are demonstrated in a vector form. Vector equation of generators oriented orthogonally to

the resulting inertia forces is set up. Components of inertia forces including gravitation force are represented in a vector form; the forces effect the mass of a high-speed vehicle in terms of non-uniform motion along the curvilinear program trajectory at the spherical earth's surface.

## References

- [1] Zhang J, Zhou S and Huang C 2010 A new method for the fault diagnosis of the train wheelset based on characteristic spectrum analysis. *Proceedings of the 29th Chinese Control Conference, CCC 10* pp. 3988–92
- [2] Ziborov K A, Protsiv V V and Fedoriachenko S O 2013 On formation of kinematical and dynamical parameters of output elements of the mine vehicles in transient *Scientific Messenger of the National Mining University* 4 pp 64–68
- [3] Bass K , Plakhotnik V and Krivda V 2013 Influence of the road profile and dump truck parameters on the exploitation. *Scientific Messenger of the National Mining University* 4 pp 59–63.
- [4] Bass K, Kuvayev S, Plakhotnik V and Krivda V 2014 Planar and spatial mathematical motion simulation of open pit mining vehicles. *Scientific Messenger of the National Mining University* 3 pp 60–65.
- [5] Sakhno V, Poliakov V, Timrov O and Kravchenko O 2016 Lorry convoy stability taking into account the skew of semitrailer axes *Transport Problems* 11 pp 70–76.
- [6] Kravets V, Bass K, Kravets T and Tokar L 2015 Dynamic design of ground transport with the help of computation experiment *MMSE Jurnal* 1 pp 105–11
- [7] Kravets V.V, Bass K.M, Kravets T.V, Zubarev M.S and Tokar L.A 2016 Kinetostatics of wheel vehicle in the category of spiral-screw routes, *MMSE Journal* 5, pp 87-99
- [8] Басс К М, Кравец В В, Кравец Т В 2012 Математическая модель дорожной поверхности скоростной автомагистрали в развязках и поворотах. *Міжвузівський збірник "Наукові нотатки"* 36 (Луцьк) pp 23-25.
- [9] Kravets V, Kravets T, Bass K and Tokar L 2014 Mathematical model of path and hodograph of surface transport *Transport problems 1* (Katowice) pp 830–41.
- [10] Martynyuk A.A, Lobas L.G, Nikitina N.V 1981 *Dynamics and sustainability of transport vehicle wheelset movement* (Kiev: Tekhnika) p 223
- [11] Kravets V, Kravets T 2008 Evaluation of centrifugal, coriolis and gyroscopic foces on a railroad vehicle moving at high speed *Int. Appl. Mech.* 44 pp 101-09.