

Large eddy simulation of flow past a 3D cylinder at $Re=3900$

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Abstract. Unsteady flow past a cylinder ($Re=3900$) was computed using large eddy simulation (LES) by OpenFOAM. The study includes the prediction of drag and lift coefficient, and pressure distribution around the cylinder. During the simulation, the one-equation sub-grid scale (SGS) model was applied. LES numerical results were compared with published RANS $k-\varepsilon$ model, $k-\omega$ model and experimental data. Results indicate that LES one-equation SGS model provides better agreement in force coefficients and pressure distribution than above mentioned RANS turbulence models. Besides, effect of spanwise discretization to numerical results is also investigated, showing that medium grid is fine enough to yield accurate results compared to the fine grid, which is very computationally expensive.

1. Introduction

The flow past circular cylinders has been extensively studied due to its importance in many practical applications, such as hydrodynamic loads on marine piles, offshore platform risers and support legs. In fact, the flow around circular cylinders includes a variety of fluid dynamics phenomena, like separation, vortex shedding and the transition to turbulence. The mechanisms of vortex shedding and its suppression have significant effects on the various fluid-mechanical properties of practical interest: flow-induced forces such as drag and lift forces and pressure coefficient [1].

Currently the numerical simulation methods to predict turbulent flows are limited. This is mainly due to their three-dimensional, unsteady and irregular characteristics. The most accurate method of solving turbulence flow is Direct Numerical Simulation (DNS), in which Navier-Stokes equations are solved on a fine grid using a small time-step. All eddy sizes (including the smallest turbulence scales) will be captured. DNS performs well for simple flows, however, it becomes computationally expensive when complicated flow is needed. The overall cost, including time step, of the computational effort is proportional to Re , which is not suitable to industrial applications with CPU resources available today.

Another numerical simulation method for turbulent flows is Reynolds-Averaged Navier-Stokes (RANS) models. RANS focuses on the mean flow and the effects of turbulence on mean flow properties. Prior to the application of numerical methods the Navier–Stokes equations are time averaged and the turbulence equations are closed by using viscosity terms. Common models of RANS including $k-\varepsilon$ method and $k-\omega$ method. Due to its modest computational requirement, this approach has been applied successfully in the industrial for the last few decades. However, it tends to fail in predicting the transient behavior of the flow and for flows involving large unsteady vortical structures. [2]

An alternative approach is called large-eddy simulation (LES) which was proposed in 1963 by Joseph Smagorinsky. In LES approach, large eddies are retained, and 3D time-dependent



Navier-Stokes equations are solved directly. Small eddies are removed and modeled using a sub-grid scale model (SGS), which results in a significant reduction in computational cost compared to DNS. LES provides higher accuracy degree than the RANS approach since the large eddies contain most of the turbulent energy and are responsible for most of the momentum transfer and turbulent mixing [3]. No model is required for the large flow structures that depend on the individual geometry of the problem. Only the fine-scale turbulence, which has more universal character, needs to be modelled. Therefore, LES method is found to be a more promising and viable tool for simulating turbulent flow and chosen for this study [2].

2. Governing equation

As mentioned above, in LES, only the influence of the small eddies has to be modeled by sub-grid scale model, whereas the large energy-carrying eddies are computed directly. Small eddies are more universal, random, homogeneous and isotropic, which simplifies the development of appropriate models.

In order to separate the large and small scale motions, the three-dimensional, time dependent Navier–Stokes equations are filtered. In the present study a box filter is applied as a filter kernel and an incompressible fluid is assumed. The governing equations are given by equation (1)-(3):

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j} (\overline{u_i u_j}) = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_i} \left[\frac{1}{\text{Re}} \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \right] - \frac{\partial \tau_{ij}}{\partial x_j} \tag{2}$$

$$\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j \tag{3}$$

where \bar{u}_i is the velocity component of the resolved scales, \bar{p} is the corresponding pressure. The filtering procedure provides the governing equations for the resolvable scales of the flow field. Although the continuity equation (1) of the resolved quantities is equal to the original unfiltered one, the filtered momentum equation (2) includes an additional term for the non-resolvable sub-grid scale stresses τ_{ij} , which results from filtering the non-linear convective fluxes. τ_{ij} describes the influence of the small-scale structures on the larger eddies. Only this effect has to be modeled by a sub-grid scale model [4].

For one-equation SGS Model, the eddy viscosity is calculated from equation (4):

$$\mu_{T,a} = \rho_a C_k \Delta k_{sgs}^{1/2} \tag{4}$$

where k_{sgs} represents the SGS kinetic energy, as show in equation (5), which is obtained by solving for transport equation for k_{sgs} ,

$$\frac{\partial}{\partial t} (\rho_r \alpha_r k_{sgs}) + \nabla (\rho_r \alpha_r u_r k_{sgs}) = -\nabla (\alpha_r \frac{\mu_{eff,r}}{\sigma_k} \nabla k_{sgs}) + \alpha_r (G_r - C_\epsilon \frac{k_{sgs}^{3/2}}{\Delta}) \tag{5}$$

where, G is the production term, defined as equation (6):

$$G_r = \mu_{T,a} \left| \overline{S_{ij}} \right| \tag{6}$$

The value of model constants ($C_\epsilon=1.05$ and $C_k=0.07$) in equation are considered on the basis of recommendation by Davidson [5].

3. Numerical implementation

3.1. Computational domain

The computational domain and grid are shown in figure 1 and figure 2. The diameter of the cylinder is D . For selection of spanwise length, according to Ref.[6], when the spanwise length of the cylinder $Z \geq \pi D$, the cylinder's flow will present obvious 3D effect. Three spanwise discretizations are chosen in order to investigate the influence of Δz to numerical results. Table 1 shows the total number of grid sizes:

Table 1. Grid Size.

Grid Type	Nx	Ny	Nz	Total
Coarse	175	114	12	239,400
Medium	175	114	18	359,100
Fine	175	114	24	478,800

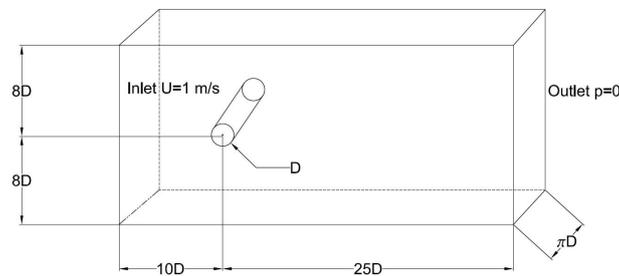


Figure 1. Computational Geometry.

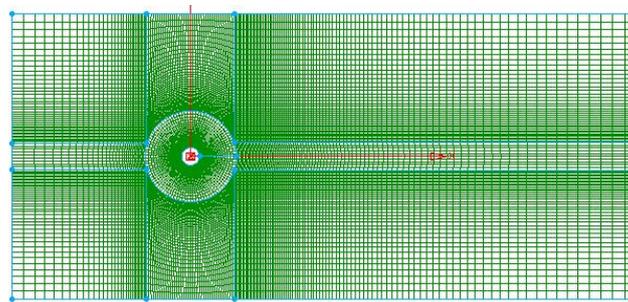


Figure 2. Computational Domain.

3.2. Boundary condition

The boundary condition for inlet is free-stream velocity $U=1$ m/s with fluctuation scale (0.02, 0.01, 0.01) and zero gradient for pressure. The pressure for outlet is specified as 0. For top, bottom and cylinder, no slip wall is applied. Symmetry Plane is set for the front and back of the domain.

3.3. Wall treatment

When LES is used to simulate the flow at high Reynolds number, a special near-wall treatment (wall-function) has to be introduced with the shortcoming that the near-wall regions cannot be properly resolved. For this project, as a relatively low Reynolds number (3900), the wall-function can be avoided by implementing the near-wall model. Specially, in order to fully resolve the viscous sub-layer, it is necessary to have $y^+ < 1$ for the first cell adjacent to the cylinder.

3.4. Computational details

Table 2 below lists the computational details for this study.

Table 2. Computational Details.

Solver	pisoFoam
Time discretization scheme	Euler backward
Δt	0.005s
Time length	30s
Pressure-velocity coupling	PISO

4. Results and discussion

4.1. Drag coefficients

The comparison of drag coefficient between different methods and experimental data is listed in table 3:

Table 3. Comparison of Drag Coefficient.

		Cd		
Exp		0.98 ± 0.05		
LES	Coarse	Medium	Fine	
	1.1075	1.0524	1.0447	
RANS($k - \epsilon$)		0.7446		
RANS($k - \omega$)		0.6208		

From figure 3 and 4, it is observed that the predicted drag and lift coefficients using LES agree well with the experimental data. While for the RANS, both $k - \epsilon$ and $k - \omega$ underestimate the Cd value. Besides, the grid refinement from coarse to medium improves the result significantly, whereas for medium to fine, the improvement becomes less obvious.

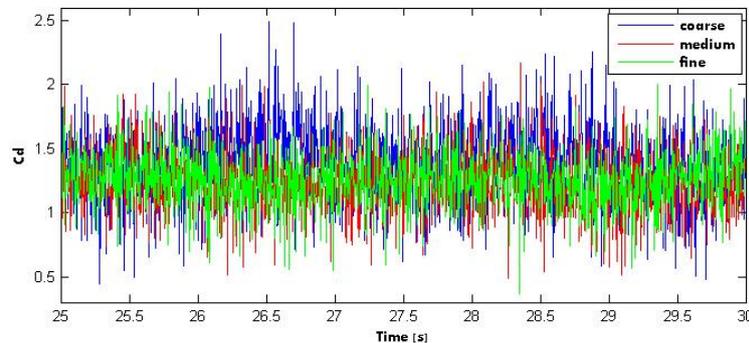


Figure 3. Time history of drag coefficient for different grid when Re=3900.

4.2. Lift coefficient

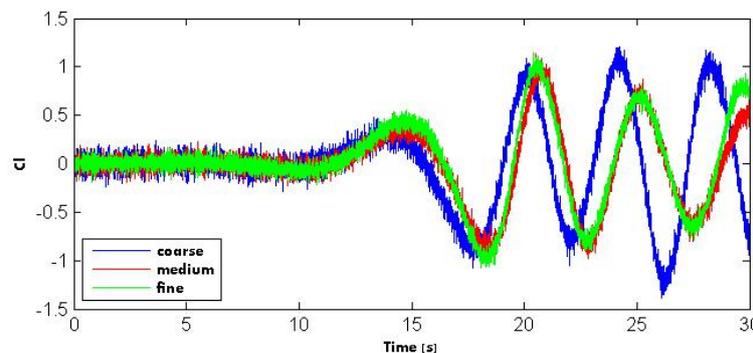


Figure 4. Time history of lift coefficient for different grid when Re=3900.

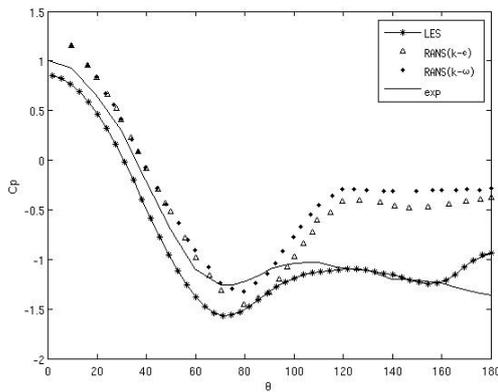


Figure 5. Comparison between different turbulence models and experimental data for pressure distribution around cylinder.

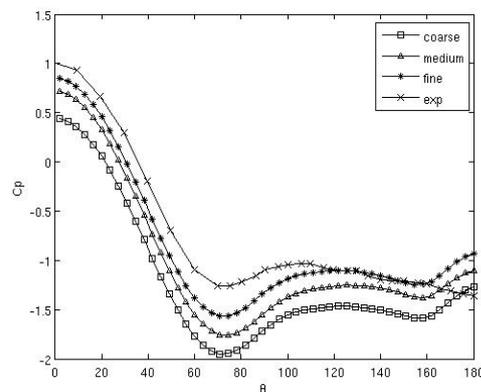


Figure 6. Comparison between different grids for pressure distribution around cylinder.

4.3. Pressure distribution

Figure 5 presents the distribution of the pressure coefficient around the cylinder at $Re = 3900$ predicted by LES, RANS ($k - \epsilon$) and RANS ($k - \omega$) models, which are compared to the experimental data. It can be observed that all three turbulence models predict the pressure distribution well from stagnation point to 60 degree. However, RANS models failed to accurately predict the pressure distribution after separation point (about 80 degree) of the flow occurred with increasing pressure coefficients. For LES, the pressure still distributes almost the same trend as experimental data, which shows that LES yields a better result than other two RANS models.

As for figure 6, it is obvious that fine grid give better prediction than medium and coarse grid. But the improvement from medium to fine is not that significant.

4.4. Mean stream-wise velocity

For mean stream-wise velocity in figure 7, it can be seen that the overall trend predicted by LES agrees the experimental data well except for the value at about $x/D=1$, which numerical results overestimate the velocity.

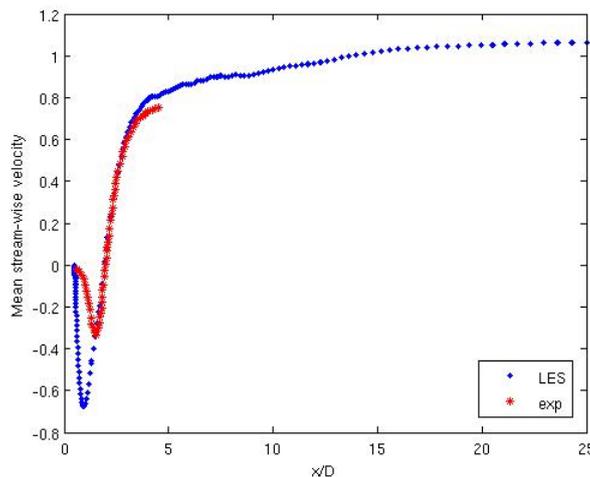


Figure 7. Mean stream-wise velocity in the wake centerline for flow over a cylinder.

5. Conclusion

Unsteady flow past a cylinder ($Re=3900$) was computed by large eddy simulation (LES). One-equation SGS model was applied, whose results validate against experimental data. Results show

that the one-equation SGS LES model predicts the force coefficient, pressure distribution and mean stream-wise velocity better than the RANS ($k - \varepsilon$ and $k - \omega$) models. For investigation of spanwise discretization, it can be seen that if restricted by time and machine condition, medium grid is accurate enough to yield good results.

References

- [1] Patel Y., *Numerical Investigation of Flow Past a Circular Cylinder and in a Staggered Tube Bundle Using Various Turbulence Models*, Master's thesis, Department of Mathematics and Physics, Lappeenranta University of Technology, 2010
- [2] Jisheng Zhang & Yakun Guo, *Large Eddy Simulation of Flow around Circular Cylinders on Structured and Unstructured Grids*, European Conference on Computational Fluid Dynamics, 2006
- [3] Zhiyin Yang, *Large-eddy simulation: Past, present and the future*, Chinese Journal of Aeronautics, (2015), 28 (1): 11–24
- [4] Breuer M., *Large Eddy Simulation of the Sub-critical Flow Past a Circular Cylinder: Numerical and Modeling Aspects*, International Journal for Numerical Methods in Fluids, Vol. **28**, pp. 1281-1302, 1998
- [5] Tabib M., & Schwarz P., *One-Equation SGS Modeling for Eulerian Large Eddy Simulation of Two Phase Solvent Extraction Pump Mixer Unit*, 17th Australasian Fluid Mechanics Conference, 2010
- [6] Chen H. L., *Three-dimensional Numerical Simulation of the Flow Past a Circular Cylinder based on LES Method*, Journal of Marine Science and Application, Vol. **8**, pp.110-16, 2009