IOP Conf. Series: Materials Science and Engineering **383** (2018) 012034 doi:10.1088/1757-899X/383/1/012034

Optimization of energy pile conductance using finite element and fractional factorial design of experiment

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Abstract. In Ground Source Heat Pumps (GSHP), Energy Piles pose as heat exchangers that transfer the heat from the buildings to the shallow ground lower temperature in order to decrease the energy consumption whilst cooling the buildings. These piles are mainly designed for highest possible thermal conductance. In this paper, nine factors influencing the thermal conductance of the energy pile are defined and statistically evaluated. These nine factors are; number of tubes, pile diameter, tube diameter, tube thickness, tube location, pile conductivity, tube conductivity, soil conductivity, and water flow rate. The thermal conductance of the energy pile is calculated using finite element model. The significance of these factors is evaluated using fractional factorial uniform design of experiment. The results show significance increase in the pile thermal conductance with the increase of the tube diameter, number of tubes, water flow rate, and tube and pile thermal conductivities. Furthermore, the tubes location near the pile outer surface show significant increase in the pile thermal conductance. On the other hand, decreasing pile diameter slightly increases the pile thermal conductance. Nevertheless, the soil thermal conductivity has shown insignificant effects on the pile thermal conductance.

1. Introduction

Ground Source Heat Pumps (GSHP) are a geothermal free form of energy utilizes the constant temperature of the shallow ground all year round to reduce energy consumption in cooling buildings through its energy piles [1]. The significant advantage of using energy piles over boreholes systems is that they require no additional structural or hydraulic measures because they are installed within elements that are already needed for structure [2]. Energy piles utilize renewable geothermal energy for buildings heating and cooling purposes and need proper design and sizing in order to end up with high plant efficiency [3].

Research on the controlling factors of the energy piles have shown that maximizing the pile surface, maximizing the concrete thermal conductivity, and maximizing the number of water tubes will increase the heat exchanging through the energy pile [4]. These reports neither include all elements affecting the thermal conductance of energy pile nor their interconnection effects. The analytical formulae proposed by [5] have shown nine factors affecting the energy pile steady state thermal conductance. These nine factors are; number of tubes, pile diameter, tube diameter, tube thickness, tube location, water flow rate, and the thermal conductivities of the pile, tube and soil. Investigating these factors together require unreachable number of experiments to evaluate the interrelation between these controlling factors. Statistical design of experiments methods like uniform fractional factorial design can solve this problem.

Based on either the finite-element method or the finite-volume method, various numerical approaches for full discretization of the Ground source heat exchangers like boreholes or energy piles have been formed. These models are employed to solve the heat-exchanging problem to optimize the heat exchanger geometry [6-9]. These models require extensive CPU time for being 3D models or for solving the transient effects. In order to decrease the time of calculations, the current analysis will be restricted to 2D steady state model.

The objective of the current work is to build a 2D steady state finite element thermal model to predict the energy pile thermal conductance at different combinations of the controlling factors. Then statistical regression method will be used to define a correlation between these controlling factors based on the significance of each of these factors on the energy pile steady state thermal conductance whilst changing other factors using uniform fractional factorial design of experiment method.

2. Energy pile factors and FE model

The current work considers the energy pile cross sectional in plane factors assuming same behaviors and relations through the pile height. Figure 1 shows schematic configurations of the studied energy pile in the current work.

The energy pile is symmetrical with repeated pattern. Half of the repeated sector can represent the whole energy pile cross section as shown in figure 1. The angle "θ" depends on the number of U-tubes in the pile and equal (90/*n*) where '*n*' is the number of U-tubes. Lines "O-A" and "O-B" are symmetry lines. The energy pile model considers the variation of five geometrical factors; number of U-tubes (*n*), pile diameter (d_p), tube inner diameter (d_i), tube thickness (*t*), and tubes spacing (*S*). In addition, it considers the variation of thermal conductivities of the pile (K_p), the HDPE tube (K_t) and the sand (K_s). The study will consider one operational factor, which is the heat transfer coefficient of the circulating water (*H*). The sand width will not affect the steady state conductance of the energy pile so that it will assumed with constant value in the current study (L_s=1.0 m). The energy pile studied factors throughout the current study are listed in table 1.

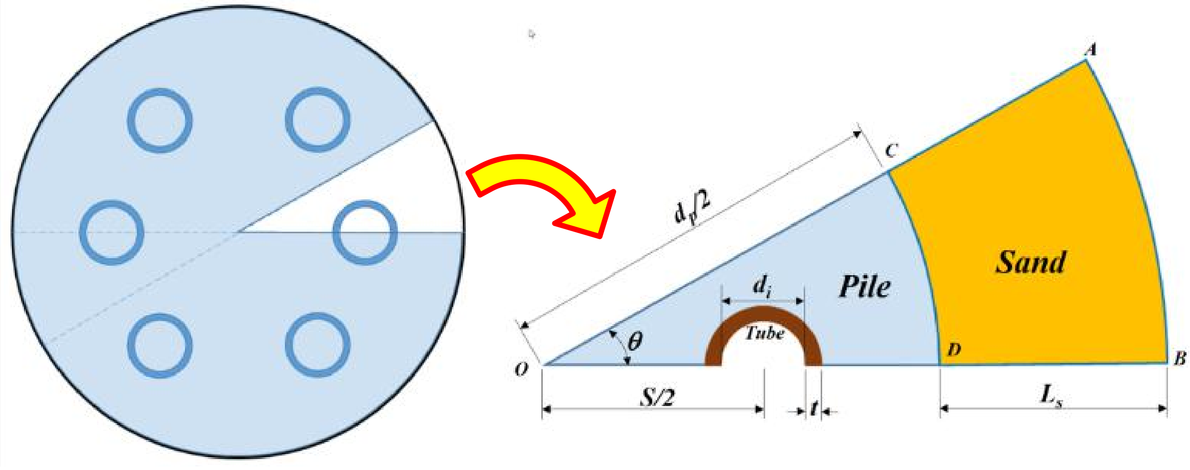


Figure 1. Energy pile geometrical factors.

Table 1. Studied factors and their levels.

Level	*n*	d_p	d_i	*T*	*S*	K_p	K_s	K_t	*H*
-1	1	0.4	0.02	0.002	0.40 d_p	1.0	0.5	0.5	10
0	2	0.7	0.03	0.003	0.60 d_p	1.75	1	16	55
+1	3	1.0	0.04	0.004	0.80 d_p	2.50	1.5	32	100

Using Galerkin method and the divergence theorem [10], the discretized 2D finite element equation of a steady state condition, with no heat generation, takes the following form;

$$\left[\iint_A [B]^T[K][B]dA + \int_S h[N^s]^T[N^s]dS\right]\{\overline{T}\} = \int_S q^s[N^s]^T dS + \int_S hT_f[N^s]^T dS \quad (1)$$

where [B] is the temperature gradient interpolation matrix and {T} is the nodal temperature vector.

The boundary conditions of this equation take the following form with respect to figure 1;

Symmetric boundary conditions at lines "O-A" and "O-B" (Adiabatic BC).

$$\frac{\partial \mathrm{T}}{\partial \mathrm{S}} = 0 \quad (2)$$

where *S* is the normal vector at symmetric lines.

Convection heat exchange at the inner surface of the HDPE tube.

$$-\mathrm{K_s}\frac{\partial \mathrm{T}}{\partial \mathrm{S}} = \mathrm{H}(\mathrm{T_s} - \mathrm{T_f}) \quad (3)$$

where;

S is the normal vector at the inner surface of the HDPE tube.

T_s is calculated temperature at the inner surface of the HDPE tube.

T_f is the bulk temperature of the circulating water (45°C).

H is the studied heat transfer coefficient.

Specified constant temperature at the outer surface of the sand line "A-B".

$$\mathrm{T_{A-B}} = 25\ {}^{\mathrm{o}}\mathrm{C} \quad (4)$$

The set of linear equations represented by equation (1) and the boundary conditions represented by equations (2-4) are assembled to form the global system matrix equations to be solved to calculate the temperatures at the inner surface of the HDPE tube "T_t" and the outer surface of the energy pile at line "C-D" "T_{C-D}". Also, the heat flow at the inner tube surface and the outer surface of the sand line "A-B" are calculated for convergence checking and for thermal conductance calculations. The energy pile steady state thermal conductance is calculated by the following equation;

$$\mathrm{C_{p,\ FE}} = \frac{\mathrm{Q_{A-B}}}{\mathrm{T_t} - \mathrm{T_{C-D}}} \quad (5)$$

2.1. FE Model Validation and Mesh Density Sensitivity

The finite element model of each design point is built and solved, autonomously, using ANSYS Parametric Design Language offered by ANSYS_MAPDL. This powerful scripting language allows parameterizing the finite element model and automating all related tasks of solution and post processing.

The current finite element model is verified against an analytical model of the steady state thermal resistance, which is the reciprocal of the thermal conductance, for these energy pile configurations. The model is proposed by [5] and approximated the thermal resistance of double U-tube energy pile as follows;

$$R_p = \frac{1}{4\pi K_p}\left[\ln\left(\frac{d_p^{2n}}{2nd_oS^{2n-1}}\left(\frac{d_p^{4n}}{d_p^{4n} - S^{4n}}\right)^{\left(\frac{\mathrm{K_p}-\mathrm{K_s}}{\mathrm{K_p}+\mathrm{K_s}}\right)}\right)\right] + \frac{1}{2\mathrm{n}\pi}\left(\frac{1}{2\mathrm{K_t}}\ln\left(\frac{\mathrm{d_o}}{\mathrm{d_i}}\right) + \frac{1}{\mathrm{d_i H}}\right)_t \quad (6)$$

where;

R_p [m.K/W]: Energy pile thermal resistance

n: Number of U-tubes

$m = 2(n-1)$

d_p [m]: Pile diameter.

d_i [m]: Tube inner diameter

t [m]: Tube thickness

S [m]: Tubes spacing.

K_p [W/m.K]: Pile thermal conductivity.

K_s [W/m.K]: Soil thermal conductivity.
K_t [W/m.K]: Tube thermal conductivity.
H [W/m^2.K]: Convection heat transfer coefficient

The FE results are usually sensitive to the mesh density. A proper mesh density that gives acceptable result within applicable CPU time is important to be achieved ahead of the investigation. A model with mid factors listed in table 1 is used in the validation and the study of mesh density sensitivity. The mesh density in this model is parameterized to the number of elements through the HDPE tube thickness. The rest of elements sizes are assigned to be within the recommended aspect ratio (< 1:5). The results of the validation and mesh density sensitivity study are shown figure 2. Domain discretization with 20 elements through HDPE thickness is the most compromised pattern between predicted results and calculation time. This pattern shows deviation percentage less than 0.02% with 10 seconds of CPU time.

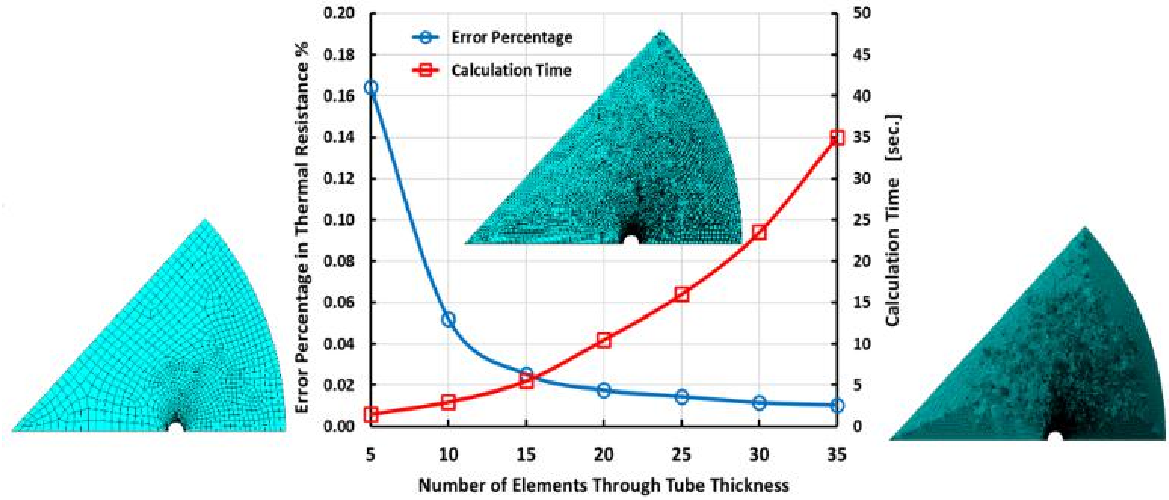


Figure 2. Deviation in the predicted energy pile thermal resistance and the CPU calculation times at different mesh densities showing the pile mesh layouts of; 5, 20, and 35 elements through tube thickness.

3. Uniform design of experiment

Design of experiment methods are widely used in factors correlations and performance optimizations of multivariable systems [11]. Fractional factorial design of experiment is optimally suitable for systems with large number of factors. The Uniform design is an efficient fractional factorial design [12]. The uniform design is one of the robust space-filling designs that is significantly important in investigating large engineering systems [13].

3.1. Design Selection

The domain of each factor of the energy pile system factors is levelled to three levels (-1, 0, +1). The related values of these levels for each factor are listed in table 1. The designs incorporated with 9 factors at three levels (3^9) can be investigated using number of simulation experimental runs as low as 9 runs and as high as 51 runs. The number of runs affect the uniformity discrepancy of the selected design. Figure 3 presents the effect of the number of runs on the discrepancy CD_2 of the design $U_n(3^9)$ [14]. The current work uses the uniform designs $U_{27}(3^9)$, $U_{36}(3^9)$ and $U_{51}(3^9)$ with 27, 36 and 51 runs respectively [14].

3.2. Signal to Noise Ratio

Measured quantities are affected by significant and insignificant factors. Significant factors produce strong signal while insignificant factors produce noise. Magnification of the signal to noise ratio emphasizes the effect of each factor on the measured data. This leads to optimizing the controlling factors for better performance. The signal to noise ratio in the current work is calculated with the Taguchi larger the better relation as follows [15];

$$S/N = -10\ \log_{10}\left(\frac{1}{m}\sum_{i=1}^{m}\frac{1}{C_{p,i}^{2}}\right) \quad (7)$$

where;

m: is the number of observations for each factor.

$C^2_{p,i}$: is the FE predicted thermal conductance of the energy pile at experiment number i.

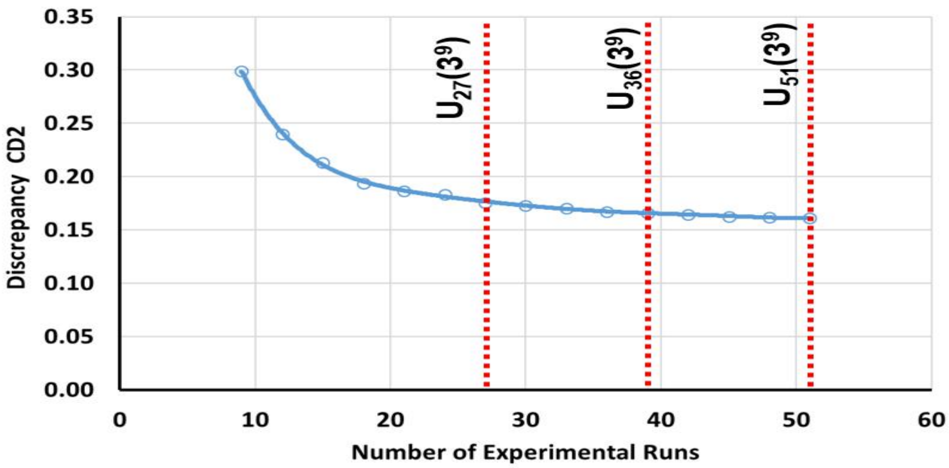


Figure 3. Effect of number of runs on the uniform design $U_n(3^9)$ discrepancy CD_2 edited based on the data published by [14].

Another method to investigate the significance for each factor is the estimation of the cubic least square regression of the observed data at all experiments. The common cubic correlation is simply expressed as follows;

$$Y = a_0 + \sum_{k=1}^{9} a_k x_k + \sum_{i=1}^{9}\sum_{j=i}^{9} b_{ij}x_i x_j + \sum_{i=1}^{9}\sum_{j=i}^{9}\sum_{k=j}^{9} c_{ijk}x_i x_j x_k \quad (8)$$

where;

Y represents a function of the equivalent thermal conductance of the energy pile including water tubes.

a_o, a_k, b_{ij} and c_{ijk} are the correlation coefficients of the cubic model. These coefficients can be calculated using stepwise least square method.

x_i represents the controlling factors as shown in table 2.

The first part is the intercept constant, the second part represents the linear weight of each factor separately, and the third part represents the cubic and the interaction between controlling factors in a pair wise manner.

Table 2. The related factors for each variable in the cubic correlation.

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
n	d_p	d_i	t	S	K_p	K_s	K_t	H

4. Results

4.1. Effect of factors means

The current work investigated the energy pile conductance using three designs $U_{27}(3^9)$, $U_{36}(3^9)$ and $U_{51}(3^9)$ with 27, 36 and 51 runs respectively [14]. Uniform design $U_{51}(3^9)$ is shown in table 3, and the other designs are available at the Uniform Design Tables book [14]. The significance of each factor is measured by calculating the mean thermal conductance for each factor with all other factors. The three designs have shown consistent behaviour with the most significant factors as shown figure 4. On the other hand, the less significant factors have shown non-consistent behaviour with the three designs. The significance of the controlling factors on the mean thermal conductance is shown in figure 5. The most five significant factors with direct proportionality are the convection heat transfer coefficient "*H*", the number of tubes "*n*", the pile thermal conductivity "K_p", the distance between tubes "*S*", and the tube inner diameter "d_i", respectively in significance order. The other four factors are either less significant or with inverse proportionality.

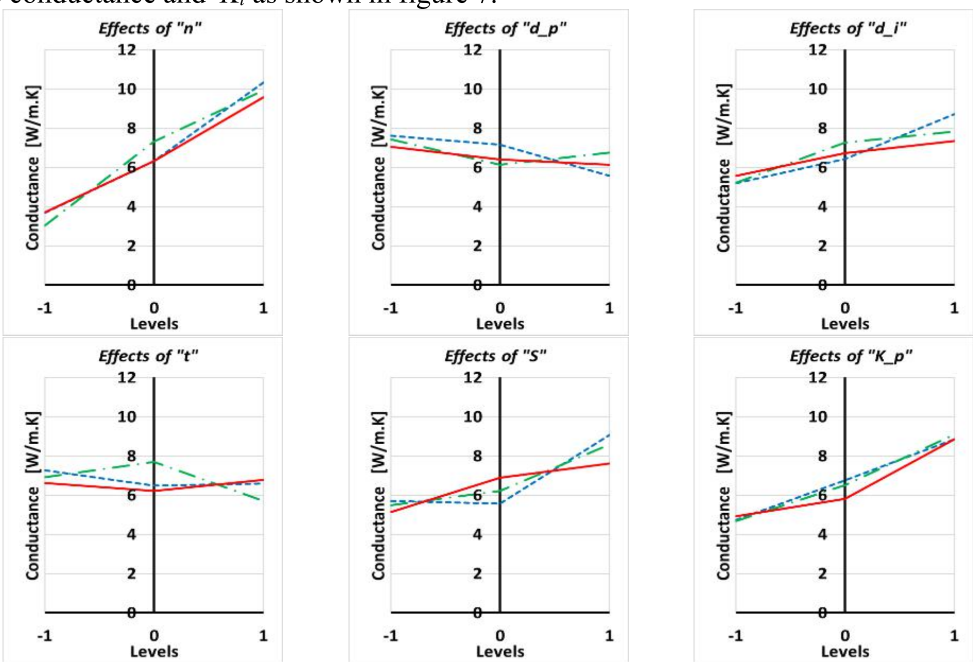
4.2. Signal to Noise Ratio measurements

The calculated thermal conductance of the energy pile at the different runs using the uniform designs of experiment are analyzed using equation (7). The significance of the factors have been better emphasized. The three designs have shown more consistence results with the Signal to Noise ratio measure than that obtained by the means measure as shown in figure 6.

The three designs have shown that the least significant factors; are tube thickness "*t*", soil thermal conductivity "K_s", and the tube thermal conductivity "K_t". The other factors have shown highest signal to noise ratio at the highest level "+1". These factors are the convection heat transfer coefficient "*H*", the number of tubes "*n*", the pile thermal conductivity "K_p", the tube inner diameter "d_i", and the distance between tubes "*S*" respectively in significance order, as shown in figure 6. On the other hand, the pile diameter has shown the highest signal to noise ratio at level "-1". This implies that the thermal conductance of the energy pile is slightly decrease with the increase of the energy pile diameter.

4.3. Cubic Least Square model

Using the stepwise least square regression, the coefficients of the cubic equation (8) are calculated using the three uniform designs $U_{27}(3^9)$, $U_{36}(3^9)$ and $U_{51}(3^9)$. As shown in figure 7, the three designs show direct proportionality of the energy pile thermal conductance with the five most significant factors; (*H*, *n*, K_p, d_i, and *S*) and inverse proportionality with d_p. However, the other three factors are controversial because they have minor effects. Design $U_{27}(3^9)$ shows dependency of the pile conductance on the *t* and K_s, however, the other two designs show no dependency. On the other hand, design $U_{27}(3^9)$ shows no dependency on K_t, however, the other two designs show direct proportionality of the pile conductance and K_t as shown in figure 7.



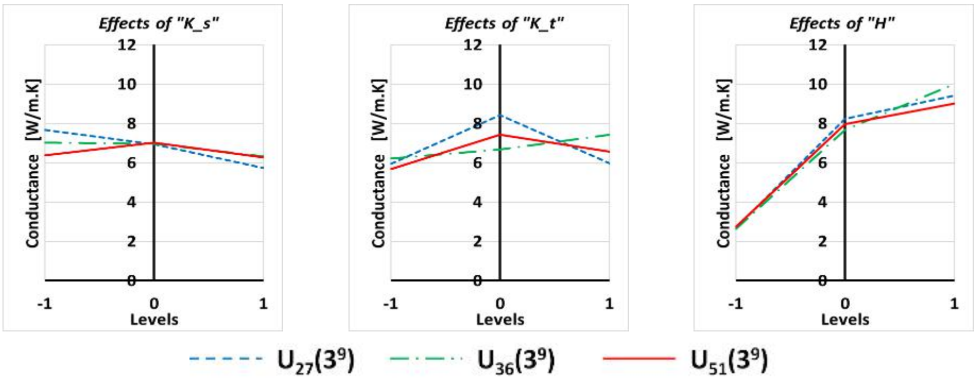


Figure 4. Effects of each factor separately on the energy pile conductance.

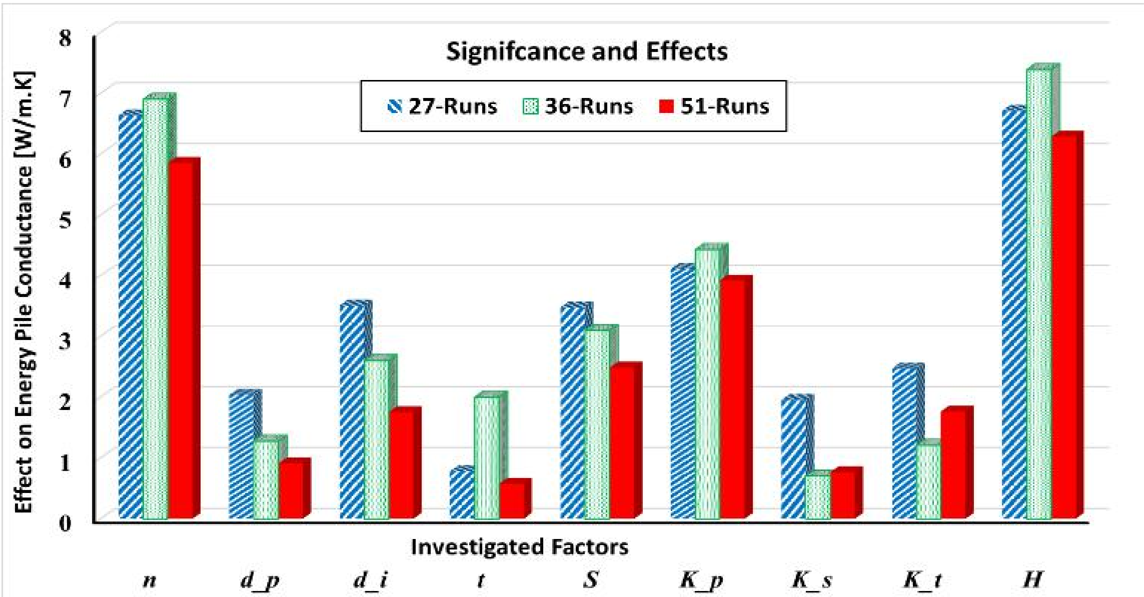
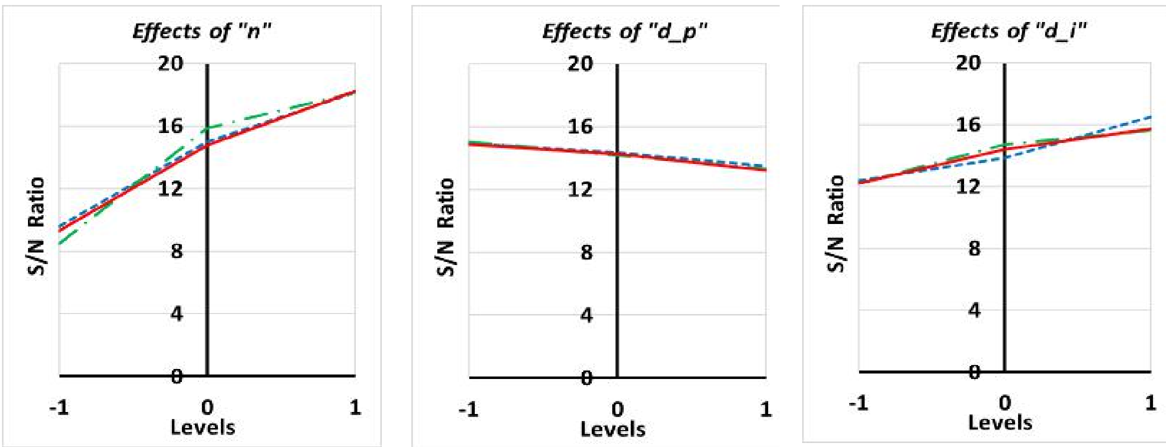


Figure 5. Effects and significance of each factor separately on the energy pile conductance.

Table 3. Uniform design $U_{51}(3^9)$ [14].

Exp.	n	d_p	d_i	t	S	K_p	K_s	K_t	H	Pile Cond. C_p
1	1	1	1	1	1	0	-1	-1	1	16.8501
2	0	1	-1	0	1	-1	0	1	1	6.3859
3	1	-1	1	1	-1	0	1	0	0	10.4425
4	0	-1	1	-1	0	1	-1	-1	-1	4.0793
5	-1	0	0	1	1	-1	1	0	1	4.1772
6	-1	1	0	1	0	-1	1	1	0	2.9434
7	1	0	0	-1	-1	1	1	-1	1	11.5765
8	0	0	0	0	0	0	0	0	0	8.3735
9	0	0	0	1	1	-1	-1	-1	0	6.3752
10	0	1	-1	-1	0	1	1	0	1	9.8099
11	0	1	0	1	-1	1	-1	0	-1	2.9456
12	0	1	1	0	-1	-1	1	-1	1	4.8182
13	1	1	0	1	1	1	0	1	0	16.6359
14	0	0	0	0	0	0	0	0	0	8.3735
15	-1	0	0	1	-1	0	0	-1	-1	1.406
16	-1	-1	1	0	1	1	1	0	1	10.1626

17	1	1	1	0	0	1	0	0	0	15.3738
18	0	0	1	1	1	1	1	0	-1	4.3231
19	0	-1	0	0	0	0	0	-1	1	10.0505
20	1	-1	0	0	-1	-1	-1	0	1	6.6228
21	1	0	1	-1	0	-1	0	0	1	9.4316
22	-1	0	-1	0	-1	1	1	1	0	3.8853
23	1	0	-1	-1	1	1	-1	0	0	12.8852
24	-1	0	-1	-1	0	-1	-1	-1	1	2.7331
25	-1	1	1	0	1	-1	-1	0	-1	1.6183
26	-1	0	1	1	0	1	-1	1	1	8.1579
27	-1	1	-1	0	1	1	0	-1	-1	1.0582
28	1	-1	0	-1	1	-1	0	-1	-1	4.3068
29	1	1	-1	1	0	-1	1	-1	-1	2.5104
30	-1	1	0	-1	1	0	1	-1	0	4.0124
31	0	1	1	0	0	0	1	1	-1	3.672
32	1	-1	0	0	0	1	1	1	-1	4.761
33	1	-1	-1	0	0	-1	-1	1	0	7.1593
34	-1	1	1	-1	-1	1	0	-1	0	5.1449
35	0	0	-1	-1	1	0	0	0	-1	2.1749
36	0	-1	-1	0	1	0	1	-1	0	7.1671
37	0	-1	1	1	-1	-1	0	1	-1	3.0084
38	1	1	-1	-1	-1	0	0	1	-1	2.7471
39	-1	-1	1	-1	-1	0	0	1	1	6.6467
40	0	1	0	-1	-1	-1	-1	1	0	4.2251
41	-1	-1	-1	-1	-1	-1	1	0	-1	0.9637
42	-1	1	-1	0	-1	0	-1	0	1	3.8843
43	1	-1	-1	1	0	1	0	0	1	15.8726
44	1	0	1	0	-1	0	-1	-1	-1	4.4323
45	0	-1	0	-1	1	1	-1	1	1	17.7434
46	-1	0	0	-1	0	0	-1	1	-1	1.5073
47	-1	-1	-1	1	1	0	-1	1	-1	1.0998
48	0	-1	-1	1	-1	1	-1	-1	0	6.1027
49	1	0	1	-1	1	-1	1	1	0	12.7236
50	1	0	-1	1	-1	0	1	1	1	8.5339
51	-1	-1	1	1	0	-1	0	-1	0	3.913



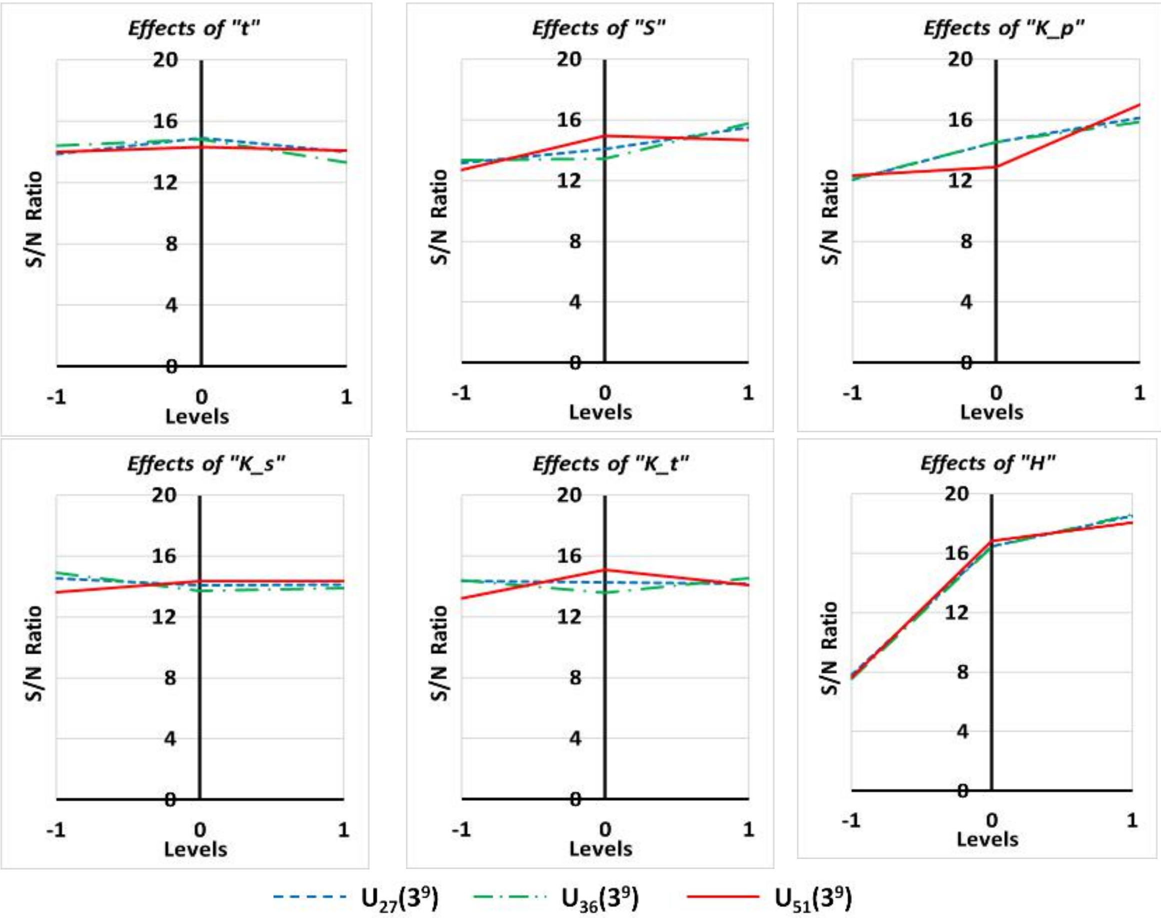
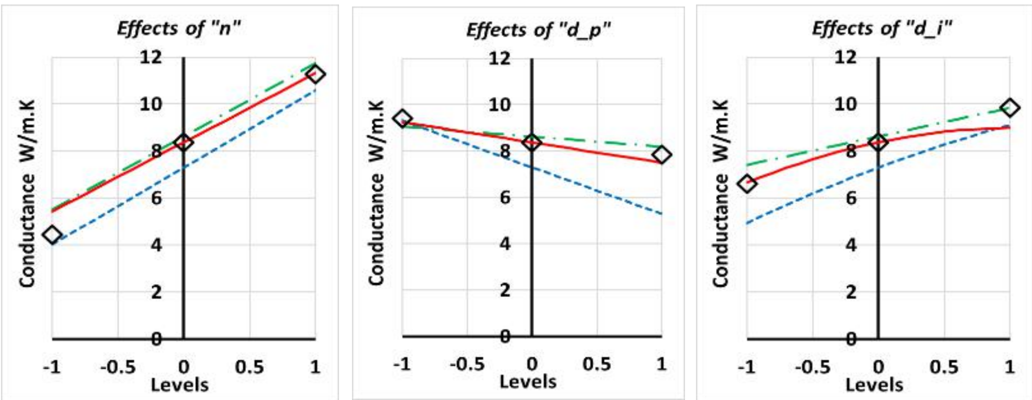


Figure 6. Signal to noise ratios of the energy pile thermal conductance predicted at different uniform designs runs.

4.4. Regression Model Verification

Three cubic regression models have been created from the three designs. To examine these models, the thermal conductance of a base case with all factors are at mid-level is calculated. Then each factor is changed to the extreme levels separately whilst other factors are at the mid-level. The energy pile conductance of these cases are compared to that predicted by the three cubic regression models. As shown in figure 7, the models achieved by designs $U_{36}(3^9)$ and $U_{51}(3^9)$ are in better correlation with the simulation experiment results. However, model achieved by design $U_{27}(3^9)$ is slightly deviated.



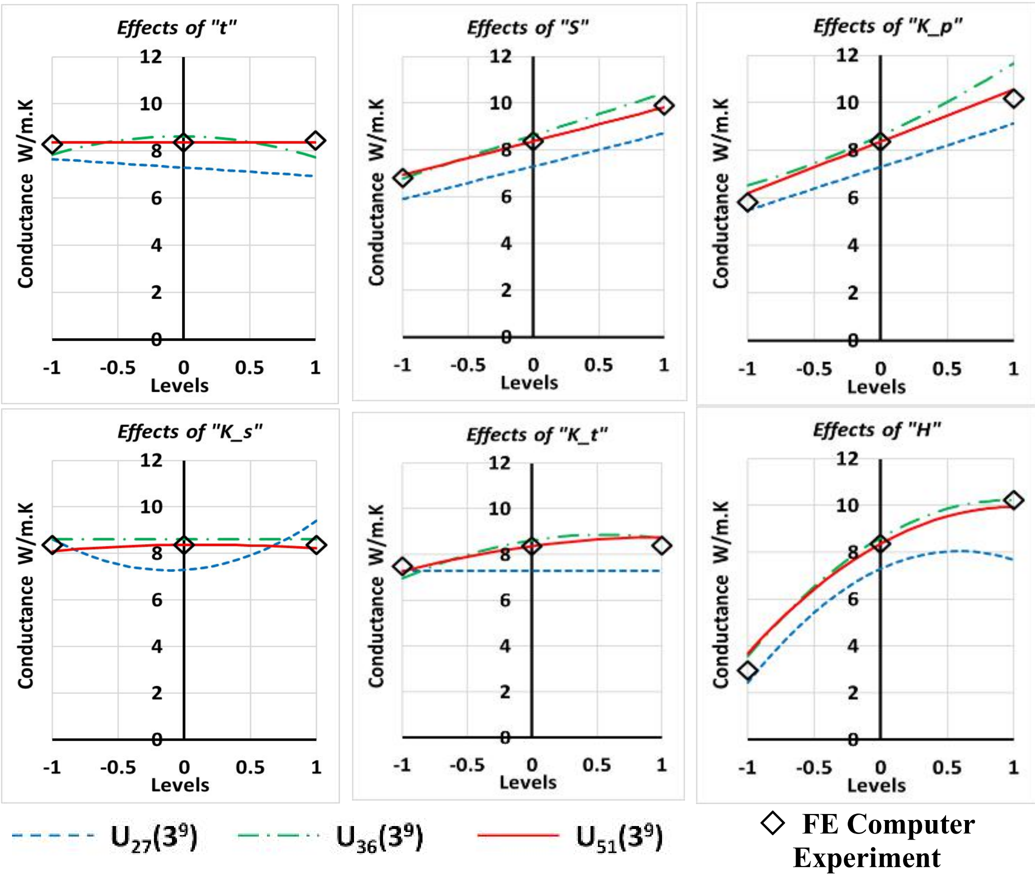


Figure 7. Predicted energy pile conductance using the cubic regression models and FE experiments.

4.5. Optimized Thermal Conductance

Each of the uniform designs have shown different maximum conductance within the investigated combinations. The combinations related to the maximum conductance for each design is shown in table 4. By using the three measures; mean of means, Signal to noise ratio, and cubic regression model, applied to the three designs $U_{27}(3^9)$, $U_{36}(3^9)$, and $U_{51}(3^9)$ we can optimize the energy pile factors for maximum thermal conductance to the values shown in table 4. The energy pile with these configurations has achieved thermal conductance of 34.52 W/m.K. To examine this result, a set of 27 simulation experiments is carried out with the optimum combination achieved as base combination. Then each factor is examined separately whilst keeping the other factors at the optimum value. As shown in table 5, the optimum combinations shows the maximum energy pile conductance in almost all cases. A slight increase (<2%) is observed with choosing thicker tube. This test also shows that using Stainless steel tube (K=16 W/m.K) instead of HDPE tubes (K=0.5 W/m.k) increases the pile conductance (27%). However, using galvanized steel (K=32 W/m.K) increases the pile conductance (1.7%). It is worth noting that these results are limited to the investigated boundaries of each factor.

Table 4. Maximum conductance achieved by each design and the optimum factors for all cases.

	$U_{27}(3^9)$	$U_{36}(3^9)$	$U_{51}(3^9)$	Optimum
Number of Tubes (n)	+1	+1	0	+1
Pile Diameter (d_p)	-1	+1	-1	-1
Tube Inner Diameter (d_i)	+1	0	0	+1
Tube Thickness (t)	-1	-1	-1	0
Distance between Tubes (S)	+1	+1	+1	+1
Pile Thermal Conductivity (K_p)	+1	+1	+1	+1

Ground Thermal Conductivity (K_s)	-1	0	-1	0
Tube thermal Conductivity (K_t)	0	0	+1	+1
Heat Transfer Coefficient (H)	0	+1	+1	+1
Energy Pile Thermal Conductance (C_p)	24.4	21.33	17.75	34.52

Table 5. Maximum thermal conductance verification.

n	d_p	d_i	*t*	*S*	K_p	K_s	K_t	*H*	C_p
-1	-1	1	0	1	1	0	1	1	9.896524
0	-1	1	0	1	1	0	1	1	22.66916
1	-1	1	0	1	1	0	1	1	34.5194
1	-1	1	0	1	1	0	1	1	34.5194
1	0	1	0	1	1	0	1	1	28.37823
1	1	1	0	1	1	0	1	1	25.5727
1	-1	-1	0	1	1	0	1	1	20.5455
1	-1	0	0	1	1	0	1	1	27.63778
1	-1	1	0	1	1	0	1	1	34.5194
1	-1	1	-1	1	1	0	1	1	33.8404
1	-1	1	0	1	1	0	1	1	34.5194
1	-1	1	1	1	1	0	1	1	35.17938
1	-1	1	0	-1	1	0	1	1	15.69314
1	-1	1	0	0	1	0	1	1	23.04178
1	-1	1	0	1	1	0	1	1	34.5194
1	-1	1	0	1	-1	0	1	1	19.84264
1	-1	1	0	1	0	0	1	1	28.39127
1	-1	1	0	1	1	0	1	1	34.5194
1	-1	1	0	1	1	-1	1	1	34.25466
1	-1	1	0	1	1	0	1	1	34.5194
1	-1	1	0	1	1	1	1	1	34.72366
1	-1	1	0	1	1	0	-1	1	26.92667
1	-1	1	0	1	1	0	0	1	34.14625
1	-1	1	0	1	1	0	1	1	34.5194
1	-1	1	0	1	1	0	1	-1	6.734525
1	-1	1	0	1	1	0	1	0	25.07089
1	-1	1	0	1	1	0	1	1	34.5194

5. Conclusions

Energy piles are crucial member of GSHP system to reject or pump heat into ground to reduce the consumption of fossil fuel and CO_2 emission. The efficiency of the energy pile increases with the increase of its steady state thermal conductance. The current work present a statistical approach to define the optimum condition with the least number of experiments using uniform design. Uniform design $U_{36}(3^9)$ has shown acceptable level of error with significantly low number of experiments. The maximum energy pile steady state thermal conductance is achieved with the highest number of tubes, largest tube diameter, largest distance between tubes, highest pile thermal conductivity and highest heat transfer coefficient. Although, smaller pile diameter slightly increase the energy pile thermal conductance, the constructional limitation might stand against reducing these factors with slight insignificant decrease in the energy pile thermal conductance.

Acknowledgement

This publication was made possible by grant No. **NPRP 7-725-2-270** from the Qatar National Research Fund (a member of Qatar Foundation). The statements made herein are solely the responsibility of the authors.

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